MAGNETISM AND ELECTRICITY

FOR STUDENTS

H. E. HADLEY

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MAGNETISM AND ELECTRICITY

FOR STUDENTS

BY

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PREFACE

TO meet the requirements of students who, having worked through the author's *Magnetism and Electricity for Beginners*, need a supplementary volume to extend their knowledge of the subjects introduced in that book, the following chapters have been prepared in response to numerous requests from teachers.

The needs of students whose mathematical attainments do not extend beyond a knowledge of the differential and integral calculus have been borne in mind throughout, and the subjects have been treated as much from the experimental as from the theoretical point of view. The scope of the volume is roughly that required by pass candidates for the B.Sc. examination of the University of London, but the book will also be found suitable for students working for the Board of Education and other examinations.

Although minor technical applications of the science are described in earlier chapters, yet for several reasons the insertion of chapters on Dynamos, Telegraphy and Telephony, Distribution of Light and Power, and similar subjects, has been considered undesirable. Such technical applications could not have been treated satisfactorily without adding unduly to the size of the volume; students, moreover, who are specialising in these branches generally attend courses of study designed to meet their requirements.

The plan adopted has allowed a fuller treatment of such subjects as Thermo-Electricity, the Discharge of Electricity through Gases, and Electric Oscillations, and an attempt has been made to give the important facts in connection with the remarkable researches of recent years in the latter subjects.

Some students may find portions of the earlier chapters to be of too elementary a character, but most will find a revision of previous work useful, and will agree that a self-contained volume is best adapted to their requirements.

Teachers know that the clear understanding of general principles is aided by their application to a wide range of questions and problems of a numerical type. It is believed that the numerous examples given at the end of most of the chapters will secure this desirable end.

The author is indebted to Mr. W. S. Tucker, B.Sc., who has read through the proof-sheets and made many important suggestions, also to Mr. G. H. Broom, B.Sc., and Mr. W. H. Clarke, B.Sc., for a similar service in special cases, to all of whom he offers his grateful thanks. The author's thanks are also due to Sir Richard Gregory and to Mr. A. T. Simmons, B.Sc., who have rendered invaluable service in criticising the manuscript and the printed proofs.

H. E. HADLEY.

KIDDERMINSTER.

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PART I

MAGNETISM

CHAPTER I

NATURAL AND ARTIFICIAL MAGNETS

The Lodestone.—Magnetite is the modern name of a mineral found in several localities, including Magnesia (in Asia Minor), Norway and Sweden, Isle of Elba, Spain, etc.; chemically it is an oxide[1] of iron, containing about 72% of iron. Many specimens of magnetite have the peculiar property of attracting small fragments of iron (*e.g.* iron filings), and, when suspended in a suitable manner, of coming to rest in a definite position relatively to the points of the compass. The latter property was observed by the Chinese as long ago as the year 2400 B.C., but the earliest record of any such knowledge in Europe is found in the writings of a Norwegian (born in 1068) in which magnetite, having the above properties, is referred to as the **lodestone**, or "leading-stone" (Fig. 1).

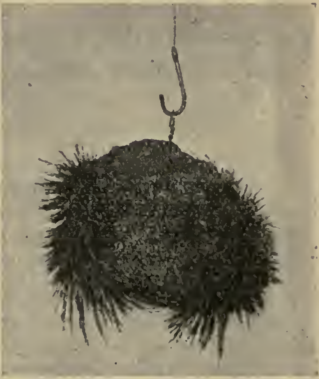

FIG. 1.—A Lodestone which has been dipped into iron filings.

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[1] Oxide of iron is a chemical compound formed by the union of iron and oxygen. The chemical formula of magnetite is Fe_3O_4.

Expt. 1.—Dip a lodestone into iron filings, and observe how the filings cling chiefly to two regions on opposite sides of the lodestone.

Suspend the lodestone, by means of copper tinsel or unspun silk, so that the line joining the two regions determined above may move freely in a horizontal plane. Note how this line almost coincides with the geographical meridian.

The regions to which iron filings are attracted may be termed the **Poles**, and the line joining the poles may be termed the **Magnetic Axis** of the lodestone. The poles which point towards the North and South points are respectively called the **North-seeking** and **South-seeking** poles.

Artificial Magnets.—The lodestone is now seldom used in experimental work, since suitable specimens are comparatively rare, and the degree of magnetisation is far less than in the case of such pieces of steel as have been magnetised by artificial methods, and are consequently termed *artificial magnets.* The more common forms of artificial magnets are (i) the **bar-magnet**, (ii) the **horse-shoe magnet.**

Expt. 2.—Repeat Expt. 1, using a bar-magnet instead of the lodestone, and observe that the characteristic properties are the same.

Law of Magnetic Attraction and Repulsion.—Either pole of a magnet will *attract* iron filings, but this result is not necessarily observed in the action of one magnet upon another.

Expt. 3.—Suspend a lodestone and a bar-magnet consecutively, and mark that end of each which points towards the North. Again suspend the lodestone, and, holding the bar-magnet in the hand, bring the two north-seeking poles close together; observe the **repulsion.** Reverse the bar-magnet, and observe the **attraction.**

Further observations with the South-seeking pole of the lodestone will indicate that all the effects may be stated in the following manner:

Unlike Poles Attract,
Like Poles Repel.

Methods of Magnetisation.—Any of the following methods may be used for preparing artificial magnets:—(i) *Method of Single Touch*, (ii) *Method of Divided Touch*, (iii) *By means of an electric current.* Methods (i) and (ii) are convenient for the magnetisation of small pieces of steel, such as sewing-needles and pieces of clock-springs ; but larger magnets can only be prepared satisfactorily by method (iii).

EXPT. 4.—**Single Touch.**—Hold a sewing-needle firmly on the table by pressing a finger of the left-hand on the eye of the needle (or, by fixing it on the table with soft-wax). Hold the lodestone in the right-hand and stroke the needle from eye to point with the N.-seeking pole of the lodestone (Fig. 2); repeat this several times. Test the needle by dipping it into iron filings, and by suspending it by means of a silk fibre. Note that the point of the needle is now the **S.-seeking** pole.

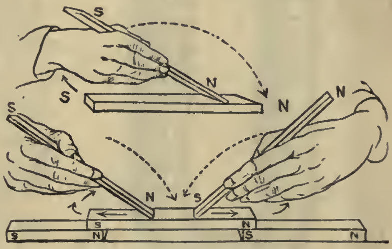

FIG. 2.—Methods of Magnetisation.

The polarity generated in that end of the needle which is last touched by the lodestone, is of opposite kind to that of the pole which is used for the process of magnetisation.

EXPT. 5.—**Divided Touch.**—Fix two bar-magnets on the table with their axes in line, and with unlike poles together. Adjust the distance between the poles so that the needle, which is to be magnetised, may be supported with its ends resting on the poles of the magnets (Fig. 2). Place the opposite poles of two bar-magnets close together, in contact with the middle of the needle, and arranged as shown in the diagram ; draw the poles apart towards opposite ends of the needle, lift them away, and repeat the process several times.

EXPT. 6.—**Magnetisation by means of an Electric Current.**—Wrap a spiral of cotton-covered copper wire round a thin glass tube (10 cms. long, 0·5 cm. bore). Place a piece of clockspring or a needle inside the tube, and pass a strong electric current through the wire for a few seconds ; tap the tube two or three times, break the circuit, remove the needle, and test it for magnetisation.

It will be found, if a piece of *soft* iron is treated in a similar manner, that the iron is highly magnetised *so long as the current continues*, but that the magnetisation almost completely disappears as soon as the current ceases. Such an arrangement is termed an **Electro-magnet** ; in this manner a magnet may be obtained which is far more powerful than an ordinary steel permanent magnet. For this reason electro-magnets are generally used in the preparation of steel magnets.

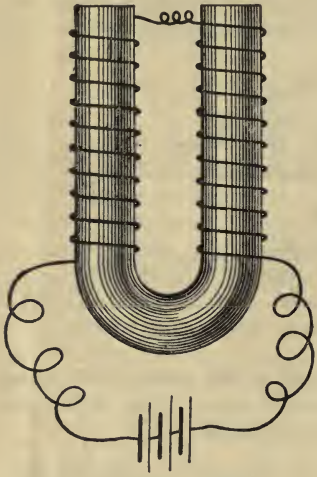

FIG. 3.—An Electro-magnet.

The more common form of electro-magnet is the *horse shoe* (Fig. 3), which consists of a thick core of soft iron, round each limb of which is wound a "bobbin" of several layers of thick cotton-covered copper wire, the direction of winding being *opposite* in the two cases.

Bars of steel may be strongly magnetised by drawing each face in turn across one of the poles of an electro-magnet, finally bringing each end of the steel bar once into contact with the unlike pole of the electro-magnet, and withdrawing it at right angles to the surface where contact is made. In an alternative method the bar is laid across the poles of the electro-magnet, the distance between the poles being so adjusted that the ends of the bar rest upon the poles ; a strong current is passed through the coils while the bar is struck several times with a wooden mallet. A steel horse-shoe is magnetised by placing it vertically over the limbs of an electro-magnet, with its ends in contact with the poles ; while in this position the current is passed three or four times.

Repulsion is the only sure Test of Magnetisation.—EXPT. 7.—Suspend an unmagnetised sewing-needle horizontally by means of a silk fibre, and observe the effect when a pole of a magnet is held near to either end of the needle. In such cases *attraction* is observed, although the needle is unmagnetised. Remove the needle from its support, and magnetise it by the method of Single Touch ; repeat the previous observations, and note that one pole of the magnet now attracts one end of the needle, but *repels the other*.

Attraction, therefore, does not prove that the steel under examination is magnetised, whereas Repulsion does prove this fact.

Consequent Poles.—Other magnetic poles, in addition to those at the ends, may be created in a piece of steel, either by using a defective method, or by intentionally modifying either of the methods described on p. 3. Such additional poles are called **Consequent Poles.** They may be detected by dipping the magnet into iron filings, or by observing a compass needle which is passed slowly along one of the faces of the magnet.

A long needle may be magnetised with a consequent pole at its middle point by magnetising one-half of it at a time by the method of Single Touch. A consequent pole may also be created by using a spiral of wire conveying a current, parts of which are wound in opposite directions.

Magnetic Substances.—Very few substances, besides iron, are attracted by a magnet. The metals nickel and cobalt resemble iron in their magnetic behaviour, but to a much less marked degree. Some compounds of iron, also paper and liquid oxygen, are feebly magnetic.

In iron or steel a more or less permanent magnetisation, with poles having opposite properties, can be created ; whereas the *magnetic substances* mentioned above only behave in a manner similar to iron filings.

Magnetic Saturation.—The degree of magnetisation acquired by any given piece of steel depends largely upon (i) the power of the electro-magnet used for the process, and (ii) the quality of the steel. The magnetisation may be increased by augmenting the power of the electro-magnet, and by prolonging the process. But, in every case, there is a limit to the magnetisation acquired. **Steel or iron is magnetically saturated when it fails to acquire a higher degree of magnetisation, however much the magnetising power is increased.**[1]

Compound Magnets.—Owing to the difficulty of magnetising to saturation the interior of a thick bar of steel, it is advantageous to subdivide the metal into several thin plates, which are separately magnetised, and united together at the ends by

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[1] For further information see Chap. XXI.

soft iron pole-pieces (Fig. 4). It is stated that the resultant pole strength is greater when the component magnets in the middle of the compound magnet are longer than those on the outside.

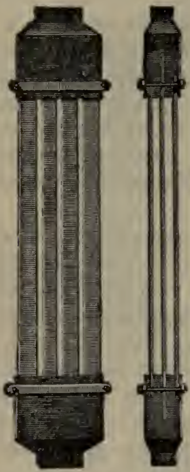

Fig. 4.—A Compound Magnet.

Destruction of Polarity.—The permanence of the magnetisation imparted to any piece of steel is largely influenced by the subsequent treatment to which it is subjected. Both heat and rough handling are detrimental to the strength of a magnet.

Expt. 8.—Heat a magnetised needle to redness in a Bunsen flame, and afterwards test its magnetisation by means of a compass-needle.

Expt. 9.—Magnetise a long wire nail by the method of single touch, and test its magnetisation. Drop it on to the table several times, and again test it.

Expt. 10.—Cut off a length of stout soft steel wire (about 14 cms. long), and bend the ends at right angles, thus, ⊓. Magnetise the middle portion, and test it. Twist the wire to and fro several times, and again test it.

Retentivity and Coercivity.—If pieces of soft iron and steel, of the same dimensions, are subjected to the same magnetising force, it will be found, when the force is removed, that the two materials do not retain the same degree of magnetisation. *If the metals are not subjected to any disturbing influence* the soft iron may retain as much as 90% of its original magnetisation, but the steel will probably retain rather less than this amount. This power of retaining magnetisation is termed **Retentivity.** If the metals are now subjected to rough handling, or to a magnetising force acting in the reverse direction to that adopted in the first instance, it will be found that the steel now retains far more magnetisation than the soft iron. This capacity for retaining magnetisation in spite of any subsequent treatment, is termed **Coercivity**; and it is usually measured by observing the degree of reversed magnetising force which is required in order to deprive the metal of the whole of its original magnetisation. (The magnitude of this force is termed the **Coercive Force.**) Hence, soft iron *may* have greater Retentivity than steel, but it certainly has far less Coercivity.

Condition of Steel suitable for making permanent Magnets.—In chemical composition soft iron differs only slightly from steel; both contain a very small percentage of carbon, thus soft iron may contain about 0.05%, and steel between 0.05% and 0.4°/₀, of carbon. The magnetic differences between the metals may be attributed to differences in the form in which the carbon exists in relation to the iron (*e.g.* it may be simply *dissolved* in the iron, or it may be *chemically combined*)[1] rather than to the actual quantity present. Steel for magnets should have great retentivity and coercivity, and these properties are found to a marked degree in *tungsten steel* (*i.e.* steel containing 10% or less of tungsten) which has been 'glass-hardened.'

The physical properties of steel are largely influenced by the operations known as **Hardening, Annealing, and Tempering.** 'Hardening' results when a red-hot piece of steel is rapidly cooled. 'Annealing' is conducted by allowing the hot metal to cool slowly. 'Tempering' consists in re-heating hardened steel to a temperature far below that to which it was heated before being hardened. The changes due to these processes may be observed by treating three similar pieces of clock-spring in the following manner:

EXPT. 11.—Heat a piece of clock-spring to bright-red heat, and quickly plunge it into cold water; it will be found to be very brittle, and its condition is technically termed *glass-hard.* Glass-harden another piece of the clock-spring; in order to temper it, clean the surface with emery paper, lay it on a sheet of iron heated above a rose-burner, and note the changes of colour (or 'sheen') on the metal surface; it will change to a 'light-straw' colour (at 220° C.), and afterwards, as the temperature rises, to 'dark straw,' 'violet,' and 'deep blue' (at 320° C.); quickly remove the metal and allow it to cool; after bending, it will be found to assume its original shape perfectly. A third piece of steel may be annealed by placing it inside a heap of red-hot ashes and allowing it to cool slowly; on bending, it will be found to remain bent, and to have lost most of its elasticity.

The degree to which steel may be 'glass-hardened' seems to depend upon the rapidity with which the steel is cooled; thus, the hardness is increased by increasing the temperature of the steel, and by using very cold water. A still greater degree of hardness is obtained if mercury is used instead of water. Occasionally a vegetable oil is used instead of mercury or water. The desirable degree of hardness in steel for magnets depends on the dimensions of the magnet. For short magnets the glass-hard condition is recommended ; and for long magnets (in which the length is equal to 20, or more, diameters) the 'blue' temper is best.

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[1] If chemically combined, it will be present as Carbide of Iron (Fe_3C).

According to the researches of Strouhal and Barus, the best procedure in making a steel magnet is as follows : Glass-harden the steel, magnetise it, and suspend it in a current of steam for 20 hours ; again magnetise it, and expose to steam for about 10 hours. This process is known as the 'ageing' of a magnet, since it is found that the magnetisation is not subject to the gradual diminution which is so evident in magnets prepared by the simplest methods. It has more recently been observed that magnets may be 'aged' by dipping them several times into liquid air.

Critical Temperature.—When iron or steel is gradually heated, it is found that, at a certain temperature, the metal ceases to exhibit any magnetic properties. The temperature at which this occurs is called the **critical temperature** of the material. This temperature corresponds to a dull-red heat, but it varies slightly according to the composition of the iron or steel. The critical temperature of pure iron is about 710° C., of low-carbon steel about 735° C., and high-carbon steel 680° C. Nickel has a critical temperature of 350° C.

Expt. 12.—Suspend a short piece of stout iron wire by means of a thin platinum wire, and fix a strong bar-magnet sufficiently near to attract the iron. Heat the iron to red heat, and notice that it is not attracted by the magnet until it has cooled considerably.

This phenomenon is perhaps intimately associated with that of **Recalescence** (observed by Professor Barrett), which is seen when a wire is heated to bright-red heat and allowed to cool slowly; at the temperature 645° C. the wire suddenly glows more brightly, indicating some internal molecular change accompanied by generation of heat. The phenomenon has been fully investigated by Osmond, who observed recalescence in all grades of iron. In the case of pure iron, it may be attributed to a change of the iron to an allotropic form of the same element; in the case of iron containing carbon it may be due to the fact that at higher temperatures the carbon is simply dissolved in the iron, and, when cooled down to the temperature of recalescence, chemical union takes place with evolution of heat. The close relationship between the loss of magnetic quality and 'recalescence' is verified by Hopkinson's discovery that recalescence does *not* occur with non-magnetisable manganese steel.

At the 'critical temperature' abrupt changes have been observed in the specific heat, the electric conductivity, and the thermo-electric power of magnetic substances.

Lifting Power of a Horse-Shoe Magnet.—The relation between the lifting power and the weight of a horse-shoe magnet may be expressed by Häcker's formula

$$P = a\sqrt[3]{W^2}$$

(where P = lifting power, W = weight, and a is a constant).

The value of the constant a depends upon the units in which P and W are expressed; if the latter are in kilograms, then $a =$ 10.3 approximately.

Magnets made by the firm of van Wetteren (in Haarlem) have twice the lifting power indicated by the above formula; in this case the constant a should have an approximate value of 21.

The important fact that the lifting power is not directly proportional to the weight may be illustrated by the consideration that if two lamellar[1] magnets of equal size and lifting power are fastened together to form a compound magnet then the total lifting power will be *less* than twice that of either of the single magnets, since each magnet tends to reverse the magnetisation of the other, according to the principle of Magnetic Induction.[2]

If the keeper of a horse-shoe magnet is placed in position before the magnet is withdrawn from the electro-magnet (employed in magnetising the horse-shoe), it is found that the lifting power is quite 30% greater than that obtained after the keeper has been once removed. Frequent removal and replacement of the keeper will at first tend to reduce the lifting power considerably.

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[1] The term *lamellar* is applied to magnets made from thin sheet steel.

[2] See p. 41.

Magnetic Strength.—The **magnetic strength** of a magnet must be clearly distinguished from the lifting-power of a magnet. One magnet, A, placed at a certain distance from a compass-needle, may have a greater effect than another magnet, B, when placed at the same distance ; in such a case the strength of A would be greater than that of B. *The strength of two magnets may be compared by observing the effect of each upon a compass-needle, the distance between the magnet and the needle being the same in each case.*

SUMMARY

The Lodestone is practically the only form of *natural* magnet. It is an oxide of iron (Fe_3O_4), and the mineral is known as *magnetite.*

The more common forms of *artificial* magnets are **The Bar-Magnet** and **The Horse-shoe Magnet.**

Laws of Magnetic Attraction and Repulsion.—(i) *Unlike Poles Attract*, (ii) *Like Poles Repel.*

Methods of Magnetisation.—(i) *Single Touch*, (ii) *Double Touch*, (iii) *Electric Current.*

Repulsion is the only sure test of magnetisation.

A magnet has **Consequent Poles** when it exhibits regions of magnetic polarity at other points besides its extreme ends.

Besides iron, *nickel* and *cobalt* are the only metals which show marked magnetic properties.

Magnetic Saturation of iron or steel is obtained when the metal fails to acquire a higher degree of magnetisation, however much the magnetising power is increased.

A Compound Magnet consists of several thin strips of magnetised steel, with similar poles held together by a pole-piece of soft iron.

Magnetisation is *diminished*, or perhaps *destroyed*, by rough handling or by extreme heat.

Retentivity is the capacity of retaining magnetisation after the magnetising power has been removed. **Coercivity** is the power of retaining magnetisation in spite of any subsequent treatment.

The best steel for making magnets is glass-hardened **tungsten-steel**

(steel + 10% tungsten). The best degree of temper depends upon the dimensions of the magnet.

The Critical Temperature approximately agrees with the temperature of **Recalescence.**

The lifting power of a magnet may be expressed by the formula $P = a\sqrt[3]{W^2}$.

QUESTIONS ON CHAPTER I

1. What conclusion would you come to when a magnetised and suspended piece of steel fails to set itself in a north and south direction? If the steel is now broken into two parts, would you expect each, when suspended separately, to behave in the same manner as the unbroken steel?

2. Explain the difference in the meaning of the terms *Retentivity* and *Coercivity.*

3. What is meant by the *Critical Temperature* of a specimen of iron or steel?

4. What would be the maximum lifting power of a horse-shoe magnet weighing 1 kilogram, assuming that the constant a is equal to 10?

CHAPTER II

VELOCITY, FORCE, WORK, ENERGY, AND POTENTIAL

THE science of Magnetism and Electricity cannot be clearly understood without a preliminary knowledge of the main principles of physical measurement: thus, Force, Work, and Energy are quantities which are frequently employed in the consideration of magnetic and electric phenomena. The present chapter is therefore devoted to a brief description of these principles, together with their applications of a more advanced nature which may be required in order to explain fully the measurements discussed in subsequent chapters.

Velocity.—When any particle of matter is in motion it traverses a certain distance in a certain time. Velocity may be defined as the rate at which the particle changes its position. Numerically, the velocity is equal to the ratio $\frac{\text{space traversed}}{\text{time occupied}}$. If the particle traverses a space l, in time t, then the velocity is given by the equation, $v=\frac{l}{t}$. If the quantities l and t are both equal to unity, then v is also equal to unity; hence **Unit Velocity is such that unit space is traversed in unit time.** In the C.G.S. system (p. 515) a particle has unit velocity when 1 centimetre is traversed in 1 second.

A complete statement of a velocity, at any moment, is obtained if we know its magnitude and its direction. Both of these quantities are capable of being represented graphically by means of a straight line, the direction of which indicates the direction of the velocity, and the length of which, drawn to a stated scale, indicates the magnitude of the velocity. Thus, in Fig. 5, the velocity of A is in the same direction as that of B, but its magnitude is twice as great.

If the velocity is not uniform, then if the body moves through a short distance *dl* in a time *dt*, which is so short that the velocity may be considered to have remained constant during that interval, the velocity of the body, at that instant, is *dl/dt*. Here, *dl* and *dt* are in the notation of the differential calculus; thus, *dl* does not mean $d \times l$, but it is a *symbol* representing the *small* increase in a length *l* which takes place in the small time-interval *dt*.

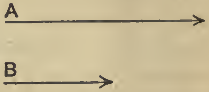


Fig. 5.

The Parallelogram of Velocities.—If a particle has two separate velocities, which are constant in magnitude and direction, imparted to it simultaneously, it will be found that the particle has a constant velocity in a straight line. This single velocity is called the *Resultant Velocity*, and the two original velocities are termed the *Component Velocities*.

For example, imagine a man standing at the point A (Fig. 6) in a railway truck which moves uniformly in one second from A to A′, and imagine that at the same moment the man begins to walk from A at such a speed that he reaches B in one second. At the termination of one second the man is neither at A′ nor at B; his position is B′, and the line AB′ truly indicates the path which he has traversed. Since AB′ is the space traversed in one second, it represents the **Resultant Velocity**, of which AA′ and AB are the components. It is evident that the Resultant Velocity is represented by the diagonal of the parallelogram of which the Component Velocities are adjacent sides.

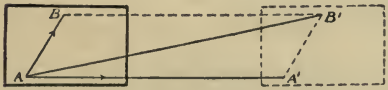


Fig. 6.

Acceleration.—When a particle is moving with a gradually increasing or decreasing velocity, then **the rate of change of the velocity** is termed the **Acceleration.** If the velocity increases uniformly from v_0 to v_t during a period of t seconds, then the acceleration $=\frac{v_t - v_o}{t}$. In C.G.S. units, this ratio states how much the velocity (in cms. per second) changes in one second; hence, an Acceleration is always expressed as 'so many cms. per second per second.'

In the notation of the differential calculus if the velocity of a body, at a time t, is equal to v cms. per second, and is equal to $v+dv$ cms. per second at a time $t+dt$, then the acceleration is equal to the ratio dv/dt.

Unit Acceleration is defined as **a change of velocity of one centimetre per second in one second.**

Force.—The visible effects of Force are evident in all cases where, acting upon a particle, it causes an increase or decrease in the previous motion of the particle. The same idea may be expressed in other words thus: **A force is that which tends to change the Quantity of Motion (or Momentum) already possessed by any particle on which the force is acting.** Also, the change in the Quantity of Motion depends upon both the **magnitude** of the Force and also on the **time** during which the Force is acting. Hence Force may be measured by **the rate at which it causes change of Momentum.**

If F=force, m=mass, v_o=initial velocity, v_t=final velocity, and t=time interval during which the force acts, and a=acceleration, then

$$F=\frac{mv_t - mv_o}{t}=m\left(\frac{v_t - v_o}{t}\right)=ma.$$

From this equation it is evident that if both m and a are equal to unity, then F also equals unity. Hence **Unit Force creates Unit Acceleration in Unit Mass.** If the units of mass and length are 1 gram and 1 centimetre, then unit force is that which, acting on a mass of 1 gram for 1 second, will give to it a velocity of 1 centimetre per second. This unit of force is called the *Dyne*.

The force due to gravitation, by which every particle is attracted towards the centre of the earth, is perhaps the most familiar example of a force; this force, acting on any given mass, is generally termed the **Weight** of that mass. Experiment has proved, when this force is allowed to act freely on a mass, that an acceleration of 981 cms. per second per second is imparted to the mass. Hence, in the formula $F = ma$, if $m = 1$ gram, and $a = 981$ cms. per sec. per sec., then

$$F = (1 \times 981) \text{ dynes, or}$$
$$\text{'The weight of 1 gram'} = 981 \text{ dynes.}$$

Hence $$1 \text{ dyne} = \frac{1}{981} \times \text{weight of 1 gram.}$$

In the British System of units, the force of gravity acting on a mass of 1 pound imparts to it an acceleration of 32.2 feet per sec. per sec.; then

the 'Weight of 1 pound' = 32.2 British units of Force,
(or Poundals).

Hence, $1 \text{ Poundal} = \frac{1}{32.2} \times$ Weight of 1 lb. ($=\frac{1}{2}$ oz. weight approx.).

Equilibrium of Forces.—Two or more forces may act at a point in a body without necessarily causing motion; if the several forces tend to produce equal motion, but in opposite directions, then the body will remain at rest. For example, a body resting on a table is acted upon by a force (the **weight** of the body) acting vertically downwards, but the tendency of this force to produce motion is neutralised by an equal force due to the resistance of the table, acting vertically upwards; so also a body supported by a thread is at rest under the action of two equal and opposite forces, viz.: the weight of the body and the tension of the thread.

The Parallelogram of Forces.—The principle of the Parallelogram of Velocities (p. 13) applies equally to cases in which the given velocities are **imparted in unit time** (*i.e.* to cases in which **accelerations** are the data); in other words, the principle of the Parallelogram of Accelerations is just as true as that of the Parallelogram of Velocities. Also, since every force is measured by the acceleration imparted to a given mass, it follows that the same principle may be applied to Forces. The principle of the Parallelogram of Forces may be stated thus: **Two forces acting on any mass will produce the same effect as though it were acted upon by a single force, represented in magnitude and direction by the diagonal of a parallelogram, the adjacent sides of which represent the two component forces.** This single force is called the **Resultant Force.**

EXPT. 13.—Clamp a drawing-board vertically, and attach light pulley wheels to nails driven in the top corners. Join three cotton threads together at the point A (Fig. 7), and attach masses of known weight to the ends of the threads, and so adjusted that F_1 is less than the sum of F_2 and F_3.

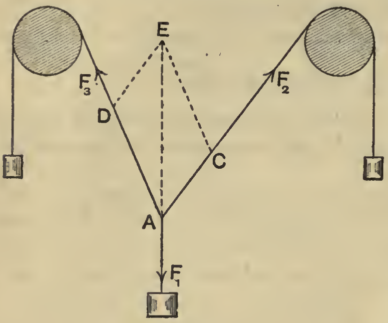
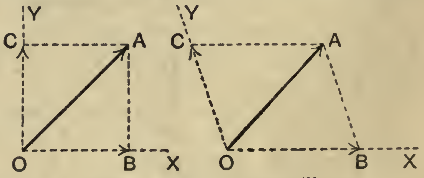


FIG. 7.

The particle A may be considered as being at rest under the action of three forces, F_1, F_2, and F_3. If we consider the force F_1 only, then, since A is at rest, the **resultant** of F_2 and F_3 should be equivalent to an **upward** force, equal in magnitude to the downward force F_1.

Pin a piece of white paper on the board and behind the threads. Mark on the paper, by means of a needle, the position of the point A and also the directions of the forces F_1, F_2, and F_3. Remove the paper, draw with a pencil the direction of the forces, and mark off lengths AC and AD, drawn to any suitable scale and proportional to the respective forces F_2 and F_3. Construct a parallelogram on AC and AD and draw the diagonal AE. AE is the *resultant* of AC and AD, and should be equal and opposite to the force F_1.

Resolution of Forces.—By reversing the process of the Parallelogram of Forces, we may resolve a single force into two others acting in directions inclined to one another at any angle not greater than two right angles, which shall be equivalent in effect to the original force. More frequently, the resolved forces are made to act in directions at right angles to each other. Thus, if the force OA (Fig. 8) is to be resolved into two forces acting in the directions OX and OY, from A draw AB



(i) (ii)

FIG. 8.

and AC parallel respectively to OY and OX. Then OB and OC represent the resolved forces in magnitude and direction.

The Moment of a Force.—When a body, free to move, is acted upon by a force, it will move in obedience to that force, and in the direction in which the force is acting. This result ceases to hold good when the body is only capable of *rotation* round a fixed point. Fig. 9 represents a disc capable of rotation round O. The torque, or turning-power, of the force F depends upon both the magnitude of F and also upon the length of the arm OP. If F is replaced by an equal force F′, the turning-power is evidently less than before, although the force is acting at the same point P; the torque of F′ depends upon the length of the arm OP′, since the point of application of F′ may be moved from P to P′ without in any way altering its effect. *The product of the magnitude of a force into the perpendicular distance between the axis of rotation and the direction of the force is called the* **Moment** *of the force.* Hence

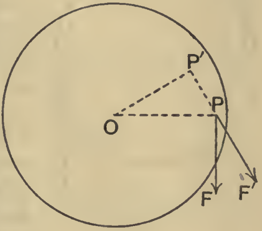


FIG. 9.

Moment of F = F × OP
" " F′ = F′ × OP′.

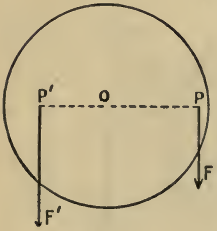


FIG. 10.

When two forces are acting on a body, and both tending to produce rotation, the resultant motion is determined by the sum of the moments of the two forces. The two directions of rotation are distinguished by regarding the anti-clockwise direction as positive and the clockwise direction as negative. In Fig. 10 the sum of the moments of F and F′ may therefore be expressed thus: (F′ × OP′) − (F × OP). If this quantity is equal to *zero* the body will remain at rest, or

$$(F' \times OP') - (F \times OP) = 0,$$

$$F' \times OP' = F \times OP,$$

$$\frac{F}{F'} = \frac{OP'}{OP}.$$

Couples.—Two equal opposite and parallel forces, whose lines of action are not the same, constitute a **Couple** (Fig. 11). The sum of the moments will now be

$$-F(OP+OP')=-(F\times PP').$$

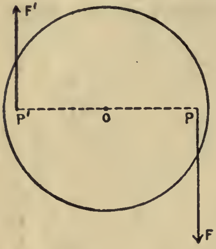


FIG. 11.

Hence, *the moment of a Couple is equal to one of the forces multiplied by the perpendicular distance between the two forces.*

The action of a Couple on a body which is capable of rotation round a fixed axis is an important problem, especially in the case of a magnet swinging in a magnetic field. Fig. 12 represents a disc acted on by a Couple F_1F_1, consisting of two equal forces acting in a constant direction. In the position F_1 the disc has *potential* energy (p. 21), which is gradually converted into *kinetic* energy (p. 21) as the disc rotates into the position F_2, where all the energy is kinetic; but the momentum of the disc carries it beyond the position F_2, and its energy again becomes potential. The disc gradually comes to rest in the position F_3, and then swings back to its original position; the amplitude of the swings gradually becomes less owing to friction with the air and in the bearings or supports, and finally comes to rest in the position F_2. The rate of swing depends upon the Moment of the Couple and upon the Moment of Inertia (p. 22) of the disc.

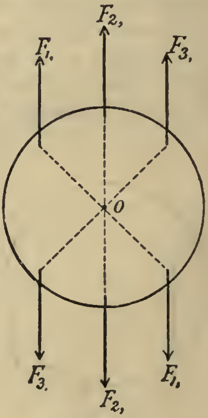


FIG. 12.

The Tangent Law.—This law may be expressed in the following words: **The tangent of the angle of deflection is proportional to the deflecting force.** The law may be proved by the following experiment:

EXPT. 14.—Fasten a sheet of white paper on a drawing-board which is clamped in a vertical position. Fix a small nail at the point O (Fig. 13). Suspend from O a string carrying a scale-pan (of known weight, p). Attach at Q, by a tight slip-knot, a second string, passing over a pulley-wheel P, and carrying a scale-pan (weight p') at the other end. Place a weight W in the pan p, so that the total weight is 20 grams. Adjust the string PQ so that it is quite horizontal, and draw the line PQ in pencil by means of a straight-edge. Measure the length OQ. Place a weight w in the pan p' so that the total weight is 5 grams. Adjust Q until PQ is again horizontal, and mark the point S. Measure QS. Increase the weight w, so that $(w+p')$ is 10 grams, and repeat the previous readings; also when $(w+p')$ is 15 grams, and so on.

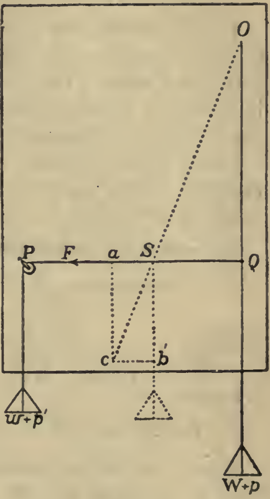


FIG. 13.

In this experiment the *deflecting force* (F) is the weight $(w+p')$, and the angle QOS is the *angle of deflection*. It will be found that the ratio $\dfrac{w+p'}{\tan QOS}$ is a constant quantity.

Work.—When a force acts upon a body and causes it to move in the direction in which the force is acting then the force is said to do *work*. The quantity of work done is proportional to the force and to the distance through which its point of application moves. Hence, if the point of application of a force F moves through a distance s, and if W is the work done, then, when suitable units are selected,

$$W = Fs.$$

The C.G.S. unit of work is the quantity of work done when a force of one Dyne acts through a distance of one centimetre. This unit is called the **Erg**. Since the Dyne is approximately equal to the weight of 1 milligram (p. 15), it follows that nearly 1 erg of work is done when 1 milligram is raised vertically through

1 centimetre. Also, in raising a mass of 1 kilogram from the floor to the table (say, 80 cms. high), the work done

$$=(1000\times 981\times 80)\text{ ergs}=78{,}480{,}000\text{ ergs}=7.848\times 10^7\text{ ergs}.$$

In practice the erg is found to be inconveniently small, and it is more usual to take 10^7 ergs as the unit of work, and this is called the **Joule.** The Board of Trade unit of work (or B.T.U.) is equal to 3,600,000 joules (this is equivalent to 1000 joules per second for a period of 1 hour).

In the British system, the absolute unit of work is the **Foot-Poundal,** and is equal to the work done when a force of 1 Poundal acts through a distance of one foot.

Conversion of Foot-Poundals into Ergs.

$$1\text{ Poundal}=\frac{1}{32.2}\times\text{Weight of }1\text{ lb.}=\left(\frac{1}{32.2}\times 453.6\right)\text{ grams}$$

$$=\left(\frac{1}{32.2}\times 453.6\times 981\right)\text{ dynes.}$$

$$1\text{ Foot}=30.48\text{ cms.}$$

Hence

$$1\text{ Foot-Poundal}=\frac{30.48\times 453.6\times 981}{32.2}\text{ ergs}=(4.211\times 10^5)\text{ ergs.}$$

Work is sometimes measured in **foot-pounds,** in which the unit of force is the weight of 1 lb. This is called the **Gravitational** unit of Work. Since the weight of 1 lb. = 32.2 Poundals, the foot-pound is equal to 32.2 foot-poundals. Also

$$1\text{ foot-pound}=(4.211\times 10^5\times 32.2)\text{ ergs}=(1.356\times 10^7)\text{ ergs.}$$

Power.—Power, Activity, and **Mean Rate of doing Work** are synonymous, and are equivalent to the ratio $\dfrac{\text{Total Work done}}{\text{Time occupied}}$.

In the C.G.S. system, **unit power is that which is capable of doing unit work in unit time** (*i.e.* one erg of work per sec.). The practical unit, in the same system, is that which is capable of doing *one joule* (10^7 ergs) *per second*; this unit is called the **Watt.**

In practical engineering the **Horse Power** is frequently used as a unit, and is equivalent to 550 ft.-lbs. of work per second.

Conversion of Horse-Power into Watts.

Since 1 foot-pound $=(1.356\times 10^7)$ ergs $=1.356$ joules,

Then

1 Horse-power (H.P.) $=(1.356\times 550)$ joules per sec. $=746$ Watts.

And 1 Kilowatt $=1.3$ H.P. approximately.

Energy.—The work expended upon a body in raising it from the ground is not lost, since, if allowed to fall freely upon some suitable mechanism, the same amount of work will reappear. Thus, if a mass of 1 lb. is raised through a height of 100 ft., then 100 ft.-lbs. of work have been expended on it ; if allowed to fall to its original position, where suitable mechanism is situated, then 100 ft.-lbs. of work will be imparted to the mechanism. *The capacity for doing work* is termed **Energy.** Before the mass is allowed to fall, the energy is *stored-up* in it although the mass is at rest ; in this case the energy is termed **Potential Energy** (or, *energy due to position*). When the mass has been allowed to fall freely, and has just arrived at its original position, then it has lost all its Potential Energy, but it still has the same total capacity for doing work, owing to its velocity of motion ; in this case the energy is termed **Kinetic Energy.**

Determination of the Kinetic Energy of a Falling Body.—A falling body (mass m) is acted upon by a constant force (the force of gravitation) which imparts to it a constant acceleration. This is expressed by the formula, $F = ma$.

$$\text{Work done on the body} = \text{Force} \times \text{distance}$$

$$= Fs = mas.$$

The Kinetic Energy possessed by the body must therefore be equal to the product mas. This may be further simplified by the following reasoning :

The Velocity acquired by the body = acceleration × time,

$$\text{or,} \qquad V = at,$$

also, the space traversed = average velocity × time,

$$\text{or,} \qquad s = \frac{V}{2} \times t = \tfrac{1}{2}at^2.$$

Hence, **Kinetic Energy** $= mas = m \times \frac{V}{t} \times \frac{V}{2} \times t = \frac{mV^2}{2}.$

Energy of Rotation.—When a particle of mass m is moving in a circle (of radius r) with angular velocity ω, then

$$\text{Kinetic Energy} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(\omega r)^2.$$

When a body, such as a magnet, is swinging to and fro, we may regard it as consisting of a large number of separate particles, m_1, m_2, etc., all travelling with the same angular velocity, but at different distances r_1, r_2, etc., from the centre of rotation. Hence

$$\text{Kinetic Energy} = \tfrac{1}{2}m_1\omega^2 r_1{}^2 + \tfrac{1}{2}m_2\omega^2 r_2{}^2 + \dots$$
$$= \frac{\omega^2}{2}(m_1 r_1{}^2 + m_2 r_2{}^2 + \dots).$$

The sum of the series included in the bracket is called the **Moment of Inertia** of the magnet.

Also, if M is the total mass of the body, and if we find a length k such that

$$\mathrm{M}k^2 = m_1 r_1{}^2 + m_2 r_2{}^2 + \dots,$$

then k is called the *Radius of Gyration.*

Transformation of Energy.—In the previous section, the case of a falling body exemplified the transformation of **Potential** into **Kinetic** Energy. So also, when a body is projected vertically upwards, its initial Kinetic Energy is gradually transformed into Potential Energy. These two forms of energy are therefore mutually convertible.

Suppose that a falling body, at the moment of returning to its original position, does not fall upon any mechanism, but is allowed to strike the ground. What now becomes of its Kinetic Energy? It is certainly not lost, but assumes another form of energy, viz.: **Heat.** With suitable apparatus this Heat might be measured, and, if the observation is repeated, it would be found that the same amount of Kinetic Energy is converted into the same amount of Heat. So also can Heat, under suitable conditions, be converted into Kinetic Energy (as exemplified in the steam engine).

Chemical Action is another form of energy; and energy is dissipated during the passage of an electric current.

All forms of Energy are mutually convertible, providing that the conditions are suitable. As a general rule, the consideration of numerous examples will indicate that the natural tendency of most forms of energy is to become resolved into **Heat,** which, when generated locally, tends to become distributed uniformly through space; but, it must be remembered, that though Heat becomes thus distributed, it is nevertheless not *lost.*

Also, no form of Energy can be generated without an **equivalent** loss in the same, or some other, form of Energy; also if Energy disappear in one form it will reappear in some other form. This generalisation is termed the **Law of Conservation of Energy,** and may be expressed in the following terms: **The total amount of Energy in the universe is a constant quantity which can never be increased or diminished, although its form may be changed.**

Surfaces of Equal Potential.—The forces acting on a smooth body resting on a smooth horizontal table are all vertical; and no work will be done against, or by, these forces if the body is moved into different positions on the surface of the table. As a general rule, **no work is done on a body when it is moved in any direction at right angles to the force (or resultant force) which may be acting on it.** In all positions on the table the mass will have the same potential energy, hence the surface of the table may be termed a **surface of equal potential** (*gravitational*). If the body is now raised up to the surface of a higher table, the work done on the body will impart to it an increased potential energy; also, the surface of the higher table will be a *surface of equal potential*, but the actual potential energy will be greater than when the body was resting on the lower table. (If m gms. = mass of body, h cms. = vertical distance between the surfaces, and g = acceleration due to gravity, then the work done in raising the body $= mgh$ ergs. This quantity is a measure of the increase of Potential Energy which the body acquires.) If the surfaces of the tables are extended in all directions, so that they are always at a constant height above the ground, and therefore always perpendicular to the direction of the force of gravitation, they will become spherical surfaces concentric with the earth's surface.

An exactly similar case is found in the force of attraction which a single N.-seeking pole, fixed in position, exerts on a single S.-seeking pole in its neighbourhood. The Surfaces of Equal Potential (magnetic) will be spheres described round the N.-seeking pole as a centre. If the movable S.-seeking pole is pulled away to a distance work is done on it, and its potential energy will be increased. If the movable pole is N.-seeking, then work must be done in order to bring it **nearer** to the fixed pole, and its Potential Energy is increased the nearer it is to the fixed pole.

Circular Motion.—**When a particle is moving in a circle with uniform velocity it is subjected to a constant acceleration (v^2/r) which is acting towards the centre of the circle.**

Let P (Fig. 14) represent a particle traversing a circle radius r with uniform velocity v. If the particle were not constrained to move in a circle its path would be along the line PQ. Hence, some force must be acting on the particle; and since the velocity is constant, any such force must always act perpendicularly to the path of the particle at any instant, *i.e.* towards the centre.

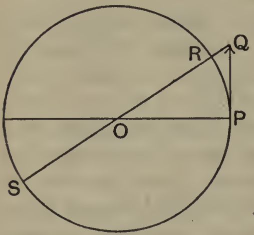


FIG. 14.

Let PR represent a very small arc, described in time t. Then, during the same period, the constraining force will pull the particle through the distance QR. If a=acceleration due to the force, then $QR=\frac{1}{2}at^2$,* also $PQ=vt$.

But (Euclid, iii. 36) $QR \times QS=(QP)^2$,

$$\text{or} \qquad QR(2r+QR)=(QP)^2,$$

$$\text{or} \qquad 2r(QR)+(QR)^2=(QP)^2.$$

Since QR is small, $(QR)^2$ may be neglected. Hence

$$2r(QR)=(QP)^2,$$

$$2r\times\tfrac{1}{2}at^2=v^2t^2,$$

$$ar=v^2,$$

$$a=\frac{v^2}{r}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The **Period** (T) of the particle is the time occupied in describing one complete revolution. Hence $T=2\pi r/v=2\pi/\omega$.

Simple Harmonic Motion.—Suppose a particle P (Fig. 15) to move uniformly round a circle, of which AOA′ is any diameter. Draw PN perpendicular to AA′.

As P travels round the circle, then N travels to-and-fro along AOA′. The point N is said to travel with **Simple Harmonic Motion** (S.H.M.).

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* From equation $s=\frac{1}{2}at^2$, see p. 21.

The **Amplitude** of N is the length OA. The **Period** of N is equal to the Period of P round the circumference of the circle.

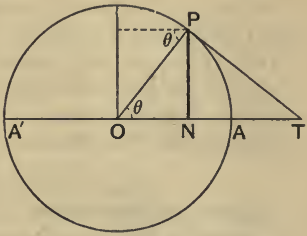

FIG. 15.

The acceleration of P may be resolved into horizontal and vertical components, and the acceleration of N will be equal to the horizontal component of that of P at the same instant. Since the total acceleration of $P = v^2/r = \omega^2 r^2/r = \omega^2 r$, therefore the acceleration of

$$N = \omega^2 r \cos\theta = \omega^2 ON,$$

and is directed towards O.

Simple Harmonic Motion may therefore be defined thus: **If a point moves in a straight line so that its acceleration is always directed towards, and varies as its distance from, a fixed point in the straight line, the point is said to move in S.H.M.**

When N is passing the point O its velocity is equal to that of P round the circle. If the Period of P = T, then

$$\text{velocity of N at the point O} = 2\pi r/T. \quad \ldots\ldots\ldots\ldots(2)$$

The Simple Pendulum.—In Fig. 16 the simple pendulum AB, consisting of a heavy particle of mass m suspended from A by a weightless thread of length l, is displaced through an angle θ. If g is the acceleration due to gravity the weight of m is mg. The weight may be resolved into two components, $mg \sin\theta$ and $mg \cos\theta$; the former acts in the direction of the tangent at C and tends to bring the bob towards B with an acceleration $g \sin\theta$, while the latter acts along the thread and only tends to keep it taut. If θ is very small, $g \sin\theta = g\theta = g \,.\, BC/l$; or, *the acceleration tending to bring the bob back to its original position is proportional to the displacement.* Hence the pendulum oscillates in S.H.M., and obeys the law stated in the previous paragraph. Consequently, if θ is so small that the path BC is

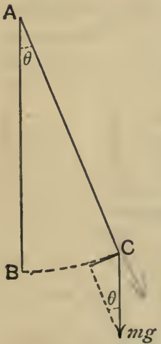

FIG. 16.

approximately a straight line, the acceleration is equal to ω^2BC. Hence

$$g.\mathrm{BC}/l=\omega^2\mathrm{BC}, \text{ or } \omega=\sqrt{g/l}.$$

But $\omega=2\pi/\mathrm{T}$, where T = the period of one complete oscillation; therefore $\mathrm{T}=2\pi\sqrt{l/g}$.

If k = the radius of gyration (p. 22), the Moment of Inertia (I) of the simple pendulum $=mk^2=ml^2$, since with a heavy *particle* we may regard the mass as concentrated at a single point at the lower end of the thread. Hence

$$\mathrm{T}=2\pi\sqrt{l/g}=2\pi\sqrt{l^2/lg}=2\pi\sqrt{\mathrm{I}/mlg}.$$

Moment of Momentum.—The product *force* × *time* is termed an **impulse**; and if an impulse acting on a body produces rotation round a fixed axis, then $\mathrm{F}t=m\omega r$, where t is the time during which the force acts, and ω is the angular velocity imparted to a mass m at a distance r from the fixed axis. The product $m\omega r$ may be termed the *angular momentum.*

The **torque**, or the tendency of a force to produce rotation, is measured by the moment of the force about the fixed axis; or $\mathrm{F}r=m\omega r^2/t$. The product $m\omega r^2$ is termed the **moment of momentum** about the axis; and it is evidently equal to the moment of the impulse about the axis of rotation.

The forces acting on the individual particles of a rotating body do not act directly on each particle; but they arise from the strain produced by forces applied externally. If the masses of the particles are m_1, m_2, ... at distances r_1, r_2, ... from the axis of rotation, then the torque is

$$\begin{aligned} f_1r_1+f_2r_2+\ldots &= (m_1\omega r_1{}^2/t)+(m_2\omega r_2{}^2/t)+\ldots \\ &= \{m_1r_1{}^2+m_2r_2{}^2+\ldots\}\omega/t \\ &= \mathrm{I}\,.\,\omega/t; \end{aligned}$$

or, *torque = moment of inertia × angular acceleration.*

SUMMARY

The **velocity of a body** may be defined as the rate at which it changes its position.

If the velocity of a body is variable, the *rate at which the velocity changes* is called the **acceleration.**

A **Force** is measured by the rate at which it causes change of momentum of a body on which the force is acting. The fundamental formula for force is $f=ma$; from this, the C.G.S. unit of force (the **Dyne**) is defined as the force which will give to a mass of 1 gram an acceleration of 1 cm. per sec. per sec.

The **Dyne** is approximately equal to $\frac{1}{981}\times$ weight of 1 gram.

The **Poundal** ,, ,, $\frac{1}{32.2}\times$,, 1 lb.

The **Moment of a Force** is the product of the magnitude of a force into the perpendicular distance between the axis of rotation and the direction of the force.

A **Couple** consists of two equal and parallel forces acting in opposite directions. *The moment of a couple* is equal to one of the forces multiplied by the perpendicular distance between the two forces.

The **Tangent Law.** *The tangent of the angle of deflection is proportional to the deflecting force.*

Work is measured by the product of the force overcome and the distance, in the direction of the force, through which the point of application is moved. An **erg** of work is done when one dyne acts through a distance of 1 cm. A **foot-pound** of work is done when a weight of 1 lb. is raised through a vertical distance of 1 ft. The **joule** is equal to 10^7 ergs.

Unit **Power** is that which is capable of doing unit work in unit time.

The **Watt** is equal to 1 *joule per second.* The **Horse-power** is equal to 33,000 *ft.-lbs. per minute.*

Mechanical Energy is *capacity for doing work.* It may be *kinetic* or *potential.* The K.E. of a mass m moving with velocity v is $\frac{1}{2}mv^2$.

If a mass, which rotates round any fixed point, consists of a large number of separate particles m_1, m_2, etc., situated at distances r_1, r_2, etc., from the axis of rotation, the expression $(m_1r_1^2+m_2r_2^2+\ldots)$ is the **Moment of Inertia** of the mass with respect to that axis.

No work is required in order to move a mass between two points situated on a **Surface of Equal Potential.**

If a point moves in a straight line so that its acceleration is always directed towards, and varies as its distance from, a fixed point in the straight line, the point is said to move in **Simple Harmonic Motion.**

QUESTIONS ON CHAPTER II

1. During what time must a constant force of 60 dynes act upon a mass of 1 kilogram in order to generate in it a velocity of 3 metres per second?

2. Calculate the force, in dynes, necessary to impart an acceleration of 1 ft. per sec. per sec. to a mass of 1 lb. .

3. How much work is done in raising a weight of 1 kilogram through a vertical height of 5 metres?

4. A body of mass 10 grams is projected vertically upwards with a velocity of 100 metres per second; what will be its potential and its kinetic energy when it has ascended to half its maximum height?

5. What is the potential energy of a mass of 25 kilograms raised to a height of 40 metres above the ground?

6. A bullet of 100 grams is discharged with a velocity of 400 metres per second from a rifle, the barrel of which is 1 metre long. Calculate the energy of the bullet when it leaves the muzzle, and the mean force exerted by the powder upon it.

7. The nominal value of a horse-power is 33,000 foot-pounds per minute. Express this (i) in kilogram-metres per minute, and (ii) in ergs per second.

8. Calculate the kinetic energy of a tram-car weighing 2.5 tons, when it moves at 6 miles an hour, and is loaded with 36 passengers, averaging 9 stone each in weight. If the resistance due to friction is equal to $\frac{1}{8}$ of the total weight, find how much work is done when the loaded car travels 3 miles along a level road.

9. A reservoir of water, of area 330,000 sq. ft., is initially of uniform depth 10 ft. How many ft.-lbs. can it supply to a turbine on a level with the bottom of the reservoir, and what horse-power can it maintain on the average if it be emptied in 10 hours? (1 c. ft. water weighs 62.4 lbs.)

CHAPTER III

THE LAW OF INVERSE SQUARES, UNIT POLE-STRENGTH, AND FIELD INTENSITY

The Law of Inverse Squares.—The lifting power of a magnet was the only method of comparing magnetic forces until the year 1780, when Coulomb introduced more accurate methods of measurement. He applied two methods, depending (i) upon the rate of swing of a magnetic needle when suspended horizontally by means of a silk thread, and (ii) upon the torsion of a thin silver or copper wire (as applied in his Torsion Balance). Coulomb applied both methods in order to compare the magnetic strengths of magnets, and also deduced therefrom the **Law of Inverse Squares,** which may be stated thus : **The force with which two magnet poles attract or repel each other is inversely proportional to the square of their distance apart.**

METHOD I (BY THE RATE OF SWING OF A MAGNET).—Suppose a short magnetised needle, suspended horizontally by means of a silk fibre, to swing freely in the earth's field; if the magnet describes n complete vibrations in 1 minute, then the magnetic force due to the earth is proportional to n^2 (p. 84). If a long bar-magnet is now placed to the north of the needle with its axis in line with that of the swinging needle and with its S.-seeking pole pointing towards the needle, and at a distance d_1 cms. from it, then, if n_1 oscillations are now made in one minute, n_1^2 is proportional to the *sum* of the forces due to the earth and the bar-magnet. The force due to the bar-magnet alone is proportional to $(n_1^2 - n^2)$.

If the distance between the bar-magnet and the needle is now increased to d_2 cms., and n_2 oscillations are described in one minute, the force due to the magnet alone is proportional to $(n_2^2 - n^2)$.

The following equation is found to be true :

$$\frac{n_1^2 - n^2}{n_2^2 - n^2} = \frac{d_2^2}{d_1^2}.$$

The Magnetoscope.—A simple piece of apparatus, termed a magnetoscope, may be used for the detection and comparison of magnetic fields of force. The form shown in Fig. 17 consists of a very short piece of a rat-tail file strongly magnetised, and suspended by a single silk fibre. The rate of vibration is observed more accurately if a beam of light is reflected from a silvered microscope cover-glass attached to the magnet.

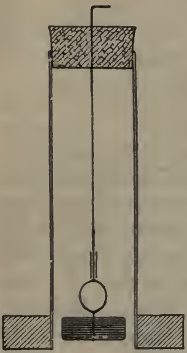

Fig. 17.—A Magnetoscope.

The Magnetometer. — A magnetometer (Fig. 18) consists of a short magnetised needle, pivoted or suspended at the centre of a circular scale; a long pointer made of thin metal wire, of foil, or of drawn-out glass, is attached to the needle. The base-board is prolonged on opposite sides of the circular scale, and a wooden centimetre scale is fixed on each of these arms and adjusted so as to measure distances from the centre of the needle.

Expt. 15.—Place a bar-magnet on the table with its axis in the meridian and with its N.-seeking pole pointing towards the north. Place a magnetoscope with its centre 10 cms. due south of the S.-seeking pole of the magnet, and observe the number n_1 vibrations in one minute. Repeat the observation at a distance of 15 cms. and observe the number n_2 vibrations in one minute. Finally remove the magnet and observe the number n vibrations due to the earth alone. Verify the formula given above.

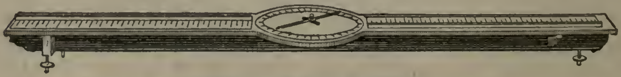

Fig. 18.—A Magnetometer.

Method 2 (by Coulomb's Torsion Balance).—When a mass is suspended by means of a thin metal wire it takes up a definite position of rest; but when the mass is rotated round its point of support *torsion* is set up in the wire, in consequence of which the wire tends to resume its original position of rest. It can be proved that the moment of the force with which the wire tends to return to its original position is proportional to the amount of torsion measured in degrees.

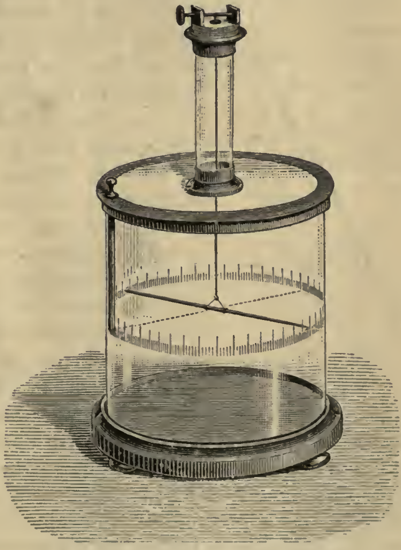

Fig. 19.—Coulomb's Torsion Balance.

The **Torsion Balance** (Fig. 19) consists of a cylindrical glass vessel round the side of which a scale is etched. A silver wire carrying the horizontal magnet is suspended inside a narrow glass tube, and its upper end is attached to a brass *torsion-head* which can be rotated, and the amount of rotation can be observed by means of a circular scale etched on the rim of the torsion-head. Provision is also made for the support of a vertical magnet of such length that its lower pole approximately coincides in position with that of the similar pole of the suspended magnet.

Suppose the torsion-head to be so adjusted that the axis of the magnet is in the meridian (NS. Fig. 20), and that the torsion-head is then rotated through 90°. The torsion of the wire will tend to rotate the magnet into a plane perpendicular to the meridian, but the earth's field will tend to keep the magnet in the meridian; the intermediate position taken up by the magnet will depend upon the relative magnitude of the moments of these forces. If the magnet is deflected through θ°, then the torsion in the wire is $(90-\theta)$. The force mH due to the earth may be resolved into two components, ma acting along the length of the magnet and ineffective in its action on the magnet, and mb acting at right angles to the magnet. Since $mb = m\text{H}\sin\theta$, the moment of the torsion is really balanced by the moment of the force $m\text{H}\sin\theta$; but, if θ is small, $\sin\theta = \theta$ approximately, hence we may say that a deflection of θ degrees in the earth's field is balanced by a torsion of $(90-\theta)$. Finally, a deflection of 1° is balanced by a torsion of $(90-\theta)/\theta$.

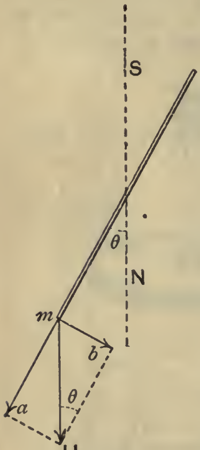

FIG. 20.

In Coulomb's original experiment, a magnetised steel wire 56 cms. long and 2 mms. diameter was used, and he found that a torsion of 35° was necessary in order to produce a deflection of 1°. On introducing the vertical magnet, the deflection was 24°. The repulsion between the poles was then neutralised by two effects, (i) the force due to the earth's field, and (ii) a torsion of 24° in the wire. The force due to the earth was equivalent to a torsion of $(24\times35)^\circ=840^\circ$, so that the repulsion was balanced by a total force equivalent to $(840+24)^\circ=864^\circ$ of torsion. In order to reduce the deflection to one-half its former value (*i.e.* to 12°), it was found necessary to rotate the torsion-head completely round 8 times, *i.e.* through $(8\times360)^\circ=2880^\circ$. The total torsion in the wire was now $(2880+12)^\circ=2892^\circ$; to this must be added the force due to the earth's field, *viz.* $(12\times35)^\circ=420^\circ$. Hence the repulsion was balanced by a total torsion of $(2892+420)^\circ=3312^\circ$.

$$\text{Hence}\quad \frac{\text{Repulsion at }12^\circ}{\text{Repulsion at }24^\circ}=\frac{3312}{864}=\frac{4}{1}\text{ approximately.}$$

It is here assumed that the *linear* distance between the poles is proportional to the *angular* distance between them. This assumption is only justified when the angular distance θ is so small that $\sin\theta = \theta$ approximately. This is discussed fully on p. 127.

METHOD 3 (DEFLECTION METHOD).—This method depends upon the principle that *the tangent of the angle of deflection is proportional to the deflecting force* (see p. 18). The necessary conditions for the experiment are obtained by placing a magnet pole due E. or W. of the centre of a suspended magnet which is controlled by the earth's field; the tangent of the angle of deflection is observed when the pole is situated at various distances from the suspended magnet. Since the two poles of a bar-magnet would produce opposite effects, it is necessary to adopt means of practically eliminating the effect of one pole; this may be approximately done by using a very long bar-magnet, so that one pole is too distant to exert an appreciable deflecting force.

Another method of eliminating the action of one pole is to support the deflecting magnet so that the one pole may be fixed *vertically over the centre of the needle* while the other pole rests on the scale of the magnetometer.

EXPT. 16.—Adjust a magnetometer (p. 30) so that the pointer is over the zero of the circular scale. Place a long magnetised knitting-needle (45 cms. long) on the right-hand scale, with its near pole 15 cms. from the needle, and note the deflection (reading both ends of the pointer, and taking the mean reading); reverse the magnet and repeat the readings. Transfer the magnet to the left-hand scale, place it at the same distance from the needle, and take the same set of readings. Make the same observations at distances of 20 cms., 25 cms., etc. Record the observations in the following manner:

Distance.	Deflections.	Mean Deflection (θ)	tan θ.	tan θ × (distance)^2.
15 cms.	E. End W. End E. End W. End			

Gauss's Proof of the Law of Inverse Squares.—A more rigorous proof, due to Gauss, of the Law of Inverse Squares is based upon a comparison of the magnetic force in the field of a short bar-magnet at two points (A and B, Fig. 21) equally distant from the centre of the magnet, one point being in line with the magnet's axis and the other situated on a line perpendicular to the axis.

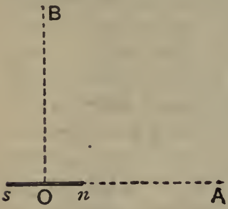

FIG. 21.

By reference to p. 78 the student will see that these relative positions correspond to the *tangent* (*A*) *and* (*B*) *positions of Gauss*; and that when M is the moment of the magnet, $2l$ cms. its length, and d is the distance AO, then

the force acting on unit pole at A is $\dfrac{2Md}{(d^2-l^2)^2}=\dfrac{2M}{d^3}$ approx., and

,, ,, ,, B is $\dfrac{M}{(d^2+l^2)^{\frac{3}{2}}}=\dfrac{M}{d^3}$ approx.

These formulae are derived *on the assumption that the Law of Inverse Squares is true.* Hence, if it can be proved experimentally that the forces at A and B are in the ratio of 2 to 1, the Law is thereby verified. The experimental method consists in observing the deflection of a delicately suspended magnet attached to the back of a small mirror (whereby the deflections are accurately determined: see p. 96), with the deflecting magnet (i) in the *A position* (ii) in the *B position*, the distance of the centre of the magnet being the same in the two cases. Since *the tangent of the angle of deflection is proportional to the deflecting force*, then the Law of Inverse Squares is verified when $\tan\theta_1/\tan\theta_2=2$.

The Unit of Magnetic Pole-strength.—Two magnet poles, of which the strengths are represented by m_1 and m_2, may be regarded as built up of m_1 and m_2 smaller poles each of unit strength. Each unit-pole in the one will exert a certain force on each unit-pole in the other, and the *total* force between the magnet poles will be $(m_1 \times m_2)$ times as great as this. Hence **the mutual force of attraction or repulsion between two magnet poles is proportional to the product of the pole-strengths**; it is also inversely proportional to the square of their distance apart. Hence the force f may be written

$$f=k\frac{m_1 m_2}{d^2},$$

where k is a constant depending for its magnitude upon the units in which f, m, and d are measured. If the poles are equal in strength then $f=k\,.\,m^2/d^2$, and the unit pole strength is so chosen that $m=1$ when f and d are both also equal to unity; the value of k will thus also be equal to unity, and the above formula may be written

$$f=\frac{m^2}{d^2}.$$

The unit magnet pole is that which, when placed at a distance of 1 cm. from a similar and equal pole, is repelled with a force of 1 dyne.

EXAMPLE.—Two long knitting needles are each suspended by two threads 30 cms. long, attached at either end, so that the needles hang in a horizontal position and just touch along their whole length. The needles are then equally magnetised, after which they repel each other, so that the distance between them is 2 cms. If each needle weighs 5 grams and g is 980 cms./secs.^{2}, calculate the strength of one of the poles of either needle.

Let A and B (Fig. 22) represent a pole of each magnet, OA and OB the threads to which they are attached. The force of repulsion between them is

$$f=\frac{m^2}{\mathrm{AB}^2}$$

and the *total* magnetic repulsion of needle A is

$$2f=\frac{2m^2}{\mathrm{AB}^2}.$$

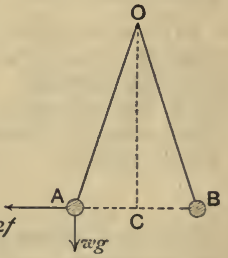

FIG. 22.

The needle A is at rest under the action of two forces, $2f$ and wg, of which the moments round O must be equal and opposite. Hence

$$\frac{2m^2}{\mathrm{AB}^2}\times \mathrm{OC}=wg\times \mathrm{AC},$$

or

$$m=\mathrm{AB}\sqrt{\frac{wg\mathrm{AC}}{2\mathrm{OC}}}=2\sqrt{\frac{5\times 980}{2\sqrt{899}}}=18.08 \text{ C.G.S. units.}$$

Intensity of a Magnetic Field (*in absolute measure*).—Both poles of a magnet, situated in a magnetic field, experience a definite force acting in a definite direction; the magnitude of the force is proportional to the pole strength (m) of the magnet and to the strength (H) of the field, or

$$f=k\,.\,m\mathrm{H}.$$

The units in which f, m, and H are measured are so chosen that $k=1$; in which case, $f=mH$. Hence, **a magnetic field has unit intensity when a unit magnet pole situated in the field is acted on by unit force (one dyne).**

This unit of intensity is sometimes called a **Gauss.**

Comparison of Magnetic Fields by means of a Swinging Magnet.—Since the square of the number of swings (n) described in any given time is proportional to mH/I (p. 84), then, for any given magnet we may say that n^2 is proportional to H, since we may assume that m and I remain constant during the period of an experiment. This fact affords a method of comparing various magnetic fields, the procedure resembling that described under *Method* 1 (p. 29). It is necessary to assume that the forces in the magnetic field are either horizontal or inclined at the same angle to the horizon.

EXPT. 17.—Compare the intensities of the magnetic field at various points of a room by means of a magnetoscope or a short bar-magnet suspended horizontally. Observe, in each case, the time required for at least 20 complete oscillations, and calculate the number (n) which would be described in one minute. Apply the formula $H_1/H_2=n_1^2/n_2^2$.

SUMMARY

The **Law of Inverse Squares** is stated thus: *The force with which two magnet-poles attract or repel each other is inversely proportional to the square of their distance apart.* It may be experimentally proved by the following methods:

(i) *By the rate of swing of a magnet.*
(ii) *By Coulomb's Torsion Balance.*
(iii) *By the deflection method.*

Gauss's proof of the above law is the most satisfactory.

A **magnet-pole of unit strength** *is such that, when placed at a distance of* 1 *cm. from a similar and equal pole, it is repelled with a force of* 1 *dyne.*

A **magnetic field of unit intensity** *is that field in which unit magnetic pole is acted on by unit force.*

The intensities of magnetic fields may be *compared* by observing the rate of swing of a suspended magnet: *the intensity in any case is proportional to the square of the number of swings described in a given time.*

QUESTIONS ON CHAPTER III

1. Describe a method of proving that the force exerted by a magnetic pole varies inversely as the square of the distance.

What is meant by a pole of unit strength?

2. Describe how a long magnet may be placed in a position such that one of its poles deflects a compass while the other does not affect it.

3. A magnetic needle is suspended horizontally at a considerable height above the centre of a bar-magnet lying horizontally in the meridian. The N.-seeking pole is towards the North, and the needle makes 10 oscillations in 1 minute; the magnet is reversed, and the needle now makes 18 oscillations in 1 minute. At what rate would it vibrate under the influence of the earth only?

4. A long thin piece of steel is uniformly magnetised parallel to its length. How would you prove that it exhibits the two opposite magnetic properties in an equal degree?

5. A small suspended magnet makes 10 oscillations per minute under the influence of the earth's field alone. A bar-magnet is brought near it so as not to disturb the *direction* of the pointing of the suspended magnet; but so that the latter now makes 14 oscillations per minute. What would the frequency be if the magnet were now reversed pole for pole?

6. A bar-magnet is placed with its centre due east of a compass needle, and with its axis parallel to the magnetic meridian. How will you determine whether the intensity of the magnetic field at the needle is increased or diminished? Further, how will you compare the field with that which existed there before the bar-magnet was brought near?

7. A horizontally suspended magnetic needle makes 13 vibrations per minute when no magnets are near. When a bar-magnet is placed north of the needle with its axis in the magnetic meridian and its N. pole towards the needle, the latter makes 7 vibrations per minute. How many will it make when the bar is similarly placed at the same distance south of the needle, its N. pole being again directed towards the needle?

8. The repulsive force between two poles is 20 dynes when they are 5 cms. apart. What will it be if the distance is increased to 6 cms.?

9. A magnet of pole strength 1500 and length 10 cms. is placed on a table. Find the intensity of field produced at (i) a point abreast of its middle point and 20 cms. distant from it, and (ii) at a point on the line of the magnet's axis and 20 cms. distant from its middle point.

10. Investigate the intensity due to a straight bar-magnet (i) at any point on its axis, (ii) at any point in the line through its centre perpendicular to its axis, the distance from the magnet being great compared with its length.

Describe how you would carry out an experiment to test the inverse square law of magnetic action, using the results of this investigation.

11. A small compass needle makes 10 oscillations per minute under the influence of the earth's magnetism. When an iron rod 80 cms. long is placed vertically with its lower end on the same level with and 60 cms. from the needle, and due (magnetic) south of it, the number of oscillations is 12 per minute. Calculate the strength of pole of the iron rod (i) neglecting, (ii) taking account of, the influence of the upper end.

CHAPTER IV

MAGNETIC INDUCTION, AND FIELDS OF MAGNETIC FORCE

Magnetic Induction.[1]—The phenomenon of Magnetic Induction is suggested by the observation that *either* end of a piece of unmagnetised iron or steel is attracted by either pole of a bar-magnet. The attraction which is observed may simply be an application of the fundamental law that *Unlike Poles Attract.* Subsequent experiments will show that the proximity of a pole of a permanent bar-magnet temporarily creates opposite polarity in the near end of the iron and also similar polarity in the distant end. Temporary polarity set up in this manner is termed **induced** magnetism.

Expt. 18.—Suspend a strip of soft iron[2] horizontally, and fix a bar-magnet near to it and with the N.-seeking pole close to one end of the soft iron (Fig. 23). In order to test the polarity set up in the iron, slowly bring the S.-seeking pole of a second bar-magnet towards the near end of the soft iron; observe the slight repulsion. Test the distant end of the soft iron by means of the N.-seeking pole, and again observe the repulsion. Remove the first

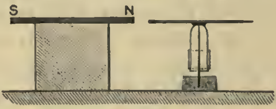

Fig. 23.

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[1] The term 'magnetic induction' is also used, in a quantitative sense, to denote *magnetic flux* (p. 69 and Chap. XXI), and it is unfortunate that the same word should be used to denote the phenomena described in this paragraph. It has been suggested that the expression 'magnetisation by influence' should be used in the present instance, but it is perhaps inadvisable to introduce a term which is not generally recognised.

[2] Narrow strips of tinned iron are very suitable for experiments on magnetic induction.

bar-magnet to a distance, and observe that both ends of the soft iron are now attracted by a neighbouring pole of a bar-magnet.

Magnetic Induction may be transmitted through several pieces of soft iron.

Expt. 19.—Clamp a bar-magnet in a vertical position, and attach to the lower pole a strip of soft iron. Bring several wire-nails into contact with the strip of iron, and notice how long a chain of nails can be supported (Fig. 24). Test the polarity of the extreme end of the chain by means of a compass-needle.

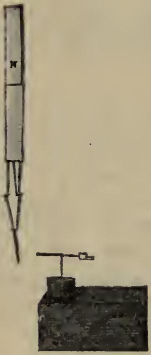

Fig. 24.

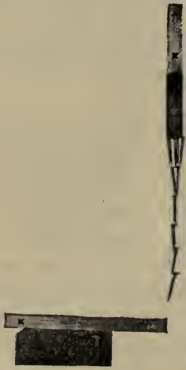

Fig. 25.

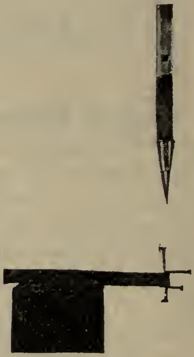

Fig. 26.

Two or more bar-magnets may either aid or oppose each other in imparting induced magnetisation to soft iron, according to the positions of the bar-magnets relatively to the soft iron.

Expt. 20.—Arrange the apparatus as in Expt. 19, with the N.-seeking pole of the bar-magnet downwards. Place the S.-seeking pole of a second bar-magnet just below the end of the chain of nails. Notice that several more iron nails can now be added to the number previously supported (Fig. 25).

The induction due to the S.-seeking pole tends to strengthen that originally present, and consequently the total induced magnetism is increased. Remove the S.-seeking pole, when several of the nails will fall off. Still more nails will fall off if the N.-seeking pole is placed close to the lower end of the chain (Fig. 26).

Magnetic induction may also be observed when steel is used instead of soft iron. Even when the steel is already permanently magnetised, induced magnetism may be imparted; the permanent magnetism will be either increased or diminished according to whether the induced polarity is similar to or opposite to the permanent polarity. In cases where the permanent polarity is very feeble the induced polarity may be sufficiently pronounced to reverse the original polarity of the steel.

Expt. 21.—Feebly magnetise a long knitting-needle, and suspend it in a stirrup. Hold the pole of a strong bar-magnet some distance away, and observe the repulsion between similar poles. Rapidly bring the magnet to within an inch of the repelled end of the needle, when the original repulsion will be converted into a strong attraction.

The previous experiment suggests the importance, when detecting feeble magnetisation, of *gradually* bringing the poles near together, and of looking carefully for the first indication of repulsion.

In the mutual action between two bar-magnets the force of repulsion between similar poles is always slightly *weaker* than the force of attraction between unlike poles; this is due to the fact that, when similar poles are approaching, each magnet tends to induce a reversed polarity in the other; on the other hand, the approach of two unlike poles tends to increase the permanent polarity.

In the case of a single bar-magnet, the influence of its own poles tends to demagnetise the magnet itself. This tendency, which is more marked in short than in long magnets, is considerably diminished by the use of *magnetic keepers* (p. 66), and it is only totally absent in the case of magnets possessing no poles (*e.g.* a steel ring magnetised in the direction of its circular axis).

Fields of Magnetic Force.—The entire space, round a magnet, in any part of which it is possible to detect magnetic force due to the magnet is termed the **Magnetic Field** of the magnet. In order to describe fully the magnetic condition of any point in a magnetic field it is necessary to determine (i) the *direction* of the force at the point, and (ii) the *magnitude*[1] of the force.

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[1] Methods of determining the magnitude of magnetic forces are described in Chap. vi.

A *map* of a magnetic field is a diagram which indicates, by means of lines drawn upon the map, the direction of the magnetic forces at all points of the field situated within the area of the diagram.

When a compass needle is placed within any magnetic field the opposite poles of the needle are acted upon by forces which are usually equal in magnitude but opposite in direction. This results in the needle coming to rest with its magnetic axis coinciding with the direction of the magnetic force at the point where the needle is situated ; if the direction of the force traces out a curved path the needle will come to rest in such a position that its magnetic axis is a *tangent* to the curved direction of the force.

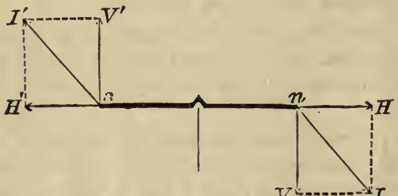

Fig. 27.

The fact that a compass needle remains horizontal does not prove that the magnetic forces are also horizontal, for, even if the forces are inclined to the horizontal, it may still be possible for them to have a directive action upon the needle. In Fig. 27 let ns represent a compass needle, and $n\mathrm{I}$, $s\mathrm{I}'$ the forces due to the magnetic field. The force $n\mathrm{I}$ may be resolved into two other forces—$n\mathrm{H}$ the *horizontal component*, and $n\mathrm{V}$ the *vertical component*; similarly $s\mathrm{I}'$ may be resolved into $s\mathrm{H}'$ and $s\mathrm{V}'$. The forces $n\mathrm{H}$ and $s\mathrm{H}'$ will pull the needle into the same vertical plane as that of the original force, while $n\mathrm{V}$ and $s\mathrm{V}'$ will simply tend to tilt the needle out of the horizontal ; the weight of the needle is usually sufficient to mask the effects of the latter forces. Hence a compass needle must be accepted only as a means of indicating the direction of the horizontal components of magnetic forces.

The actual direction of the magnetic force is arbitrarily chosen as that in which the N.-seeking pole tends to travel : this direction is termed *positive*, and the direction in which the S.-seeking pole tends to travel is termed *negative*.

Maps of Magnetic Fields.—Maps of magnetic fields may be determined either by means of a compass-needle, or by means of iron filings; the former method is the more accurate, although somewhat tedious. Care should be taken that there are no iron pipes or girders near the table on which the experiment is conducted.

(1) COMPASS-NEEDLE METHOD.—When a compass-needle is placed near to a bar-magnet it is, as a general rule, acted upon partly by the magnet, and partly by the earth; the final direction in which the needle points will indicate the direction of the *resultant* force, of which the forces due to the magnet and to the earth are the components.

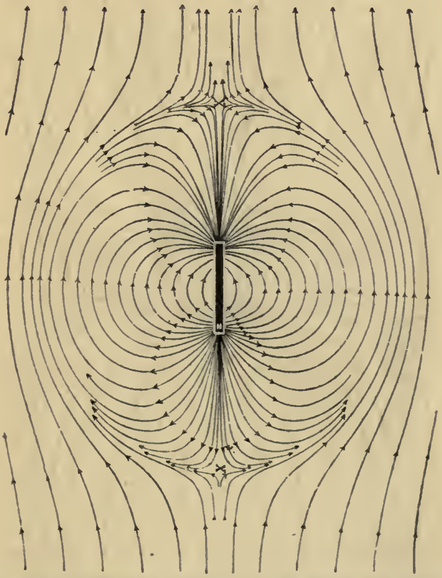

FIG. 28.—Resultant Magnetic Field due to the Earth and a Bar-Magnet (north pole pointing south).

EXPT. 22.—Fasten a square sheet of paper on the table, with one edge pointing approximately north and south. Determine accurately the north and south line by means of a compass-needle, and place a bar-magnet at the centre of the paper with its axis pointing north and south. Mark off one of the edges pointing east and west into spaces about 5 cms. wide. Place a compass-needle so that one of its poles is just over one of the marks, and indicate by means of a pencil the direction in which the other pole is pointing. Move the needle until its first pole is exactly over the second pencil mark; continue this process until a series of marks have been obtained completely across the paper. Join up these points by a continuous line. Plot out other lines in a similar manner. Map out the lines of force (i) *with the N.-seeking pole pointing south* (Fig. 28), (ii) *with the N.-seeking pole pointing north* (Fig. 29), and (iii) *with the axis of the magnet lying east and west* (Fig. 30).

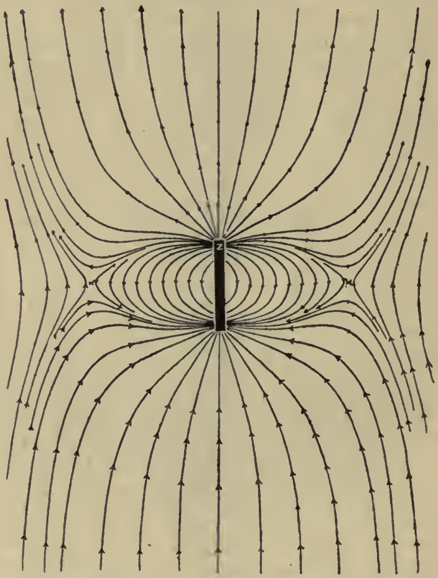

FIG. 29.—Resultant Field (north pole pointing north).

The lines which constitute a map of a magnetic field are usually termed **magnetic lines of force**—a term which was suggested by Faraday in 1837.

It will be noticed in Figs. 28-30 that, at points near to the magnet, the forces due to the magnet predominate, while at greater distances these become weaker in effect, and appear to be simply those due to the earth, and slightly distorted by the presence of the magnet. Also, there are two regions (*marked X*) in which the forces due to the magnet and the earth exactly neutralise each other, and in which the needle will come to rest in all positions with equal readiness ; these regions are termed **neutral points.**

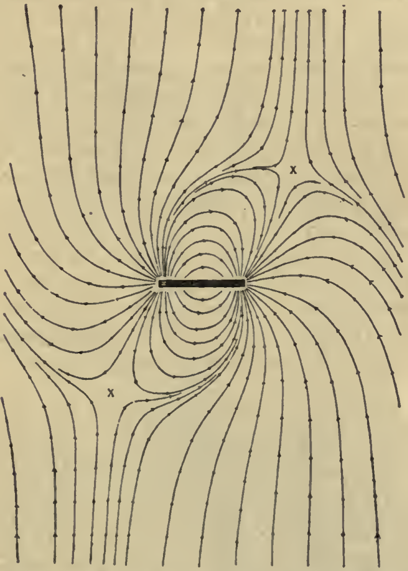

FIG. 30.—Resultant Field (north pole pointing west).

Since the positive direction of lines of force is that in which a N.-seeking pole tends to travel, the lines of force are represented as **emerging** from the N.-seeking pole of a magnet, and *re-entering* at the S.-seeking pôle ; also, the direction of the lines of force due to the earth is represented as proceeding from geographical south towards geographical north. The latter statement suggests that the earth's field may be due to a region of N.-seeking polarity near to the south geographical pole, and *vice-versâ.*

An important point to remember is that lines of magnetic force always connect poles of opposite polarity.

A close analogy exists between the phenomena due to magnetic lines of force and those which might be brought about if the lines of force were replaced by stretched elastic threads, whose directions coincide with those of the lines of force ; such threads tend to shorten and to become simultaneously thicker, while lines of force also tend to shorten and to repel each other from side to side. Faraday, to whom this analogy is due, even regarded lines of force as physical realities and not merely as imaginary conceptions ; and he assumed that these lines of force were of the nature of springs, in a state of tension, which mutually repelled each other. It may be that, in the near future, the advance of knowledge will prove that Faraday's idea of the reality of lines of force is fully justified, and the same remark applies to the electrostatic lines of force of an electric field.

(2) IRON FILINGS METHOD.—A far more rapid method of obtaining maps of magnetic fields depends upon the fact that when fragments of soft iron are placed in a magnetic field they become temporarily magnetised, and, if free to move, arrange

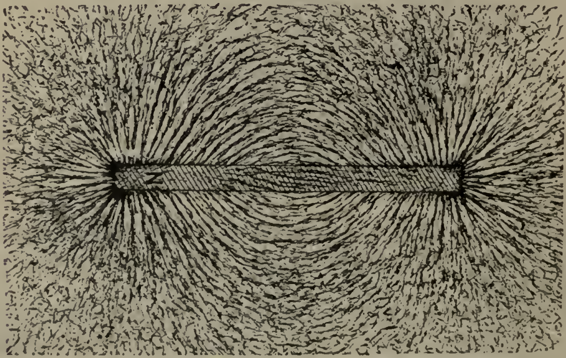

FIG. 31.—Map of the Magnetic Field due to a single Bar-Magnet.

themselves in such a manner that their longer axes coincide with the direction of the magnetic force. In this manner a single iron filing will behave similarly to a compass-needle, the chief disadvantage being that the filing has far less freedom of movement than the needle ; this difficulty is partly overcome by distributing the filings over a smooth surface (*e.g.* glass or smooth paper), and afterwards gently tapping the support. This smaller freedom of movement prevents the filings from being influenced by a weak field ; the method is therefore only suitable for mapping fields comparatively near to magnets. Permanent iron-filing maps can be obtained by the method described in the following experiment :

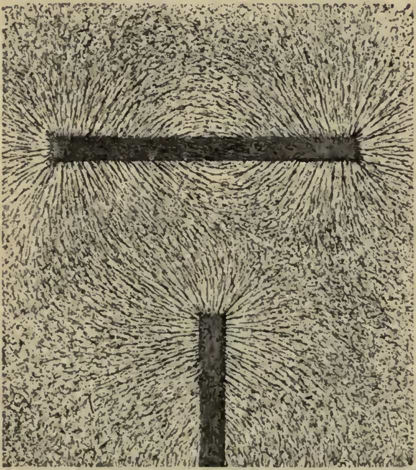

FIG. 32.—Map of the Magnetic Field due to two Bar-Magnets.

EXPT. 23.—Lay a sheet of paraffined paper[1] over a magnet and support it horizontally on pieces of wood placed on either side of the magnet. Sprinkle filings uniformly over the paper, and gently tap the paper with a pencil or a copper-wire. Fix the filings by passing a Bunsen flame over the surface until the wax has melted slightly, and allow to cool (Figs. 31-33).

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[1] Paraffined-paper may be prepared by passing sheets of thin white paper through melted paraffin-wax contained in a shallow tin.

It is, of course, impossible, owing to the weight of the filings, to obtain an iron-filing map of a magnetic field in any other plane than the horizontal ; but, if a bar-magnet is resting on its side instead of on the broad face, a map would be obtained which is similar to that obtained in the first case, so also a cylindrical magnet would give the same distribution of lines in whatever horizontal position it may be placed. Hence the map of any single bar-magnet may be regarded as correct for all planes passing through its axis, and the whole space surrounding the magnet is permeated by lines of magnetic force, the distribution of which is indicated by the map obtained in the horizontal plane only. The contour of the lines of force in a vertical plane may be observed by holding a small **dip-needle** in a series of positions vertically over and under a long bar-magnet which is clamped horizontally.

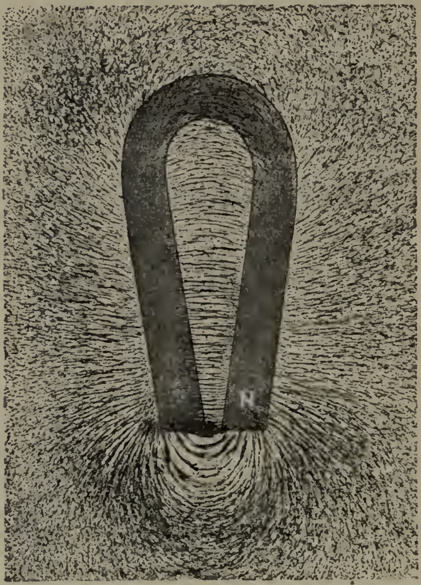

FIG. 33.—Map of the Magnetic Field due to a Horse-shoe Magnet.

The Poles of a Magnet.—In magnetic measurements it is necessary to regard the poles of a magnet as being situated at well-defined *points*, whereas the manner in which iron filings are attracted would suggest that a bar-magnet has a pole-area of considerable extent and with the force of attraction more pronounced towards the extreme ends of the magnet. Fig. 34 represents the forces acting on the poles of a magnet NS suspended in a uniform magnetic field. These forces may be imagined to be resolved into two single forces PR and P′R′ acting at P and P′. The points P and P′ are the **poles** of the bar-magnet, and they may be defined as **the points of application of the resultant magnetic forces acting on the magnet when suspended in a uniform magnetic field.**

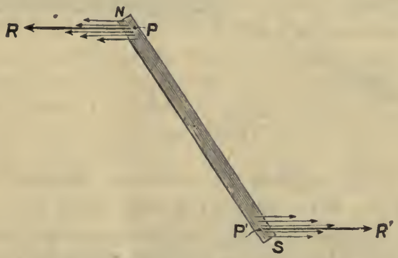

FIG. 34.—The points P and P′ are the Poles of the Magnet NS.

The poles of a magnet may be separately located by applying the fact that a compass-needle comes to rest pointing in the direction of the resultant magnetic force; but the experiment must be so arranged that (i) the compass-needle is always in the magnetic meridian, and therefore free from any deflection due to the earth, and (ii) at a sufficient distance from the other pole of the magnet to ensure the effect of that pole being negligible.

EXPT. 24.—Place a long bar-magnet on a sheet of paper supported on a drawing-board, and mark its position by passing a pencil-point round its edge (Fig. 35). Place a compass-needle at n_1s_1, rotate the board until the needle is in the magnetic meridian, and put pencil marks in line with the axis of the needle; repeat the process in the positions

n_2s_2 and n_3s_3. Remove the magnet, and produce by means of a straight edge the three directions obtained. The point of intersection of these lines indicates the position of the pole of the magnet.

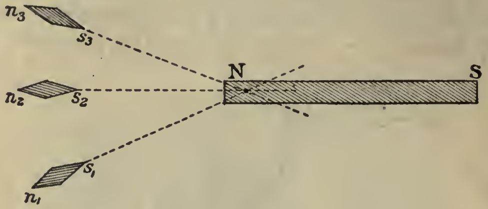

FIG. 35.

In short thick magnets the poles are situated about 1 cm. from each end. If the magnet is only about 1 or 2 mms. wide and at least 20 times as long, the poles approximately coincide with the ends. The distance between the poles of a magnet is termed the **Magnetic Length** of the magnet.

Geometrical Method of Determining the Direction of the Magnetic Force due to a Bar-Magnet.—The direction of the magnetic force, at any point n (Fig. 36), due to a bar-magnet is determined geometrically by the fact that the forces acting on a single N.-seeking pole at n due to N and S

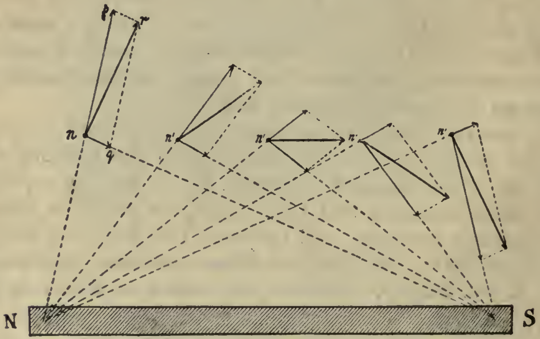

FIG. 36.

are inversely proportional to the square of the distances Nn and Sn, or $np/nq=(r\text{S})^2/(n\text{N})^2$. The direction of the resultant force nr is obtained by drawing np and nq to any suitable scale and completing the parallelogram $nprq$.

If a compass-needle is placed at n it will point in the direction nr, providing that the needle is very short, since the resultant force acting on the S.-seeking pole of the needle will be equal and opposite to that acting on the N.-seeking pole.

The Intensity of a Magnetic Field Represented by Lines of Force.—It is possible to represent the intensity of a magnetic field at any point by the number of lines of force intersecting a surface of unit area (1 sq. cm.) placed perpendicularly to the direction of the lines of force. A field of unit intensity is represented by a single line of force passing through each unit area.

Since the intensity of the field at a distance of 1 cm. from a unit pole is equal to unity (p. 36), it therefore follows, if a sphere of unit radius is described round the pole, that the intensity of the field on the surface of the sphere would be represented graphically by a single line of force passing through each square cm. of the sphere's surface. The area of the surface of any sphere of radius r cms. is equal to $4\pi r^2$ sq. cms., hence, if $r=1$, then area $=4\pi$ sq. cms. Consequently, the number of lines of force proceeding from a unit pole would be equal to 4π; also, the number of lines proceeding from a pole of strength m would be equal to $4\pi m$.

Geometrical Construction for a Magnetic Field.—It would be easy to represent a magnetic field due to a single pole if it were only necessary to indicate a uniform distribution of the lines of force *in the plane of the diagram;* but the uniform distribution *in space*, which is actually the case, is not easily represented with accuracy. We can imagine a spherical surface to be described round the magnet-pole as centre, and that the lines of force penetrate this surface at equi-distant intervals; and if the surface is divided up into a number of small portions of equal area we may assume that each of these small areas will be penetrated by the *same* number of lines of force. This division into equal areas is obtained by dividing a diameter into a number of equal parts by points through which planes are drawn perpendicular to the diameter; these planes will divide the surface of the sphere into equal portions.

Fig. 37 represents a spherical surface divided up in this manner, and the radial lines mr_1, mr_2, ... will separate the total number of lines of force into groups each containing the same number of lines. If the number of groups is sufficiently numerous we may assume that the lines mr_1, mr_2, ... graphically represent, with satisfactory approximation, the distribution of the lines of force originating from the pole at the centre. Strictly speaking, if the pole strength is m units, and if the axis is divided into m equal parts, then 2π lines of force will penetrate each portion into which the spherical surface is sub-divided.

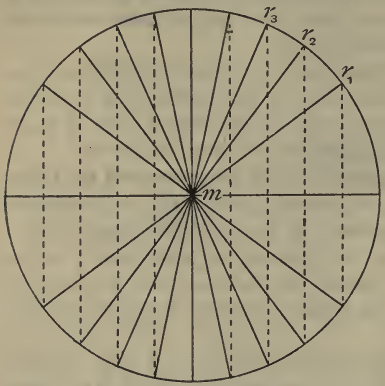

Fig. 37.

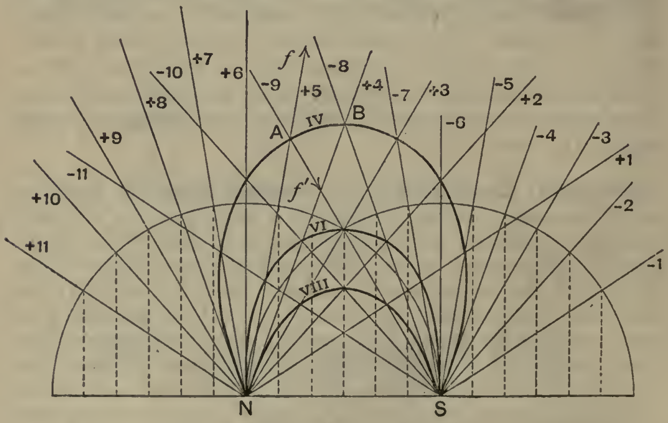

Fig. 38.—Geometrical Method of Mapping the Field due to two Poles.

Fig. 38 represents the method of constructing the field of force due to two opposite poles each of strength 12 units. The system of lines due to each pole is constructed as in Fig. 37; the lines due to N are marked +1, +2, ..., and those due to S are marked −1, −2, ... The resultant lines of force are drawn by connecting together points of intersection of component lines which have the same algebraic sum; thus the line marked IV is obtained by joining the intersections of the lines −8 and +4, −9 and +5, −10 and +6, and so on. The process really consists in drawing the diagonals of the four-sided figures into which the field is divided; for, the forces acting on a single north-seeking pole at A will be f and f' respectively, and the direction of the resultant force is along the diagonal AB. This is rendered more evident when the component lines are so numerous that contiguous lines are approximately parallel, and then applying the principle that the intensity of the force at any point is inversely proportional to the distance between two contiguous lines. Thus, in Fig. 39, if b and b' are the distances apart of contiguous lines from N and S respectively, then $f : f' :: b' : b$; but $b' : b :: An : As$, hence $f : f' :: An : As$, and the diagonal AB represents the direction of the resultant force at A.

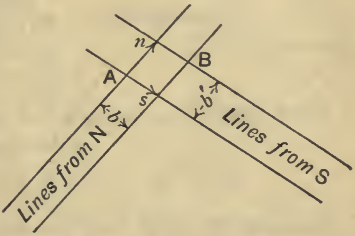

FIG. 39.

Magnetic Potential.—The Magnetic Potential at any point is equal to the work done against the magnetic force when a unit magnet-pole (N.-seeking) is conveyed from an infinite distance up to the point. At the point N (Fig. 40) let there be a single N.-seeking pole of strength m, and let RN represent the path along which a unit pole (N.-seeking) is conveyed, R representing a point at an infinite distance from N. The symbols r_1, r_2, etc., represent points which are very close together, and situated at distances r_1, r_2, etc., respectively from N.

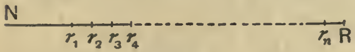

FIG. 40.

$$\text{Force of repulsion at } r_1 = \frac{m \times 1}{r_1^2} = \frac{m}{r_1^2}.$$

$$\text{,,} \qquad \text{,,} \qquad r_2 \qquad = \frac{m}{r_2^2}.$$

Average force along $r_1r_2=\frac{m}{2}\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}\right)=\frac{m}{2}\left(\frac{r_2^2+r_1^2}{r_1^2r_2^2}\right)$

$$=\frac{m}{2}\left\{\frac{(r_1+\delta)^2+r_1^2}{r_1^2r_2^2}\right\}\text{ where the distance } r_2=(r_1+\delta)$$

$$=\frac{m}{2}\left(\frac{2r_1^2+2r_1\delta+\delta^2}{r_1^2r_2^2}\right).$$

But, since δ is very small, δ^2 may be neglected, and $r_2=(r_1+\delta)$. Hence

$$\text{Average force}=m\left(\frac{r_1(r_1+\delta)}{r_1^2r_2^2}\right)$$

$$=\frac{m}{r_1r_2}.$$

Work done in conveying a unit pole from r_2 to r_1

$$=\text{Average Force}\times\text{Distance}$$

$$=\frac{m}{r_1r_2}(r_2-r_1)=m\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Similarly, the work done between r_3 and r_2

$$=m\left(\frac{1}{r_2}-\frac{1}{r_3}\right).$$

And the work done between r_4 and r_3

$$=m\left(\frac{1}{r_3}-\frac{1}{r_4}\right).$$

And the work done between R and r_n

$$=m\left(\frac{1}{r_n}-\frac{1}{R}\right).$$

By addition, the total work done between the points R and r_1

$$=m\left(\frac{1}{r_1}-\frac{1}{r_2}+\frac{1}{r_2}-\frac{1}{r_3}+\ldots+\frac{1}{r_n}-\frac{1}{R}\right)$$

$$=m\left(\frac{1}{r_1}-\frac{1}{R}\right).$$

If R is at an infinite distance from N, then $\frac{1}{R}=0$.

Hence, **the Magnetic Potential at** $r_1=\frac{m}{r_1}$.

If the pole at N is S.-seeking, then its strength would be indicated by the symbol $-m$, and the Magnetic Potential at r_1 would be $-\frac{m}{r_1}$.

Potential at any point due to a Short Magnet.—Let NS (Fig. 41) be a short magnet of length $2l$ and pole-strength m. Let A be any point in space, the line AO making an angle θ with the axis of the magnet.

The potential at A, due to N, $=m/\text{AN}$; also, the potential at A, due to S, $=-m/\text{AS}$. Hence the resultant potential $=m/\text{AN}-m/\text{AS}$.

Drop perpendiculars Nn and Ss from N and S on AO. Then, if the angles NAO and SAO are small, $\text{AN}=\text{A}n$ approximately, also $\text{AS}=\text{A}s$. Therefore,

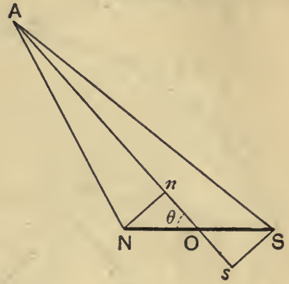

FIG. 41.

$$\frac{m}{\text{AN}}-\frac{m}{\text{AS}}=\frac{m}{\text{AO}-\text{O}n}-\frac{m}{\text{AO}+\text{O}n}=\frac{2m\,.\,\text{O}n}{\text{AO}^2-\text{O}n^2}$$

$$=\frac{2m\,.\,\text{O}n}{\text{AO}^2}\ \text{(since O}n\text{ is very small compared with AO)}$$

$$=\frac{2m\,.\,\text{ON}\,.\,\cos\theta}{\text{AO}^2}=\frac{2ml\cos\theta}{\text{AO}^2}.$$

If $\theta=90°$, the potential is consequently zero; also, for any given value of AO, the potential is a maximum when $\theta=0$.

Surfaces of Equal Magnetic Potential.—Since the magnetic potential at a point r cms. distant from a single magnet-pole of strength m is equal to m/r, it follows that all points equally distant from the pole will have the same potential. Hence, if a spherical surface is described round the pole as a centre, all points on the surface will have the same potential, and the sphere would be termed a **Surface of Equal Magnetic Potential.**

The potential at a distance r from a single magnet-pole, of strength m, is $V=m/r$; hence the radius of the sphere for which the potential is V is given by the equation $r=m/V$. If we now give to V the values 1, 2, 3, etc., and draw the corresponding spheres, we obtain a series of equipotential surfaces. Fig. 42 represents the sections of such spheres by a plane passing through their common centre; the pole-strength is here taken as 4.35 units. The relative values of the potential at various points along the line *ah* is represented by the thick line AH, the vertical distance of which above any point on *ah* is proportional to the potential at that point.

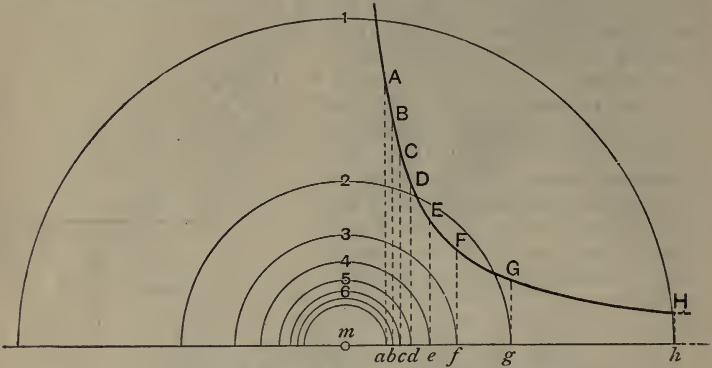

FIG. 42.—Surfaces of Equal Magnetic Potential due to a single Pole.

The construction of the equipotential surfaces due to *two* neighbouring magnet-poles, is based upon the fact that the potential at any point is equal to the algebraic sum of the potentials at that point due to the two poles. Fig. 43 represents the sections of equipotential surfaces near to two equal and opposite poles, of pole-strength + 25 and − 25 units. The thin lines represent the surfaces due to each pole, and are constructed in the manner explained in Fig. 42. The resultant potential surfaces are constructed by joining together those points at which the algebraic sum V of the potentials are the same. It is evident that the potential at any point along the central line of symmetry, marked $V=0$, will be zero; for, if a single magnet-pole is placed at any point on this line the resultant force due to m and m' will be *perpendicular* to that line, and no work will therefore be done on or by the pole in moving along it. The point where any given surface intersects the line joining the poles is given by the equation

$$\frac{m}{x}-\frac{m}{d-x}=V$$

where x is the distance from the pole $+m$, and d is the distance between the poles; the point of intersection *outside* the line joining the two poles is given by the equation

$$\frac{m}{x}-\frac{m}{d+x}=V.$$

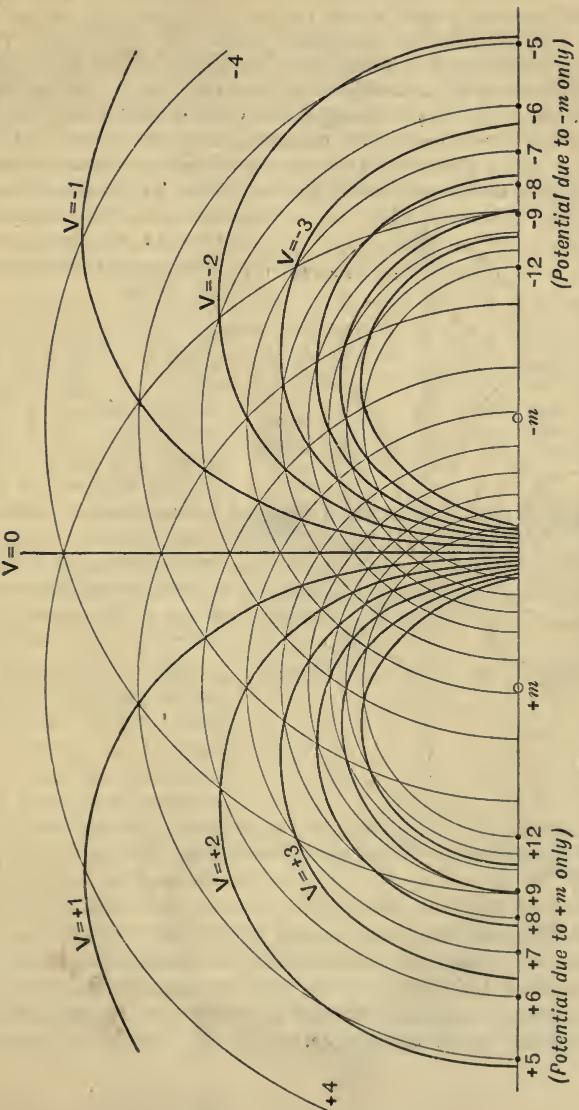

FIG. 43.—Method of Mapping the Surfaces of Equal Magnetic Potential due to two opposite Poles.

It is important to bear in mind that in any magnetic field the lines of force always intersect the equipotential surfaces at right angles. This may be demonstrated by assuming that a given line of force intersects an equipotential surface at an angle other than a right angle; the magnetic force, acting on a pole situated at the point of intersection, may be resolved into two components, one acting *along* the surface, and the other at right angles to it. In order to move the pole to another point on the same surface work would have to be done by, or against, magnetic force—which is contrary to the fundamental definition of an Equipotential Surface.

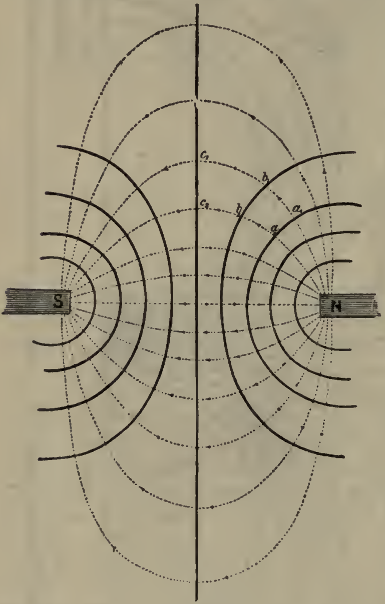

Fig. 44.—Lines of Force and Surfaces of Equal Potential mapped out by means of a Compass-needle.

The student is referred to Fig. 110, which, although intended to represent the lines of force and the equipotential surfaces in an electrostatic field, may equally well represent the *magnetic* field of force due to two poles, of strength + 30 units and − 10; the lines of force and surfaces in this diagram have been drawn in the manner described on pp. 51 and 52.

Fig. 44 represents a series of equipotential lines which may be mapped cut experimentally, with the aid of a compass-needle provided with a brass cross-bar fixed to the centre of the needle, and at right angles to its axis. Since the axis of the needle always coincides with the direction of the lines of force, which latter are always perpendicular to the equipotential lines, it follows that the brass cross-bar will always coincide with the equipotential lines, and these may thereby be mapped out in the manner described on p. 43.

The potential difference between any two equipotential lines is, by definition, measured by the work done against the magnetic forces in conveying a unit magnet-pole from one surface to the other. Hence, whatever path may be traversed, the work done will always be the same; thus the work required to convey the unit-pole from b_1 to c_1 (Fig. 44), is equal to the work required between b_2 and c_2. This fact can be readily understood when it is remembered that although b_1c_1 is longer than b_2c_2, nevertheless, the average force along b_1c_1 is less than that along b_2c_2, and that the work required = (Average force × Distance).

EXPT. 25.—Map out, by means of a compass-needle (fitted with a brass cross-bar), a small portion of a magnetic field, *e.g.* $a_1c_1c_2a_2$ (Fig. 44). Measure the lengths of the curved lines a_1c_1 and a_2c_2. Find the middle points of these lines, and determine the rate of vibration of a magnetometer needle situated at these points. If l is the length of any line of force connecting two equipotential surfaces, and n is the number of vibrations per minute described by the suspended needle, then $(l \times n^2)$ should be a constant quantity.

The Relationship between Magnetic Intensity and Potential.—If a magnet-pole of strength m is placed at a point in a magnetic field of intensity H, the force f acting on the pole is equal to mH. If the pole is allowed to move through a short distance s, then the work W done by the magnetic force is

$$W = fs = mHs.$$

But, if the potentials at the initial and final positions are V_1 and V_2, then also $W = m(V_1 - V_2)$. Hence

$$mHs = m(V_1 - V_2),$$

or

$$H = \frac{V_1 - V_2}{s}.$$

This result may be expressed in words thus: *The intensity of the field is equal to the rate of change of the magnetic potential with respect to distance.* Also, for any given values of V_1 and V_2, H is a maximum when s is a minimum; the intensity of the field is therefore greatest in those regions in which the equipotential surfaces are closest together.

Molecular Magnets.—When a long magnet is broken into several pieces, it is found that each portion is a complete magnet having opposite poles at opposite ends. It will also be observed that, at each point of fracture, the two broken ends have *opposite* polarity: since uniformity of magnetisation may be assumed, the contiguity of equal and opposite poles will explain why no polarity was exhibited at the point of fracture previously to the magnet being broken, for all the lines of force proceeding from the N.-seeking end of any fragment will proceed directly to the S.-seeking pole in contact with it, and no lines of force will be present outside the steel.

EXPT. 26.—Magnetise a piece of clock-spring, and break it into several pieces. Test the polarity at the ends of each fragment by means of a compass needle.

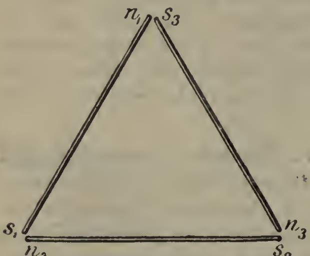

FIG. 45.

Modern theory suggests that the molecules of any unmagnetised piece of steel are initially magnetised but connected together in no regular order, so that the magnetic forces due to any one molecule are masked by those of neighbouring molecules. We may say that the molecules are grouped together into a number of independent *closed magnetic chains*, which may be roughly imitated by grouping together two or more magnetised sewing-needles (Fig. 45). According to the same theory, the process of magnetisation consists in rotating the molecules until the axis of each coincides more or less accurately

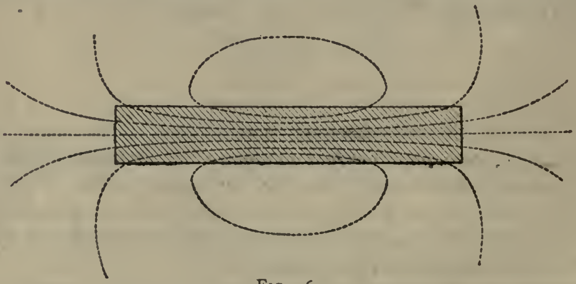

FIG. 46.

with the direction in which the steel is magnetised and so that the similar poles of each molecule point towards the same end

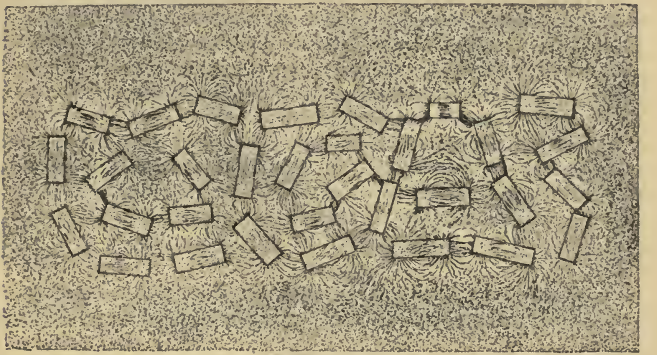

FIG. 47.

of the steel; this process involves the conversion of the 'closed magnetic chains' into a smaller number of *open magnetic chains*, the lines of force due to each of which are, in part of their path,

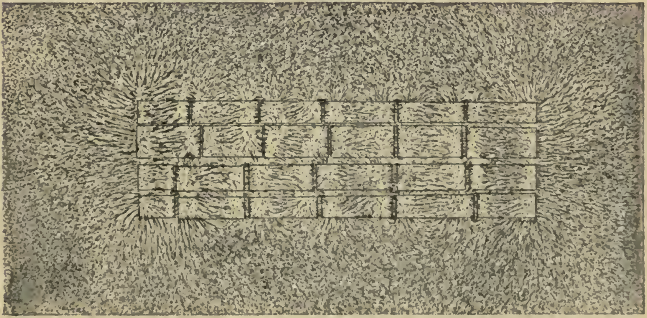

FIG. 48.

traversing the air space outside the steel (Fig. 46). Fig. 47 represents the magnetic field due to a number of short pieces of magnetised steel distributed without any sense of uniformity; Fig. 48 represents the field due to the same fragments when arranged in order, with similar poles pointing in the same direction.

Expt. 26a.—Fill a glass test-tube with *steel* filings, and insert a cork in the open end. Magnetise the tube (by means of a spiral of wire and battery, if available). Observe that the tube now has opposite polarity at the ends. Empty the filings out on to a sheet of paper, mix well together, and again transfer them to the tube; the ends of the tube will be found to have lost their polarity.

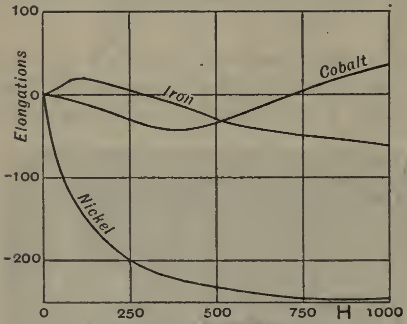

Fig. 49.—Magnetostriction in Iron, Cobalt, and Nickel.

The degree of magnetisation may be attributed to the degree to which the alignment of the constituent molecules is perfect, and that magnetic saturation is obtained when the alignment is perfect. Joule observed that a bar of steel elongates, when strongly magnetised, to the extent of $\frac{1}{720{,}000}$ of its length; this would appear to verify the above theory of magnetisation, since each molecule, providing that it is not spherical, might be regarded as having poles at the ends of its longest diameter. It has also been observed that if a short piece of iron is immersed in water contained in a glass-tube (resembling the bulb of a thermometer) and then strongly magnetised, the *volume* of the iron is slightly increased. On the other hand, Shelford Bidwell has more recently observed that although iron elongates when placed in a magnetic field of moderate intensity, yet, as the field is gradually increased, the iron gradually returns to its original length, and becomes even shorter than before if the field is made sufficiently intense (Fig. 49).[1] Cobalt and nickel, under similar conditions, exhibit a special behaviour: thus, as the field increases, cobalt shortens then elongates to its former length and finally acquires a gradual elongation, while nickel, under intensity of the same conditions, gradually shortens, and remains so whatever the field may be. In Fig. 49 the elongations are expressed in ten-millionths of the length of the material.

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[1] Fig. 49 is adapted from Mr. Shelford Bidwell's article on 'Magnetism' in the *Encyclopædia Brit.* (10th edn.).

The term *Magnetostriction* is usually applied to this phenomenon. The following simple method of observing the effect is due to Prof. W. S. Franklin:[1] A small weight is suspended by an iron wire constituting a torsion pendulum. A direct current is sent through the wire and through a magnetising coil which surrounds it; the former magnetises the surface of the wire transversely, and the latter magnetises it longitudinally, hence the resultant magnetisation is along a helical line, and the accompanying change of length causes the wire to twist slightly. If the current in the coil (or in the wire) is reversed in rhythm with the free oscillations of the pendulum, the magnetisation along right and left helical lines creates oscillations of the pendulum which are easily perceptible.

Behaviour of Soft Iron in Magnetic Fields.—The theory of molecular magnets, explained in the previous section, may also be adopted in order to explain the behaviour of soft iron when placed in a magnetic field. The molecules, constituting the soft iron, will tend to set themselves with their magnetic axes coinciding with the direction of the magnetic force, and with their N.-seeking poles pointing in the direction of the

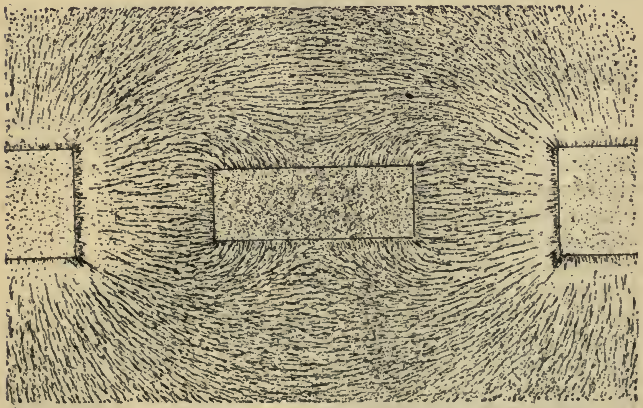

Fig. 50.—Effect of Soft Iron on a Magnetic Field.

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[1] *Nature*, Vol. 69, p. 582.

force; soft iron will consequently develop S.-seeking and N.-seeking polarities in the regions where lines of force enter and emerge from the iron. Also, if we consider a free N.-seeking pole, moving from a distance and towards the iron, the path traversed will be bent inwards towards the iron (this is more evident if we consider the effect of the S.-seeking polarity induced in the end of the iron). Fig. 50 represents an iron-filing map of a magnetic field in which a piece of soft iron is placed, and it clearly indicates this *concentration* of the lines of force through the iron. It is a useful convention to regard magnetic substances as *good conductors* of magnetic lines of force, and that lines of force tend to trace out a path of *least magnetic resistance* (*i.e.* of greatest magnetic conductivity); consequently, in the neighbourhood of iron, the lines of force appear to converge towards it.[1]

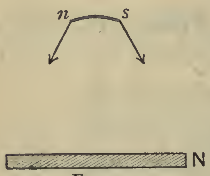

FIG. 51.

The movements of a piece of soft iron, when suspended in a magnetic field, depend upon two principles: (i) *The iron tends to move from weaker to stronger portions of the field.* This may be understood by considering the special case of a thin soft-iron wire *ns* (Fig. 51), bent so as to coincide with the lines of force due to the magnet SN. The forces acting upon the induced poles in the soft iron will tend to draw the wire into positions where the field is stronger. (ii) *The iron tends to move so that its longest axis coincides with the direction of the lines of force*; for, imagine that the iron consists of a number of iron spheres (Fig. 52), and that the arrow indicates the direction of the lines of force. Each sphere will primarily be magnetised in the direction *sn*, but the position of the poles will be slightly modified by the inductive action of neighbouring spheres, thus resulting in the poles being situated at n_1 and s_1. Each sphere will consequently

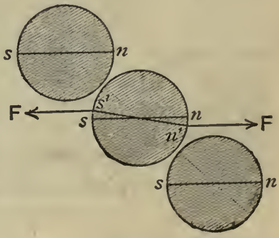

FIG. 52.

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[1] The distribution of lines of force near to iron is analogous to the lines of flow of water in a stream the open channel of which is contracted by growths of weeds or rushes on either bank.

be acted upon by a couple (FF), which will tend to move the iron so that its length coincides with the direction of the magnetic field.

Magnetic Screening.—A point is said to be *magnetically screened* when the magnetic force at that point is reduced or destroyed owing to the presence of soft iron near to it. Thus, a bar of soft iron placed with its axis in the direction of the lines of force of a uniform field *partially* screens points on either side of the iron, except towards the extreme ends. A point situated in line with the axis of a magnet is partially screened by interposing a thick slab of soft iron (Fig. 53); but the screening will be far more effective if the iron is placed near and parallel to the magnet.

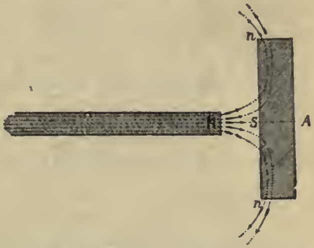


FIG. 53.—A Slab of Soft Iron held near to a Magnet.

Fig. 54 indicates how the space within a soft-iron cylinder is satisfactorily screened from the effects of a neighbouring magnet, and Fig. 55 represents an iron-filing map of a magnetic field in which a soft-iron ring is placed. This method has been adopted by Lord Kelvin in order to protect galvanometers from external magnetic fields when used on ironclads.

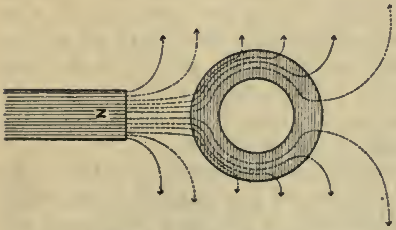


FIG. 54.—A Ring of Soft Iron near to a Magnet.

EXPT. 27.—Support a bar-magnet horizontally on the E. or W. side of a magnetometer, with its axis in line with the centre of the needle. Note the deflection. Interpose a thick slab of soft iron with its face perpendicular to the magnet's axis, and again note the deflection. Repeat the observation with the iron in different positions relatively to the magnet.

Obtain the same results by observing the changes in the rate of swing of a magnetoscope situated at the point where the screening is to be observed.

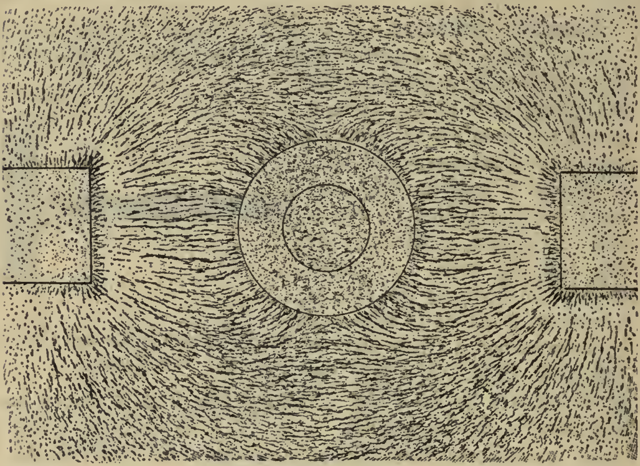


Fig. 55.—A Ring of Soft Iron screens the space within it.

Magnetic Keepers.—Every horse-shoe magnet and every pair of bar-magnets is supplied with short bars of *soft* iron which, when the magnets are not in use, are placed in contact with opposite poles. Such bars are termed *keepers*, since they serve the purpose of aiding the magnet to retain as much as possible its original degree of magnetisation.

Every magnet has a tendency to *de-magnetise* itself, for it may be regarded as a piece of steel with free magnetic poles at each end, which will act inductively on the steel itself and tend to reverse the direction of the magnetisation already existing in the steel (p. 41). This effect is only absent in the case of magnets which have no free poles, as in the case of a steel ring magnetised along its circular axis. The addition of keepers to any magnet serves to make the iron circuit approximately continuous, and the free poles of the magnet are neutralised by the opposite polarity induced in the soft iron, hence the demagnetising effect is reduced to a minimum.

The addition of a keeper largely *screens* the surrounding space from the magnetic lines of force originating from the magnet.

Distribution of Free Magnetism along a Bar-Magnet.—A map of the magnetic field of a bar-magnet suggests that the number of lines of force emerging from a given area of the magnet's surface increases from zero, at the centre of the magnet, to a maximum in the region near to the poles. Several methods have been devised for determining the relative amounts of this so-called 'free' magnetism. In the *test-nail* method a soft-iron nail is allowed to touch various points of the magnet, and the force required to separate the nail from the magnet is measured at each point: it is assumed that the magnetisation set up in the nail is proportional to the amount of free magnetism at the point of contact, and that consequently the force required is proportional to the square of the amount of free magnetism; but this assumption is scarcely permissible, since the magnetisation of the iron does not increase so rapidly as the magnetising force, especially when nearly saturated. Another source of error is due to the probability that the presence of the soft iron will disturb the original distribution of the free magnetism.

The only method free from serious objections is that devised by Prof. Rowland, in which a small flat coil of thin wire embraces the magnet and is moved rapidly between various pairs of points along the magnet. The cutting of the lines of force creates an induced current (Chap. XXII.), the magnitude of which is observed by means of a galvanometer.

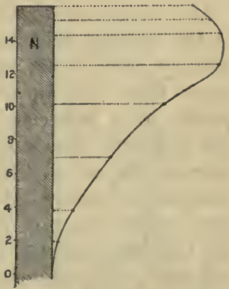


FIG. 56.

Another method, depending upon the rate of swing of a small magnet, is only open to the objection that the presence of even a small magnet will disturb the distribution of the free magnetism on the bar-magnet under examination: Expt. 28 is based upon this method. Fig. 56 represents one-half of a bar-magnet, to the right of which are drawn horizontal lines proportional in length to the square of the number of oscillations described in a given time when the swinging magnet is placed opposite various points of the magnet. It will be noticed that the maximum force is not obtained at the extreme end of the magnet (cf. p. 50).

Expt. 28.—Clamp a long bar-magnet vertically, with its N.-seeking pole downwards. Support a magnetoscope vertically at the same level as the centre of the magnet. Remove the magnet to a distance, and observe the number of oscillations (n) due to the earth alone. Place the magnet as near as possible to the magnetoscope, and due South of it; observe the number of oscillations (n_1). Raise the magnet two centimetres, and again determine n_1. Repeat for various other portions of the magnet. In each position, the free magnetism is proportional to $(n_1^2 - n^2)$.

Distinction between 'Lines of Force,' 'Lines of Magnetisation,' and 'Lines of Induction.'—A rod of soft iron becomes magnetised when placed parallel to the lines of force of a uniform field, and the number of lines of force per sq. cm. of cross-section of the field is increased in the space occupied by the iron, the increase being due to the lines of force added by the molecular magnets constituting the iron bar. The intensity of magnetisation, I, of the soft iron may be defined as the pole-strength per sq. cm. of cross-section placed at right angles to the direction of magnetisation; hence, since $4\pi m$ lines of force proceed from a pole of strength m, the number of lines originating from each such sq. cm. of the iron itself is $4\pi I$.

It is now usual to discriminate between the lines of force due to the original field and those added by the iron, for the name *lines of force* is not appropriate to the lines within the iron, since these do not give a measure of the 'magnetic force' there. It is better to apply the term *lines of magnetisation* to the magnetic lines due to the iron and within the iron, and to restrict the term 'lines of force' to the magnetic lines due to external magnetised bodies. Strictly speaking, this magnetic force within the iron must be taken to include the force due to the free poles created in the iron and of which the magnitude is calculated from the inverse-square law *as though no iron were present.*

Finally, if a narrow transverse gap is cut across the iron, the force acting on a single north-seeking pole placed within the gap would be due partly to the original field and partly to the magnetisation set up in the iron. If the original field has an intensity H, the total force acting on the unit-pole would be $H+4\pi I$, and we may regard the whole group of lines traversing the gap as being partly 'lines of force,' and partly 'lines of magnetisation.' The term *magnetic induction* is applied to the whole group of lines, and is denoted by the symbol B, and we refer to them collectively as **lines of induction.**

Just as the force acting on a unit-pole within a narrow transverse gap gives a measure of B, so the force acting on the pole when placed in a long narrow cylindrical cavity bored through the iron in the direction of magnetisation gives a measure of H, since no free magnetism will then be developed on the sides of the hole.

In the case of soft iron the magnetisation is always parallel to, and in the same direction as, the magnetic force; hence the equation $B=H+4\pi I$ always holds good. But, in the case of a bar-magnet, the magnetisation is merely residual magnetism produced by a magnetic force which has been removed; the value of H is therefore that which arises from the free poles in the magnet itself only, and this is directed *from* the north-seeking pole *to* the south-seeking pole both within the magnet and in the space outside; on the other hand, the lines of induction are directed from the south- to the north-seeking pole within the iron, and are continued by the lines of force in the opposite direction outside the magnet. Hence, if the magnetic force is restricted to that due to the poles of the magnet, the induction within the magnet is represented by the equation $B=4\pi I-H$. This again introduces the principle of the self-demagnetising force of a permanent magnet.

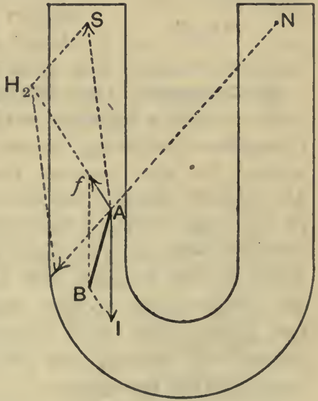


FIG. 57.

In the case of a horse-shoe magnet (Fig. 57), at any point A the lines of magnetisation will be in the direction AI, and will be numerically equal to $4\pi I$. The magnetic force at A is due to the poles N. and S., and by the law of inverse squares will be in the direction AH_2; this force will produce a magnetic induction B_2 in the same direction, and equal to $4\pi I_2 + H_2$, where I_2 is the intensity of magnetisation in the direction AH_2. The resultant induction is obtained by combining B_2 and $4\pi I_1$; thus, if Af represents the magnitude of B_2, the resultant induction will be AB.

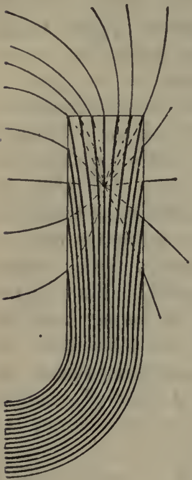


FIG. 58.

The distinction between 'lines of force' and 'lines of induction' within a horse-shoe magnet is indicated in Fig. 58; the dotted lines are lines of force, and the thick continuous lines are lines of induction. In the surrounding air lines of induction and lines of force coincide, and a line of induction can therefore only leave the iron where a line of force within the iron emerges into the air. It is assumed in Fig. 58 that the pole of a magnet may be regarded as a point; also, it is necessary to mention that the diagram is not in any sense intended to be quantitative.

Paramagnetic and Diamagnetic Substances. — Faraday observed that a fragment of bismuth, when suspended between the poles of a strong electro-magnet, is feebly repelled outwards; he also observed that a bar of bismuth, suspended horizontally between the poles, tends to set itself equatorially to the lines of force between the poles. In these respects bismuth behaves in an exactly opposite manner to iron. Provided that the magnetic field is sufficiently strong, *all* substances are found to behave either like iron or like bismuth. To these two groups the terms *Paramagnetic* and *Diamagnetic* are respectively applied, and the following are the principal substances in the two groups:

Paramagnetic.—Iron, Nickel, Cobalt, Manganese, Platinum.

Diamagnetic.—Bismuth, Antimony, Zinc, Silver, Copper.

Diamagnetism is fundamentally due to the fact that the polarity created in the diamagnetic metal is the reverse of that created in a paramagnetic metal. Applying this fact to Fig. 51, it will be seen that a diamagnetic substance tends to travel from stronger into weaker magnetic fields. The tendency of a bar of diamagnetic material to set equatorially is due to the same cause, and the tendency is therefore not evident when the field is uniform—in fact, the poles of the electro-magnet should be conical, so as to concentrate the field towards the centre as much as possible.

All diamagnetic phenomena are extremely feeble as compared with those exhibited by paramagnetic substances.

SUMMARY

Magnetic Induction.—The temporary magnetisation set up in a piece of soft iron held near to a magnet is termed *induced* magnetism. The total magnetisation set up in the presence of more than one magnet is the sum of the effects due to each individual magnet.

Induced magnetisation may be superimposed upon permanent magnetisation in steel. The repulsion between like poles of two magnets is therefore slightly weaker than the attraction between unlike poles of the same initial strength. Also, a bar-magnet tends to demagnetise itself.

Maps of Magnetic Fields.—A diagram consisting of lines which indicate the direction of the magnetic forces at all points of a magnetic field is termed a *map* of the field. The direction of the forces is arbitrarily chosen as that in which a *north*-seeking pole tends to travel. Maps may be constructed by means either of (i) a compass-needle or (ii) iron filings.

In any general case the map of the field of a magnet is really the *resultant* field due to that of the magnet and that of the earth.

The Poles of a Magnet may be defined as *the points of application of the resultant forces of attraction and repulsion which the magnet exerts on any magnet pole near to it.*

If a magnetic field of unit intensity is defined as the intensity at a distance of 1 cm. from a single unit pole, then the number of lines of force originating from a pole of strength m is $4\pi m$.

The **Magnetic Potential** at any point is equal to the work done against the magnetic force when a unit north-seeking pole is conveyed from an infinite distance up to the point. The potential at a point distant r cms. from a pole of strength m is m/r. A surface passing through points for which the value of m/r is the same is termed an *equipotential surface.*

The rate of change of the magnetic potential per unit distance is equal to the intensity of the field.

Molecular Magnets.—The individual molecules of a piece of steel may be regarded as complete magnets. The process of magnetisation consists in rotating the molecules into alignment with similar poles pointing in the same direction. The same theory explains the magnetisation of a piece of soft iron when placed in a magnetic field.

The movements of soft iron in a magnetic field depend upon two principles: (i) *Iron tends to move from weaker to stronger fields*, and (ii) *so that its longest axis coincides with the direction of the lines of force.*

A point is *magnetically screened* when the magnetic force at that point is reduced or destroyed owing to the presence of soft iron near to it.

Magnetic keepers attached to the poles of a magnet tend to mask the poles of the magnet, and therefore to remove the de-magnetising effect.

The **distribution of free magnetism** along a bar-magnet may be investigated by either of three methods: (i) 'Test-nail' method; (ii) Rowland's induced-current method; (iii) by rate of swing of suspended magnet.

Magnetic substances may be divided into two groups—*paramagnetic* and *diamagnetic.* Iron is a typical example of the former group, while bismuth and antimony are the chief examples of diamagnetic metals. A diamagnetic metal tends to travel from stronger to weaker parts of a magnetic field.

QUESTIONS ON CHAPTER IV

1. A bar-magnet is placed on a table. Describe how you would proceed to determine the directions of the lines of magnetic force in the plane of the table due to the magnet and to the earth's magnetism.

2. A bar-magnet is laid on a sheet of paper on a drawing board. Supposing it to have its poles concentrated at two given points, how would you determine by measurement and calculation the direction of the force at a given point on the paper? and, given a compass-needle, how would you test the result?

3. A N.-seeking and S.-seeking pole, of strengths 100 and 50 respectively, are situated 10 cms. apart. Find the point between them at which the potential is zero. Determine a few other points in the field at which the potential is also zero.

4. A bar-magnet 15 cms. long and of pole strength 50 is in a given position. Find the magnetic potential due to this magnet at a point distant 5 cms. from its N.-seeking pole and in line with the magnet's axis.

5. In what way can the distribution of magnetism in a magnet be experimentally investigated?

6. What effect has a magnet on a piece of soft iron in its neighbourhood? What conditions determine the direction of the mechanical force acting on a small piece of iron? Does the direction of this force always coincide with that which would act on a magnet pole if it occupied the same position as the soft iron?

7. A plate of iron of uniform thickness, but broader at one end than at the other, is magnetised by drawing from the narrow to the broad end a magnet which is itself wider than the broad end. Describe the magnetic state of the plate, and explain how it exhibits the two opposite magnetic properties in an equal degree.

8. Two magnets are placed horizontally on a large sheet of white paper. You are supplied with a small compass, a pencil, and a watch. Assuming that the earth's field may be neglected, how would you trace the lines of force due to the magnets and determine points at which the intensity of the magnetic field was equal?

9. A magnet is placed horizontally in the magnetic meridian due south of a compass needle. How will its action on the latter be affected if (i) a plate of soft iron is interposed between the two, (ii) a rod of soft iron is placed along the line which joins their centres? Give reasons.

10. Two circular rings of iron are magnetised, the first by being placed between the poles of a strong horse-shoe magnet, so that the line joining the poles of the magnet is a diameter of the ring, the second by having one pole of a bar-magnet drawn round it several times. Describe the magnetic state of each ring.

11. A magnet is placed so that its centre is due magnetic east of the centre of a compass needle, which is so mounted that it cannot dip. The magnet is made to rotate in a vertical plane about a horizontal axis which lies east and west, and passes through its centre. Describe the behaviour of the compass needle during one complete revolution of the magnet.

12. A vertical magnet (the north pole of which is the lower) is moved slowly from north to south in the magnetic meridian which passes through the centre of a compass needle. The magnet passes over the compass, being always at the same height above it, but is not near enough to reverse the direction in which the needle points. What will be the effect on the time of vibration of the compass needle when the magnet occupies different positions?

13. A short bar-magnet is placed at the centre of a circle 3 feet in diameter, its axis being in the magnetic meridian. Trace the changes in the direction in which a compass needle points as it is carried round the circumference of the circle.

14. Describe the changes that are supposed to take place in the molecules of a steel bar when it is magnetised. How would you propose to determine whether this had any effect on the size of the bar?

CHAPTER V

MOMENTS OF MAGNETS

Couple acting on a Magnet swinging horizontally in a uniform Field.—Let NS (Fig. 59) represent a magnet, of length $2l$ and pole-strength m, suspended at O in a uniform field of intensity H, the direction of which is indicated by the line XX. Equal and opposite forces, mH, act on the poles of the magnet, and therefore constitute a *couple*.

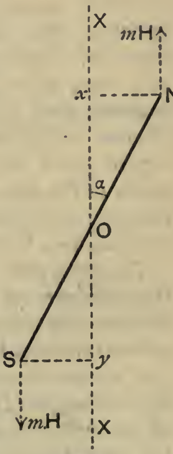


FIG. 59.

The moment of force, acting at N,

$$= mH \times Nx = mH \times l \sin \alpha.$$

The moment of force, acting at S,

$$= mH \times Sy = mH \times l \sin \alpha.$$

Hence, the total moment of the couple

$$= 2mH \times l \sin \alpha.$$

If the axis of the magnet is perpendicular to the lines of force (so that $\alpha = 90°$), and if $H = 1$, then the total moment of the couple $= 2ml$ (*i.e.* the strength of one pole × the length); this product is termed the **Moment of the Magnet**, which may be defined thus:

The moment of a magnet is equal to the moment of the mechanical couple which must be applied to a magnet in order to retain it in a position perpendicular to the lines of force of a field of unit intensity.

Intensity of Magnetisation.—By the previous section, the moment of a magnet is $2ml$. If two such magnets are fixed together end-to-end, then the total magnetic moment $=4ml$, and so on; hence *the total moment is equal to the sum of the moments of the constituent magnets.*

If the length of any magnet is L cms., and its cross-section is s sq. cms., we may regard it as being built up of Ls separate magnets, each 1 cm. long and 1 sq. cm. cross-section. The moment of each of these constituent magnets would be

$$\frac{\text{Total Moment}}{\text{Volume}}=\frac{Lm}{Ls}=\frac{m}{s}=\frac{\text{Pole-strength}}{\text{Cross-section}}.$$

This ratio is termed the **Intensity of Magnetisation**, which may be defined in either of two ways: (i) **The Total Moment divided by the Volume**, or (ii) **The Pole-strength divided by the Cross-section.**

The Moment of a Magnetic Shell.—If a thin sheet of steel or iron is magnetised so that opposite polarities are distributed uniformly over the flat faces, the distribution is then said to be *lamellar*, as compared with the *solenoidal* distribution obtained in a long thin wire. Such a magnetised sheet is termed a **magnetic shell**, and it may be regarded as being constituted of a number of very short magnetised bars placed side by side with similar poles pointing in the same direction. It is usual to speak of the pole-strength distributed on either face as the total *quantity* of magnetism on the face of the shell, and the quantity per unit area is termed the *density* of the magnetisation.

The *Magnetic Moment* of a shell is equal to the sum of all the moments of the various portions into which it may be divided. If the shell is divided into *unit* areas, then each constituent magnet has a length equal to the thickness of the shell and a cross-section of 1 sq. cm. *The moment of each such constituent magnet is termed the* **strength** *of the shell.* If $\phi=$ strength of the shell, $\sigma=$ surface density of magnetisation, $t=$ thickness of the shell, and A = area of shell, then

$$\phi=\sigma\times t$$

$$\text{and, Moment of the Shell}=A\times\sigma\times t.$$

Potential due to a Magnetic Shell.—Fig. 60 represents a small portion of a Magnetic Shell, of area A and thickness t.

P is a point situated at distances d_1 and d_2 from the centres of the faces of the shell. If ω is the solid angle subtended by the face of the shell at P, then (p. 563)

$$\frac{A\cos\theta}{\omega}=d_1^2$$

or

$$\omega=\frac{A\cos\theta}{d_1^2}. \quad \ldots(1)$$

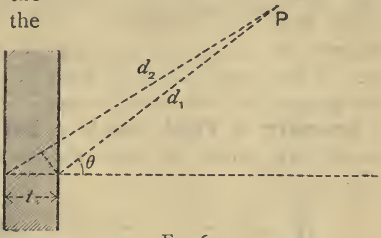


FIG. 60.

If Q = the quantity of magnetism on each face of the shell, and if the face nearer to P has N-seeking polarity, then (p. 54)

Potential at P due to the near face $=\frac{Q}{d_1}$

,, ,, ,, distant ,, $=-\frac{Q}{d_2}$

$$\text{Resultant Potential}=\frac{Q}{d_1}-\frac{Q}{d_2}=\frac{Q(d_2-d_1)}{d_1d_2}.$$

But

$\frac{d_2-d_1}{t}=\cos\theta$ (approximately), and $d_1d_2=d_1^2$ (approx.) $=\frac{A\cos\theta}{\omega}$ (by equation 1).

Hence

$$\text{Resultant Potential}=\frac{Q\times t\cos\theta}{A\cos\theta}\times\omega=\frac{Q}{A}\times t\times\omega=\sigma\times t\times\omega.$$

By the previous section, the strength ϕ of the shell $=\sigma t$.

Hence, the resultant potential $=\phi\omega$; or, in words, **the potential at any point due to a magnetic shell is equal to the product of the strength of the shell and the solid angle subtended at the point by the boundary of the shell.**[1]

---

[1] It will be explained in a subsequent chapter that the lines of force and equipotential surfaces surrounding a magnetic shell are identical in character with those surrounding a wire (of the same contour) conveying a current. Such a wire may therefore be said to have its *equivalent magnetic shell*, the strength of the current being adjusted so that the magnetic fields are identical. If i = strength of current, then the potential at any distant point may be written ωi, where $\omega\phi=\omega i$, or $A/d^2\times i=A/d^2\times\phi$, or $A\times i=A\times\phi$. The strength of the current may therefore be measured in terms of the strength of the equivalent magnetic shell. This forms the basis of one method (*Electromagnetic*) of measuring electric currents.

If P is situated very near to the N-seeking face of the shell, then $\omega = 2\pi$, and the potential $= 2\pi\phi$. If situated near to the other face, the potential $= -2\pi\phi$. Consequently, if a unit magnet pole is moved from the N.-seeking to the S.-seeking face of the shell, the work done (*i.e.* the change in potential) will be equal to $4\pi\phi$ units of work.

Intensity of Field, due to a Bar-Magnet.—(i) *At a point on the axis of the bar-magnet produced.* Let NS (Fig. 61, i) represent a bar-magnet of pole-strength m and length $2l$. Let A be a point on the axis of the magnet produced, and distant d cms. from the centre of the magnet. The intensity of the field at A, due to the pole S, is $m/(d-l)^2$; and that due to the pole N is $-m/(d+l)^2$; hence the **resultant intensity** at A is

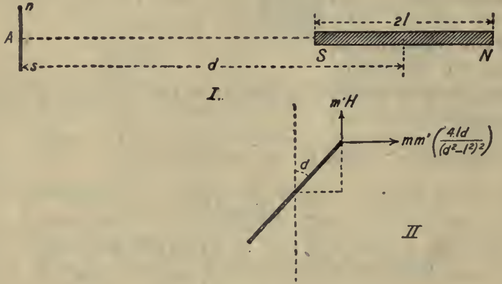


FIG. 61.

$$m/(d-l)^2 - m/(d+l)^2 = m \,.\, 4ld/(d^2-l^2)^2$$
$$= 2\text{M}d/(d^2-l^2)^2. \quad \dots\dots\dots\dots\dots (1)$$

If l is very small relatively to d, then this may be written

$$2\text{M}/d^3, \text{ approximately.} \quad \dots\dots\dots\dots\dots\dots (2)$$

(ii) *At a point on the line drawn through the centre of the magnet, and at right angles to its axis.* The intensity of the field at A (Fig. 61 (B)), due to the pole N, is m/NA^2, and in the direction Ap; and the intensity at A, due to S, is m/SA^2, and in the direction Aq.

Since NA = SA, these intensities are equal in magnitude. If each is resolved into two components along, and at right angles to, OA, the components along OA neutralise each other, and the resultant intensity is represented in magnitude and direction by $2An$.

Since the triangles pAn and ANO are similar,

$$An/Ap = NO/NA,$$

$$\text{or}\quad An = Ap \,.\, NO/NA$$

$$= m \,.\, NO/NA^3$$

$$= ml/(d^2+l^2)^{\frac{3}{2}}.$$

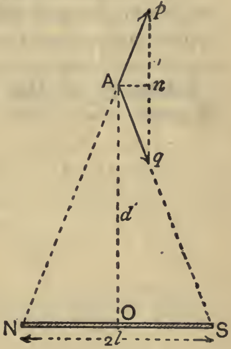


Fig. 61 (B).

Hence, the resultant intensity ($2An$) is equal to

$$2ml/(d^2+l^2)^{\frac{3}{2}}$$

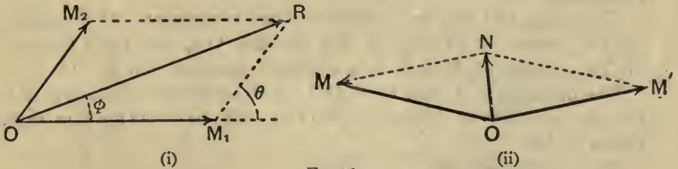
$$= M/(d^2+l^2)^{\frac{3}{2}}. \quad\text{..........(3)}$$

If l is very small relatively to d, then this may be written

$$M/d^3. \quad\text{..................(4)}$$

It is evident that the intensity of the field at any point is not altered when another magnet is substituted, providing that the centres of the magnets are at the same point and that the moments are numerically equal. Thus the intensity due to a magnet of pole-strength 100 and length 4 cms. is the same as that due to a magnet of pole-strength 400 and length 1 cm.

Composition and Resolution of Magnetic Moments.—It has been proved experimentally that the moments of two or more neighbouring magnets may be resolved or combined in the same manner as in the case of two or more forces; for the moment of a magnet may be represented in magnitude and direction by a line, and it is therefore a *vector* quantity. In Fig. 62 (i) let OM_1 and OM_2 represent the moments of two neighbouring magnets; the distant field



(i) (ii)

Fig. 62.

is the same as that due to a single magnet the moment of which is represented in magnitude and direction by OR. If θ is the angle between the components, the direction of the resultant moment is given by the expression $\tan\phi = M_2 \sin\theta/(M_1 + M_2 \cos\theta)$.

If the magnets of an astatic pair are not in the same vertical plane (Fig. 62 (ii)), it is evident that the system will come to rest with the magnets approximately at right angles to the magnetic meridian, the combination being equivalent to a single magnet of moment ON.

Intensity of the Field at any Point, due to a Small Magnet.—Let A (Fig. 63) be a point which is d cms. from the centre of a small magnet NS, of moment M; and let the angle AON be represented by θ.

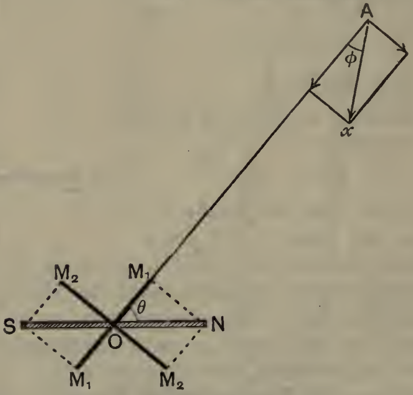


FIG. 63.

The moment M may be resolved into two components—(i) a moment M_1OM_1, equal to $M\cos\theta$, in the direction OA, and (ii) a moment M_2OM_2, equal to $M\sin\theta$, in a direction perpendicular to OA.

The intensity at A, due to M_1OM_1, will be $2M\cos\theta/d^3$, along AO, and the intensity at A, due to M_2OM_2, will be $M\sin\theta/d^3$, at right angles to AO.

The resultant intensity (Ax) will be

$$\sqrt{\left(\frac{2M\cos\theta}{d^3}\right)^2 + \left(\frac{M\sin\theta}{d^3}\right)^2} = \frac{M}{d^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{M}{d^3}\sqrt{3\cos^2\theta + 1}.$$

Also, if ϕ be the angle between OA and the direction of the resultant intensity, then $\tan\phi = \frac{M\sin\theta}{d^3} \Big/ \frac{2M\cos\theta}{d^3} = \frac{1}{2}\tan\theta$.

Couple acting on a Magnet, due to another Magnet.—(i) **The Tangent (A) Position of Gauss.** In Fig. 61 (i) let NS be a bar-magnet with its axis perpendicular to the magnetic meridian, and *ns* a small magnet suspended at a point on the axis of NS produced. Let m' and 2λ be the pole-strength and length of *ns*, M the moment of NS, and H the horizontal intensity of the earth's field.

The magnet *ns* will be deflected into a position such that the moment of the couple due to the magnet is equal and opposite to that due to the earth's field. Hence, from equation (1) (p. 78),

$$\frac{2Md \, . \, m'}{(d^2 - l^2)^2} \, . \, 2\lambda \cos\alpha = m'H \, . \, 2\lambda \sin\alpha,$$

or $\quad M/H = (d^2 - l^2)^2 \, . \, \tan\alpha / 2d = (d^3 \, . \, \tan\alpha)/2,$ approximately.

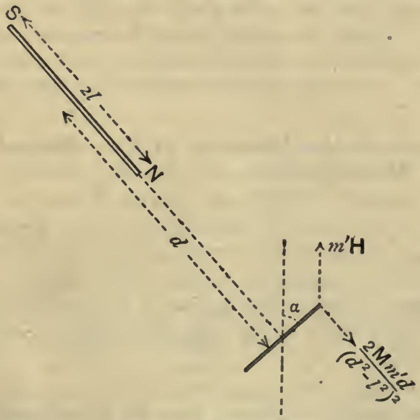


FIG. 64.—The Sine Position.

(ii) **The Tangent (B) Position of Gauss.** In this case the suspended magnet is due north (or south) of the centre of a bar-magnet the axis of which is perpendicular to the meridian.

From equation (3) (p. 79), the deflection α is such that

$$\frac{Mm'}{(d^2+l^2)^{\frac{3}{2}}} \,.\, 2\lambda \cos \alpha = m'H \,.\, 2\lambda \sin \alpha,$$

or
$$M/H = (d^2+l^2)^{\frac{3}{2}} \tan \alpha = d^3 \,.\, \tan \alpha, \text{ approximately.}$$

(iii) **The Sine Position.** If the deflecting magnet is placed in the *tangent* (*A*) *position*, and the whole instrument rotated until the axis of the deflecting magnet is *perpendicular* to that of the suspended magnet (Fig. 64), then

$$\frac{2Md \,.\, m'}{(d^2-l^2)^2} \,.\, 2\lambda = m'H \,.\, 2\lambda \sin \alpha,$$

or
$$M/H = \{(d^2+l^2)^2 \,.\, \sin \alpha\}/2d.$$

In this position, therefore, the ratio M/H is proportional to the *sine* of the angle of deflection.

Expt. 29.—Place the deflecting magnet in position, and rotate the instrument until the pointer attached to the needle coincides with the zero of the scale. Remove the magnet; the needle will now point north and south, and the reading of the pointer will give the angle of deflection. Repeat the readings with the magnet reversed; and also take a similar series with the magnet on the other side of the magnetometer. From the *average* deflection calculate the value of M/H.

Comparison of the Moments of Two Magnets by the Deflection Magnetometer.—In the experiments described below, the Tangent (A) Position of Gauss is used in preference to the (B) Position since the adjustments in the former position can be carried out more accurately, and the deflection obtained with any given magnet is greater. The comparison of the moments requires the determination of the ratio M/H for each magnet, and the ratio of the values obtained gives the ratio of the moments.

Expt. 30.—Adjust the magnetometer so that its arms are perpendicular to the magnetic meridian. Place the bar-magnet (marked A) on the right-hand arm of the magnetometer, and with its centre 15 cms. away from the middle. Note the mean scale-reading of the two ends of the pointer. Reverse the magnet, and again note the deflection. Repeat these observations when the magnet is on the left-hand arm of the magnetometer.

Calculate the mean value of the ratio M_A/H from the equation $M/H = (d^2 - l^2)^2 \tan a/2d$. Repeat the observations, using the magnet marked B. Calculate the ratio M_A/M_B.

EXPT. 31.—The moments of the magnets A and B may also be compared in the following manner: Place the two magnets on opposite sides of the magnetometer so that they tend simultaneously to deflect the needle in opposite directions. Alter the distances d_1 and d_2 until the deflection is zero. Then

$$(M_A \times 2d_1)/(d_1^2 - l_1^2)^2 = (M_B \times 2d_2)/(d_2^2 - l_2^2)^2,$$

or

$$M_A/M_B = (d_1^2 - l_1^2)^2 d_2/(d_2^2 - l_2^2)d_1.$$

The Magnetic Length of a Bar-Magnet.—In the experiments of the previous paragraph it is assumed that the length $2l$ is equal to the length of the steel bar of which the magnet consists. But, as a general rule, the poles do *not* coincide with the extreme ends of the bar. The distance between the poles, or *the magnetic length*, of the magnet may be approximately determined by taking two sets of observations, similar to Expt. 30, with the magnet at different distances from the magnetometer needle. If the distances are d_1 and d_2, and the corresponding deflections are α_1 and α_2, then

$$(d_1^2 - l^2)^2 . \tan \alpha_1 / 2d_1 = (d_2^2 - l^2)^2 . \tan \alpha_2 / 2d_2.$$

From this equation, neglecting higher powers of l,

$$l^2 = \frac{d_1^3 \tan \alpha_1 - d_2^3 \tan \alpha_2}{2(d_1 \tan \alpha_1 - d_2 \tan \alpha_2)},$$

whence the value of $2l$ is directly obtained.

With ordinary bar-magnets the ratio of the magnetic length to the length of the steel bar is about 0.85.

Mutual Force between Two Bar-Magnets, with their Axes along the same Straight Line.—Let N_1S_1 and N_2S_2 (Fig. 65) be two bar-magnets, of moments M_1 and M_2 respectively, and placed with their axes in the same straight line, and with their centres d cms. apart. Also, let $2l_1$ and $2l_2$ be their respective lengths, and m the pole-strength of N_2S_2.

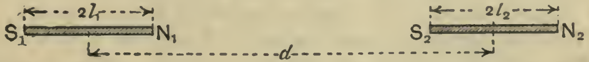


FIG. 65.

The intensity of the field due to N_1S_1, at the middle point of N_2S_2, is $2M_1/d^3$; and

the force, acting on S_2, is $2M_1m/(d-l_2)^3$,
and „ „ N_2, is $2M_1m/(d+l_2)^3$.

Hence, the resultant force $=2M_1m\{(d-l_2)^{-3}-(d+l_2)^{-3}\}$

$$=2M_1m\{(6d^2l_2+2l_2^3)/(d^2-l_2^2)^3\}$$
$$=M_1M_2\times 6d^2/d^6 \text{ (approximately)}$$
$$=6M_1M_2/d^4.$$

The Period of Oscillation of a Bar-Magnet.—Fig. 66 represents a bar-magnet NS suspended horizontally at its centre. If m = the pole-strength, H = the horizontal intensity of the magnetic field, and $2l$ = the length of the magnet, then the force acting on each pole of the magnet is mH. If θ is the angle between the magnet's axis and the meridian, this force may be resolved into two components, $mH \sin\theta$ and $mH\cos\theta$, only the former of which is effective in tending to produce movement round O. If θ is very small and is expressed in circular measure, then $\sin\theta=\theta=AN/l$, and the effective force acting on the pole $=mH\,.\,AN/l$. Hence the force, and consequently the acceleration, is proportional to the displacement AN, and the magnet moves with *Simple Harmonic Motion* (p. 24).

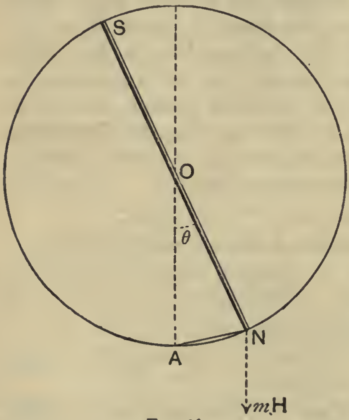


FIG. 66.

The mode of vibration of a magnet in a uniform field is quite analogous to that of a simple pendulum (p. 25), and the formula for the time of vibration may be directly derived from that for the simple pendulum, viz. $T=2\pi\sqrt{I/mlg}$. The product mg will be replaced by an attraction of one pole $+mH$ and a repulsion $-mH$ of the other, or by a total force $2mH$. Also, the magnet swings like a couple of simultaneously oscillating pendulums, each of length l cms. Hence

$$T = 2\pi\sqrt{\frac{I}{2mHl}} = 2\pi\sqrt{\frac{I}{MH}}.$$

Comparison of the Moments of Two Magnets by the Method of Vibration.—If a magnet is swinging freely, the time of one complete oscillation is given by the formula

$$t = 2\pi\sqrt{\frac{I}{MH}},$$

or $$MH = \frac{4\pi^2 I}{t^2}$$

(where I = the Moment of Inertia, which can be calculated from the weight and dimensions of the magnet).[1]

The moments of two magnets may be compared by determining the ratio of the value of MH determined separately for each magnet.

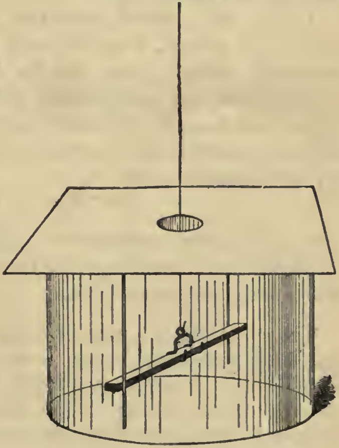


FIG. 67.

EXPT. 32.—Carefully measure the dimensions and weights of the two magnets (A and B), and calculate their Moments of Inertia (I_A and I_B). In order to determine the period of oscillation, suspend the magnet horizontally, by means of unspun silk, within a glass vessel (Fig. 67) or box with glass sides, and mark on the glass vertical lines which are in line with the centres of the pole faces of the magnet. Set the magnet swinging through a *small* angle, determine the time required for 50 oscillations, and calculate the times occupied by one oscillation (t_1 and t_2).

Then $$M_A H = \frac{4\pi^2 I_A}{t_1^2}, \quad \text{and} \quad M_B H = \frac{4\pi^2 I_B}{t_2^2}.$$

Hence $$\frac{M_A}{M_B} = \frac{I_A}{t_1^2} \times \frac{t_2^2}{I_B}.$$

---

[1] For a cylindrical magnet, $I = \left(\frac{(\text{length})^2}{12} + \frac{(\text{radius})^2}{4}\right) \times \text{mass}.$

" rectangular " $I = \left(\frac{(\text{length})^2 + (\text{breadth})^2}{12}\right) \times \text{mass}.$

(The measurements must be in C.G.S. units.)

The Influence of Temperature on the Moment of a Magnet.—The moment of a magnet gradually diminishes as the temperature increases, and increases again, but at a smaller rate, when the temperature is lowered; consequently the moment is lowered permanently if an ordinary magnet is subjected to changes in temperature.

It has been found, however, if a magnet is heated for a prolonged period, then strongly magnetised, and the process repeated several times (p. 8) that the moment is rendered constant for any temperature within the range to which it was previously heated. Within this range the moment at any given temperature may be expressed in terms of the moment at 0° C by the simple formula:

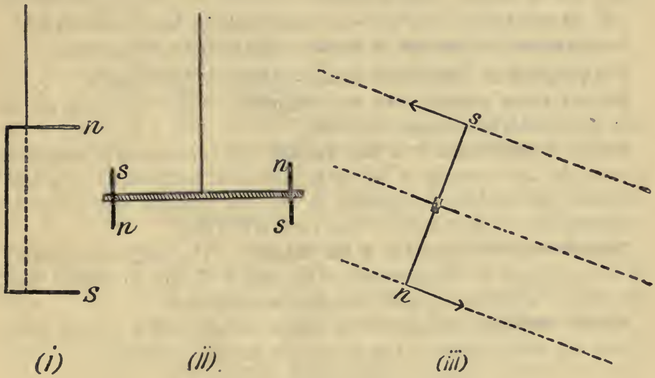
$$M_t = M_0(1 - \alpha t),$$

where α is the *temperature coefficient*: the value of α seldom exceeds 0.001, providing that the magnet has been prepared by the method described above.

Expt. 33.—Place a magnetometer with its arms north and south, and raise it until the needle is at the same horizontal level as the magnet, which is contained in a dish of oil supported on a copper tripod immediately to the east of the magnetometer needle. Support the magnet so that its axis lies east and west. Clamp a thermometer so that its bulb is immersed in the oil. Take a series of simultaneous readings of the deflection and the temperature, both when the oil is being heated and when it is cooling.

Astatic Needles.—A magnetic needle, or system of needles, is astatic when, if suspended so as to turn freely round its point of support, it does not tend to come to rest in any one definite position. Nobili's pattern (Fig. 197), the most frequent type of astatic needle, consists of two similar steel needles, magnetised to an equal degree, and rigidly connected with similar poles pointing in opposite directions. As a general rule the *moments* of the magnets are not exactly equal: if the moments are m and m', then, if the system is deflected θ degrees from the meridian, the turning couples (acting in opposite directions) are $m\text{H} \sin \theta$ and $m'\text{H} \sin \theta$, and the resultant turning couple is $(m - m')\text{H} \sin \theta$. The astatic system will therefore behave like a single magnet of small moment, $m - m'$, and the pair will come to rest in the meridian in obedience to the stronger magnet. This result only holds good if the magnets are in the same vertical plane.

Fig. 68 represents three other forms of astatic needles. Fig. 68 (i) consists of a magnetised needle bent so as to form three sides of a rectangle. Fig. 68 (ii) is due to Prof. S. P. Thompson; and Fig. 68 (iii), which consists of a needle supported on a rigid axle which coincides with the direction of the magnetic lines of force, is due to Ampère.



(i) (ii) (iii)

Fig. 68.—Types of Astatic Needles.

SUMMARY.

The **Moment** of a magnet is equal to *the moment of the mechanical couple which must be applied to a magnet in order to retain it in a position perpendicular to the lines of force of a field of unit intensity.* The moment of the couple required to produce a deflection α in a field of intensity H is HM sin α.

The **Intensity of Magnetisation** may be defined in two ways: (i) The magnetic moment per unit volume; or (ii) the pole-strength per unit cross-section.

Magnetic Shell.—The *strength* of a magnetic shell is the moment per unit cross-section. The *magnetic moment* of the entire shell is equal to the product: area × strength of shell.

The *potential* at any point due to a magnetic shell is equal to the product of the strength of the shell and the solid angle subtended at the point by the boundary of the shell.

The Intensity of the Field, due to a Bar-Magnet. (i) *At a point on the axis produced.*—The intensity at a point distant d cms. from a magnet of moment M and length $2l$ is equal to $2Md/(d^2-l^2)^2$.

(ii) *At a point on the equatorial.*—The intensity at a point distant d cms. from a magnet is $M/(d^2+l^2)^{\frac{3}{2}}$.

(iii) *At any point.*—The intensity at any point is $M\sqrt{3\cos^2\theta+1}/d^3$.

The Tangent (A) Position of Gauss.—$M/H=(d^2-l^2)^2\tan\alpha/2d$.

The Tangent (B) Position of Gauss.—$M/H=(d^2+l^2)^{\frac{3}{2}}\tan\alpha$.

Mutual Force between two Bar-Magnets.—When their axes are in line, force $=6M_1M_2/d^4$, approximately.

Period of Oscillation of a Bar-Magnet.—A bar-magnet of magnetic moment M and moment of inertia I, suspended horizontally in a field of which the horizontal intensity is H, will describe one complete oscillation in a period t secs., where $t=2\pi\sqrt{I/MH}$.

Temperature Coefficient of a Bar-Magnet.—The magnetic moments of a bar-magnet at temperatures $t°$ C. and 0° C. are so related that $M_t=M_0(1-\alpha t)$, where α is the *temperature-coefficient*.

Astatic Needles.—A magnet, or system of magnets, is *astatic* when it does not tend to come to rest in any one definite position.

QUESTIONS ON CHAPTER V

1. A short horizontal bar-magnet is movable so as always at its middle point to touch a horizontal circle, at the centre of which is a compass-needle. Determine the positions of the magnet in which the needle's deflection is the largest.

2. Two small magnets, each of length l and moment M, are placed with their axes in line and their centres at a distance r apart. Find the force of attraction or repulsion between them.

3. What is the *magnetic moment* of a magnet? Show how the magnetic moments of two magnets may be compared by the use of the torsion balance.

4. A suspended magnet is deflected (i) through an angle of 45°, (ii) through an angle of 60°. Compare the moments of the couples tending to bring the needle to its original position.

5. A magnet 10 cms. long is suspended in a magnetic field of which $H=0.18$. If the strength of each pole is 10 units, find the moment of the couple required to deflect the magnet (i) through 30°, (ii) through 90°.

6. The moment of a magnet is 1000 in C.G.S. units. How much work is done in turning it through 90° from the magnetic meridian in a

horizontal plane at a place where the horizontal intensity is 0.16 C.G.S. units?

7. What data are necessary in order that the moments of two magnets may be compared by observation of their times of oscillation in a constant magnetic field? Describe the experiment.

8. A bar-magnet is suspended by a wire so as to hang horizontally. By how much must the top end of the wire be twisted for the magnet to be deflected 90° from the magnetic meridian, when for a deflection of 30° it has to be twisted through 120°?

9. Being given a small compass and a bar-magnet, and knowing the horizontal intensity of the earth's magnetic field, how would you determine the moment of the bar-magnet?

10. A bar-magnet is suspended horizontally in the magnetic meridian by a wire without torsion. To deflect the bar 10° from the meridian the top of the wire has to be turned through 180°. The bar is removed, remagnetised, and restored, and the top of the wire has now to be turned through 250° to deflect the bar as much as before. Compare the magnetic moments of the bar before and after remagnetisation.

11. Two equal and equally magnetised bar-magnets are fastened together so as to cross each other at right angles at their centres, and the cross can turn freely about a vertical line through its centre; find the direction in which it will set in the earth's magnetic field.

12. Two pieces of steel, of which one is a little longer than the other, are magnetised so that when attached by their centres to a piece of wire and suspended so that both are horizontal and in the same vertical plane, the combination is astatic. Will it be astatic if it be suspended by its centre of gravity so that the magnets are in a horizontal plane? Give reasons.

13. Show how the magnetic moments of two unequally strong magnets may be compared by mounting them in the manner of astatic needles, (i) with like (ii) with unlike poles together, and observing their oscillations when so mounted.

14. The magnetic moment of a *short* magnet is 2000 C.G.S. units. Find the intensity of its field at a point on its axis produced and 20 cms. from its centre. Find the intensity if the magnet is 10 cms. long.

15. Two magnets, the moment of one of which is double that of the other, are rigidly connected, with the centre and axis of the one vertically above the centre and axis of the other, and the whole is suspended by a fine metallic wire. When the magnets are oppositely directed the top

end of the wire has to be twisted through 75° in order to deflect the combination through 30° from the magnetic meridian. By how much must the top end of the wire be twisted when the magnets are similarly directed in order that the combination may take up a position perpendicular to the magnetic meridian?

16. A small compass-needle is pivoted on a point fixed to a horizontal table which is movable about a vertical axis. A bar-magnet is now laid on the table with its centre in, and its length perpendicular to, the magnetic axis of the needle in its undeflected position, and the needle is thereby deflected. It is then found that the table must be turned through 30° for the needle to point to the centre of the bar. Show that the magnetic moment of the magnet is $\frac{1}{2}HD^3$, where H is the horizontal component of the earth's magnetic intensity and D is the distance of a pole of the magnet from the needle's centre.

17. Show that the force exerted by a bar-magnet, of magnetic moment M, on a unit pole placed at any point in the plane which bisects the magnet at right angles is Mr^{-3}, where r is the distance of the point from either pole, it being assumed that the magnetic strength of the magnet is concentrated at its poles. How may this result be applied in the comparison of the horizontal component of the earth's magnetic field at different points?

18. Prove that the magnetic force exerted by a short magnet at a point A on the line passing through its centre and perpendicular to its axis, is the same as the force exerted at a point on the axis, the distance of which from the centre of the magnet is $\sqrt[3]{2}$ times the distance of A from the centre.

19. The centres of two short magnets AB and CD are at a distance r apart. AB lies along the line joining their centres and CD is at right angles to it. Show that the couple due to AB tending to make CD twist round is $2MM'/r^3$, where M and M′ are the magnetic moments of the magnets.

20. Two thin short bar-magnets, whose magnetic moments are as 1 to 2, are placed horizontally with their centres together and their axes perpendicular. Show that a compass-needle, placed with its centre at any fairly large distance along the axis of the former magnet, would take up a position equally inclined to both magnets if the earth's influence upon it were compensated.

21. The magnetic moment of a bar-magnet, which weighs 65 grams, is found to be 2500. If the density of steel is 7.9, find the intensity of magnetisation.

22. The intensity of magnetisation of a bar-magnet, 12 cms. long and 0.75 sq. cm. cross-section, is 200 C.G.S. units. Find the tangent of the

greatest angle of deflection of a magnetometer needle which this magnet could produce if its centre is 20 cms. from the needle, $H = 0.20$.

23. A short bar-magnet is placed at Gibraltar, perpendicular to the magnetic meridian, and 'end on' towards a compass-needle from which it is distant 100 cms. When the experiment is repeated at Portsmouth the magnet has to be placed at a distance of 110 cms. from the compass to produce the same deflection of the needle. Compare the horizontal forces of the earth's magnetism at Gibraltar and Portsmouth.

24. Two magnets have the same pole strength, but one is three times as long as the other. The shorter is placed 10 cms. from a magnetometer needle (using the A Position of Gauss). At what distance must the other be placed in order that there may be no deflection?

25. Explain how you could compare the earth's horizontal magnetic forces at two stations, by noting at each station the deflection of a compass-needle produced by a given small bar-magnet placed in a definite position with regard to the needle.

26. Two short bar-magnets the moments of which are 108 and 192 respectively are placed along two lines drawn on the table at right angles to each other. Find the intensity of the field due to the two magnets at the point of intersection of the lines, the centres of the magnets being respectively 30 and 40 cms. from this point.

27. A small compass-needle is observed to be in neutral equilibrium when laid on a table at a distance of 20 cms. due north (magnetic) of the south pole of a bar-magnet lying on the table with its axis in the magnetic meridian. If the horizontal component of the earth's field is 0.180 C.G.S. unit, and the distance between the poles of the magnet is 10 cms., find the moment of the magnet.

28. Find an expression for the couple acting on a magnet of moment M, when its axis makes an angle θ with the lines of force of a magnetic field of strength H. If the magnet is suspended by a fine fibre, deflected until θ is 30°, and then released, find an expression for the kinetic energy which it will possess at the moment when it is passing through the magnetic meridian.

CHAPTER VI

TERRESTRIAL MAGNETISM

THE presence of a magnetic field distributed over the surface of the earth is rendered evident by the fact that a compass-needle, or any magnet suspended horizontally, swings to-and-fro in obedience to a magnetic couple, even when the neighbourhood is free from all other magnets or magnetic material. Moreover, if the magnet is suspended from its centre of gravity—so as to enable it to come to rest with its magnetic axis in the direction of the lines of force—it is observed that in nearly all localities the magnetic axis is inclined to the horizontal. Finally, it is only in a few places that the magnetic axis points towards the geographic north, and this *variation* from true north is not only different in different localities but also varies from time to time.

In any complete statement of the earth's magnetic field at any point it is therefore necessary to determine

(i) **the variation from true north,**
(ii) **the inclination to the horizontal plane, and**
(iii) **the Intensity of either the Total Magnetic Force or its Horizontal Component.**

In Fig. 69 the vertical plane GOY represents the **Geographical Meridian,** *i.e. the vertical plane passing through the point of observation* O *and the earth's geographical pole*; the vertical plane MOY represents the **Magnetic Meridian,** *i.e. the vertical plane in which the magnetic axis of a freely suspended magnet* *comes to rest*; OI represents in magnitude and direction the earth's magnetic field, and is contained within the vertical plane MOY.

The Declination, or **Variation, is the angle between the geographical and magnetic meridians** (GOM, Fig. 69).

The Dip is the angle between the direction of the lines of force of the earth's field and the horizontal (MOI, Fig. 69).

The Horizontal and Vertical Intensities (OH and OV) **are the horizontal and vertical components of the Total Intensity** (OI). Hence

$$\frac{OH}{OI}=\cos i, \text{ and } \frac{OV}{OI}=\sin i.$$

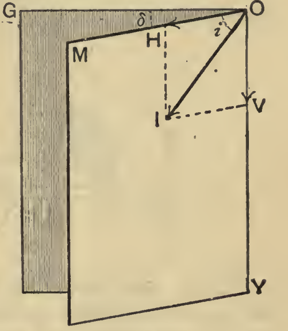


Fig. 69.

Declination.—The measurement of the declination involves two determinations, those of (i) the geographical meridian and (ii) the magnetic meridian. The former can be determined by careful astronomical observations. The determination of the latter is complicated by the fact that the magnetic axis of a magnet seldom coincides with the geometrical axis: this error is neutralised by repeating the observation with the magnet reversed, so that the surface previously uppermost is now underneath; if the geometrical axis points to the E. of magnetic north in the first position it will point to the W. of north in the reversed position (cf. Fig. 75), and the *mean* position of the geometrical axis gives the direction of the magnetic meridian. The process may be demonstrated by the following simple experiment.

Expt. 34.—Bore circular holes through two square pieces of cardboard, and fasten silk fibres across the holes (Fig. 70). Attach these to opposite end-faces of a bar-magnet, and suspend the magnet above the table by means of a silk loop and a bundle of unspun silk fibres. Bring the magnet to rest, and mark the direction of the line *ab* joining the points of intersection of the silk fibres by means of *brass* pins fixed into the table. Reverse the magnet so that the cross-fibres are now below the magnet, and mark the direction *a′b′*. The line bisecting the angle between *ab* and *a′b′* is the magnetic meridian. Also, since the magnet comes to rest with its *magnetic axis* coinciding with the magnetic meridian, a line drawn on the face of the magnet in the same vertical plane as the meridian will indicate the direction of the magnet's magnetic axis.

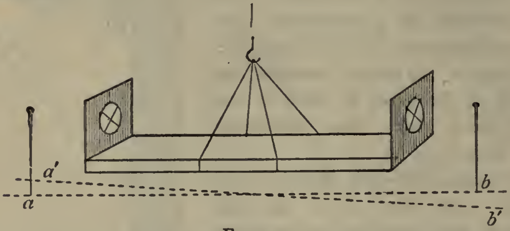


FIG. 70.

Fig. 71 represents the instrument used in Kew Observatory for the measurement of declination, except that the telescope T, the brass bar SS^1, and the magnet m^1 are removed; also the magnet M_1 is replaced by one provided with a simple means of reversing its position. The essential parts of the apparatus are the glass tube A which carries the magnet by means of silk fibres, the telescope T_1, the circular scale E round the centre of which the whole apparatus rotates, and the mirror C which rotates on a horizontal axis and is supported on a stand which can be adjusted so that the plane of the mirror is perpendicular to the axis of T_1. The magnet consists of a hollow steel tube of which the end distant from T_1 is closed by a piece of plate glass on which a scale is etched, and the near end carries a lens of focal-length equal to the length of the magnet. The magnet is adjusted so that the scale can be focussed clearly in the telescope, and the instrument is rotated until the cross-fibres in the eye-piece of the telescope coincide with the centre of the scale; the reading of the circular scale is then observed by means of the lens *l*. The magnet is now reversed and the observation is repeated. The mean of the two observations corresponds to the scale reading of the magnetic meridian.

The geographical meridian is determined by means of the telescope T_1, the mirror C, and a chronometer of which the error is known. The mirror is adjusted so that its axis is horizontal, so that its normal is perpendicular to its axis, and so that when fixed vertically its normal coincides with the axis of the telescope. The instrument is then rotated until an image of the sun is reflected from the mirror along the telescope, and the exact time is noted when the centre of the sun crosses the point of intersection of the fibres in the eye-piece. The *azimuth*, that is, the angular distance E. or W. of the meridian, of the sun at the instant of transit can be calculated from the time, the latitude, and the north-polar distance of the sun, which can be found by reference to the *Nautical Almanac*. The scale E is read, and the instrument rotated through an angle equal to the azimuth, calculated as above, when the axis of the telescope will now coincide with the geographical meridian; the scale is again read. The **Declination** is equal to the difference between the last scale-reading and that corresponding to the magnetic meridian.

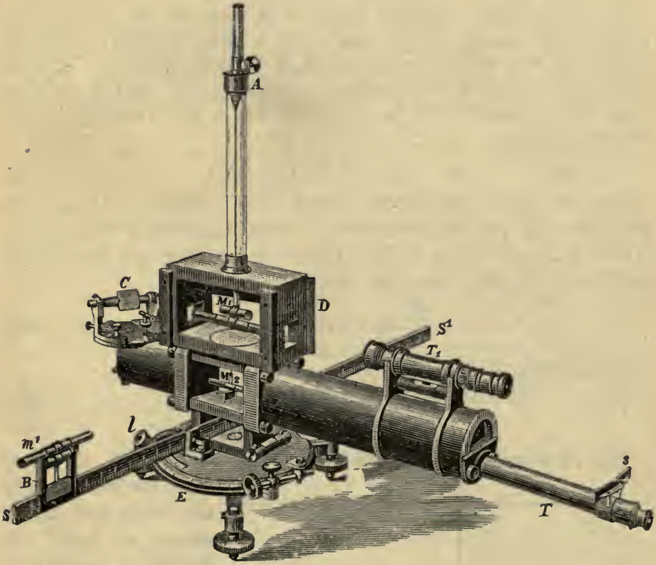


FIG. 71.—The Kew Magnetometer.

The Horizontal Intensity.—The measurement of the horizontal intensity involves two series of observations :

(i) A magnet is suspended horizontally, and its period of swing is observed accurately, then (p. 84)

$$t=2\pi\sqrt{\frac{I}{MH}}, \text{ or } MH=\frac{4\pi^2 I}{t^2} \dots\dots\dots\dots\dots\dots (1)$$

In this formula a *complete vibration* is taken as the time-interval between the passage of the magnet in a *given* direction across its position of rest and the next passage in the *same* direction. In some text-books the time of vibration is taken as the interval between *consecutive* passages across the position of rest, in which case equation (i) should be written,

$$t=\pi\sqrt{\frac{I}{MH}}.$$

(ii) The same magnet is placed in the tangent (B) position of Gauss (p. 79) relatively to a short suspended magnet, to which a small mirror is attached in order that the deflections may be accurately observed, then

$$\frac{M}{H}=(d^2+l^2)^{\frac{3}{2}} \tan \alpha. \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

By dividing (i) by (ii),

$$H^2=\frac{4\pi^2 I}{t^2(d^2+l^2)^{\frac{3}{2}} \tan \alpha}. \dots\dots\dots\dots\dots\dots\dots\dots (3)$$

Fig. 72 explains this method of observing deflections. M_1 represents the mirror, S_1S_2 the scale, and L the source of light. If the mirror is deflected through an angle α, the beam of light is rotated through an angle δ, and $\delta=2\alpha$. Now $\tan \delta=S_1S_2/OS_1$, and the value of 2α can be obtained from tables of natural tangents, thus giving the value of α.

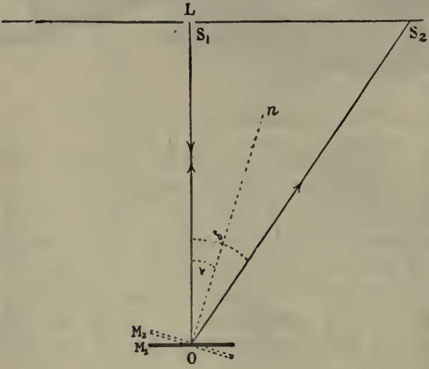


FIG. 72.

EXPT. 35.[1]—Prepare a small cylindrical magnet, and calculate its moment of inertia (p. 22) from the formula

$$I=\left(\frac{4l^2}{12}+\frac{r^2}{4}\right)w,$$

where $2l$=length, r=radius, and w=weight in grams.

Suspend the magnet horizontally by means of a silk fibre, using apparatus similar to Fig. 66. At least three independent observations of the time required for 50 complete vibrations should be observed, and the time of one vibration calculated from the mean of these observations. From these observations calculate the value of MH by means of formula (1).

Place a mirror magnetometer so that its face coincides with the magnetic meridian. Place a lamp and scale about 1 metre distant from, and its centre exactly opposite to, the centre of the mirror; and adjust the scale so that it coincides with the magnetic meridian. Focus the cross-wire on to the scale by means of a lens. Measure the distance of the centre of the scale from the mirror, and also the length of each scale division. Note the scale reading. Fix the same magnet as used in the vibration-experiment, by means of its cork support, with its centre at a known distance from the mirror, and observe the deflection; repeat, with the magnet reversed, but at the same distance. Repeat both observations with the magnet on the other side of the magnetometer. Record the observations thus:

Distance (D) of scale from mirror=
Length ($2l$) of magnet=
Distance (d) of middle of magnet from mirror=

$(d^2+l^2)^{\frac{3}{2}}$	Deflection in cms. (s).	$\tan 2\alpha=\frac{s}{D}$.	$\tan \alpha$.	$\frac{M}{H}=(d^2+l^2)^{\frac{3}{2}} \tan \alpha$.

From the values of MH and M/H calculate the value of H by means of equation (3).

---

[1] Full details of this experiment are given in Stewart and Gee's *Practical Physics*, vol. ii. (Macmillan), and in Hadley's *Practical Exercises in Magnetism and Electricity* (Macmillan).

Determination of the Moment of a Magnet.—If equations (1) and (2) are multiplied together, then

$$M^2 = \frac{4\pi^2 I}{t^2} \times (d^2 + l^2)^{\frac{3}{2}} \tan \alpha.$$

Hence the moment of a magnet may be calculated from the same observations as would be required for a determination of H.

The Determination of H at Kew Observatory.—The observations are taken in the following manner:—

(i) **The Time of Vibration of the Magnet.**—The suspended magnet M_1 (Fig. 71) consists of a hollow steel tube with a glass scale at the distant end and a lens at the near end; its *moment of inertia* (p. 22) is determined by supporting a brass cylinder in the metal tube immediately over the magnet, and by observing the time of vibration of the magnet both with and without the additional brass cylinder. If these times of vibration are t' and t seconds respectively, and if i is the moment of inertia of the brass cylinder, then

$$t' = 2\pi\sqrt{\frac{I+i}{MH}}, \text{ and } t = 2\pi\sqrt{\frac{I}{MH}}.$$

Hence

$$t'^2/t^2 = (I+i)/I,$$

or

$$I = i \times \frac{t^2}{t'^2 - t^2}.$$

(ii) **The Measurement of $\frac{M}{H}$.**—The wooden case D and the telescope T_1 are removed, and the glass tube A is screwed into the top of the lower case. A hollow steel magnet M_2, to which is fixed a mirror perpendicular to the magnet's axis, is suspended from the torsion-head. A scale s is fixed above the telescope T, and a graduated brass bar SS^1 is clamped in a position perpendicular to the axis of the telescope. The divisions of the scale are reflected from the mirror down the telescope, and the instrument is rotated until the central division of the scale coincides with the cross-fibres in the eye-piece. The reading of the horizontal scale is then noted. The magnet M_1, used in the previous part of the observation, is supported on the carrier B as shown at m^1, causing a deflection of M_2. The instrument is then rotated until the scale centre is again coincident with the cross-fibres. The angle through which the instrument is rotated measures the angle of deflection.

In this position the magnet M_1 is perpendicular to the axis of the magnet M_2. The formula for the calculation of $\frac{M}{H}$ will therefore be $\frac{M}{H}=\frac{(d^2-l^2)^2}{2d}\cdot\sin\alpha$. This corresponds to the *sine position*, described on p. 80.

Magnetic Dip.—The magnetic dip is determined by the angle which the magnetic axis of a magnet, supported through its centre of gravity, and free to move in a vertical plane, makes with a horizontal plane when the plane of motion coincides with the magnetic meridian. An accurate measurement is only possible with an elaborate instrument, owing to errors due to the axle of the needle not coinciding with the centre of gravity, and to the magnetic and geometrical axes not coinciding.

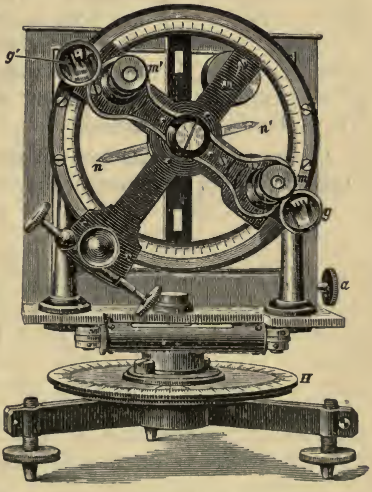


Fig. 73.—The Kew Dip Circle.

Fig. 73 represents the type of instrument used in Kew Observatory. The essential parts of the instrument are the *needle*, *nn'*, supported by a cylindrical steel axle resting on agate knife-edges, the *microscopes m* and *m'* for determining the positions of the ends of the magnet, the *magnifying glasses g* and *g'* for reading the vertical circular scale, and the *horizontal scale* H. The observations must be carried out systematically in order to avoid the following possible errors:

(*a*) **The line joining the 90° reading on the upper and lower scale may not be vertical.**—This error is corrected by taking one reading when the needle is in position, the instrument is then turned through 180° (by means of the horizontal scale) so as to bring the needle into different quadrants of the vertical circle; if the needle reads too low in the first case, it will now read too high, and the error is corrected by taking the mean of these readings.

(*b*) **The needle's axle may not coincide with the centre of the scale.**—Let *ns* (Fig. 74) represent the axis of the needle, giving scale readings at *n* and *s*. The correct readings should be *n'* and *s'*. The error is corrected by reading both *n* and *s*, and taking the mean of these readings, for it is evident from the figure that, if the reading *n* is too high, the reading of *s* is too low to an equal extent.

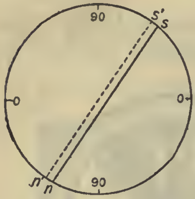
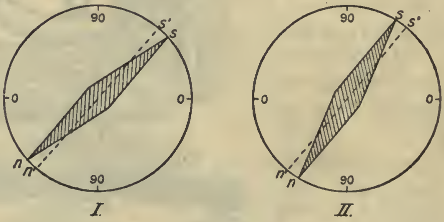


FIG. 74.

(*c*) **The magnetic axis does not coincide with the axis of figure.**—Let *n's'* (Fig. 75, i) represent the magnetic axis of the needle *ns*.



I.

II.

FIG. 75.

The readings *n* and *s* are incorrect—they are both too low. If the needle is reversed in its bearings (Fig. 75, ii), then both readings are too high. The error is therefore corrected by taking the average of these four readings.

(*d*) **The centre of mass may be situated on one side of the centre of motion.**—In the position indicated in Fig. 76, i, this error will cause the reading to be too high. But if the needle is reversed in its bearings, as shown in Fig. 76, ii, the reading will be too low. The correct reading is obtained by taking the mean of the two readings.

(*e*) **The centre of mass may be displaced along the axis of the needle.**—This error is removed by reversing the poles of the needle. For if, in the first observation, the centre of mass should be above the axis of motion (Fig. 77, i), thus causing the dip to diminish, the reversal of the poles will bring the centre of mass below the axis of motion (Fig. 77, ii), thus causing the dip to increase. The mean of these readings eliminates the error.

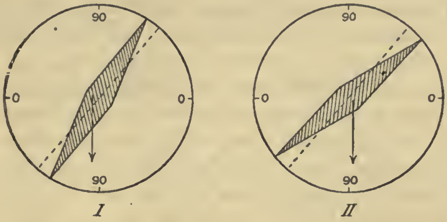


FIG. 76.

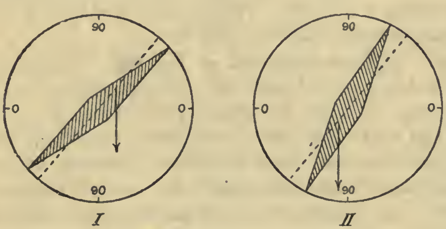


FIG. 77.

EXPT. 36.[1] (i) **Determination of the Magnetic Meridian.**—This line is determined by rotating the plane of free motion of the needle until the needle is vertical. This position indicates that the needle is only influenced by the vertical component of the earth's magnetic force—the horizontal component is only *tending* to strain the axle of the needle. If the needle is now rotated through 90°, as observed on the horizontal scale, the plane of free motion will coincide exactly with the magnetic meridian.

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[1] For laboratory and class-room purposes, a much simpler type of instrument than Fig. 73 may be used with advantage.

All the enumerated errors, except (*e*), are involved, and they are eliminated by taking the following eight readings:

Face of Instrument.	Face of Needle.	Readings of horizontal scale, when needle points to 90°.
South	South	North pole (i).............. South pole (ii) ....
	North	(iii).............. (iv).. ....
North	South	(v).............. (vi)..............
	North	(vii).............. (viii)..............

Take the mean of these readings, θ, and rotate the instrument until the scale-reading is $(90+\theta)$. The plane of free motion of the needle now coincides with the magnetic meridian.

EXPT. 36. (ii) **Determination of the Dip.**—Commence with the instrument facing *east*, and with the needle also facing *east*: read both poles of needle. Reverse the needle on its bearings, so that it now faces *west*, and read both poles. Rotate the instrument through 180°, so that it now faces *west*; read both poles. Reverse the needle on its bearings, and again read both poles. So far eight readings have been obtained, but the possible error (*e*) has not been eliminated. The polarity of the needle must now be reversed by the method of "double touch," and all the previous observations repeated. Tabulate the results in the following manner:

Face of Instrument.	Face of Needle.	North Pole. South Pole.	North Pole. South Pole. (Polarity reversed.)
East	East	....	....
	West	....	....
West	East	....	....
	West	....	....

The true dip is obtained by calculating the mean value of these observations.

Continuous Magnetic Records.—Since the magnetic elements are subject to slight periodical changes, it is usual in observatories permanently to equip apparatus for the purpose of obtaining continuous records. The declination, horizontal intensity, and the vertical intensity are the three elements usually recorded ; the latter is substituted for the dip since the modification of the dip circle for recording purposes involves great practical difficulty. The record in each case is obtained by reflecting a narrow beam of light from a mirror (attached to the movable portion of the apparatus) to sensitized paper wrapped round a drum, which is rotated at a constant speed by means of clock-work.

The changes in **Declination** are recorded by means of a magnet suspended from a single fibre. It has been found preferable to use several small magnets rigidly fixed together instead of a single magnet, since, weight for weight, a much greater magnetic moment is thereby obtained. Halves of a circular mirror (Fig. 78) are situated below the magnet, the upper half is rigidly connected to the magnet, and the lower half, the direction of which relatively to the meridian is known, is permanently fixed to the base. The beam of light is reflected partly from the upper and partly from the lower mirror, the latter tracing out on the sensitized paper a straight line, which serves as a base-line from which the amount of variation may be measured.

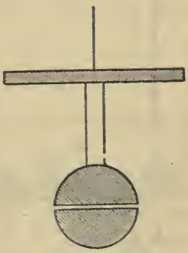
FIG. 78.

The variations in **Horizontal Intensity** are recorded by means of a magnet, supported by a *bifilar suspension*.[1] (In a recent form of apparatus a single fibre of quartz has been used for the purpose.) The upper end of the suspension is rotated so that the magnetic axis of the magnet is nearly perpendicular to the meridian, when the moment (MH), due to the earth's field, will be balanced by the moment due to the bifilar suspension. Any slight alteration in the magnitude of H will cause a slight rotation of the magnet. The method of recording the variations is identical with that used in the case of the declination instrument.

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[1] See Bifilar Suspension, p. 104.

The **Vertical Intensity** instrument consists of several magnetised steel rods fixed horizontally and parallel to each other through a vertical aluminium frame, which is supported on agate knife edges—the arrangement resembling the beam of a chemical balance. A mirror is attached to the aluminium frame. Brass weights are added, so that the system comes to rest in a position nearly horizontal, and so that the centre of gravity is just below the plane of the knife edges. The moment of the magnetic forces is balanced by that of the weight acting at the centre of gravity, and any variation of the former produces a slight movement of the system in a vertical plane.

Bifilar Suspension.—Fig. 79 represents a magnet supported by two fibres (AB and CD) of equal length. Each fibre supports one-half the weight of the magnet. If the magnet is deflected through a small angle the fibres are inclined to the vertical, and the tension in each fibre may be resolved into vertical and horizontal components; the vertical components support the magnet while the horizontal components tend to rotate the magnet into its original position. The lower portion of Fig. 79 represents a bird's-eye view of the magnet retained in a position with its axis perpendicular to the meridian owing to a rotation of the torsion head through a small angle (α).

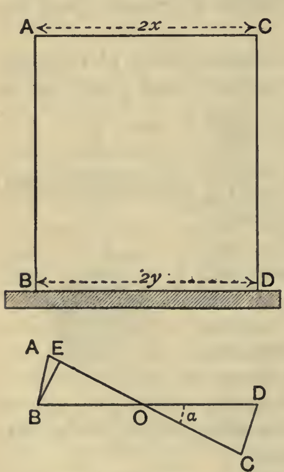

Fig. 79.—Bifilar Suspension.

If l = length of each thread, T = tension, W = weight of magnet, and β = inclination of each thread to the vertical, then

Vertical component of tension $= T \cos\beta$.

Horizontal ,, ,, $= T \sin\beta = T \times \frac{AB}{l}$ (where AB is the horizontal projection of the fibre).

Moment due to $T \sin\beta = \left(T \times \frac{AB}{l}\right) \times$ (the perpendicular distance from O)

$$= 2 \times \text{area OAB} \times \frac{T}{l}$$

$$= 2 \times \frac{OA \times BE}{2} \times \frac{T}{l} = 2 \times \frac{OA \times OB \sin\alpha}{2} \times \frac{T}{l}$$

$$= xy \,.\, \sin\alpha \times \frac{T}{l}.$$

Total moment, due to both fibres, $= 2xy \sin\alpha \times \frac{T}{l}$

$$= \frac{2xy \sin\alpha}{l} \times \frac{W}{2\cos\beta} \left(\text{since } T\cos\beta = \frac{W}{2}\right)$$

$$= \frac{W \,.\, xy \,.\, \sin\alpha}{l \cos\beta}.$$

If the magnet is perpendicular to the meridian, the moment due to magnetic forces $= MH$. Hence

$$\frac{W \,.\, xy \,.\, \sin\alpha}{l \cos\beta} = MH.$$

(The angle β is usually so small, that $\cos\beta$ may be regarded as unity.)

Magnetic Maps.—Since the magnetic elements have different values in different localities, much information may be derived from geographical maps, on which lines are drawn connecting together those localities in which a magnetic element has equal values. Figs. 80, 83, and 84 represent the distribution of the lines joining together localities having equal Declination, Dip, and Total Force respectively.

Fig. 80 represents a map intersected by lines of *Equal Declination*, or of *Equal Variation*, which are termed **Isogonic Lines.** There are two lines of no Declination, termed **Agonic Lines**, passing from North to South, also an Agonic Line in the form of an oval located near to Siberia, and usually termed the *Siberian Oval*; these are indicated by thick continuous lines. The dotted lines indicate regions of Easterly Declination, and the thin continuous lines indicate regions of Westerly Declination.

The distribution of the isogonic lines may be fixed in the memory by attributing the earth's magnetism to two magnets;

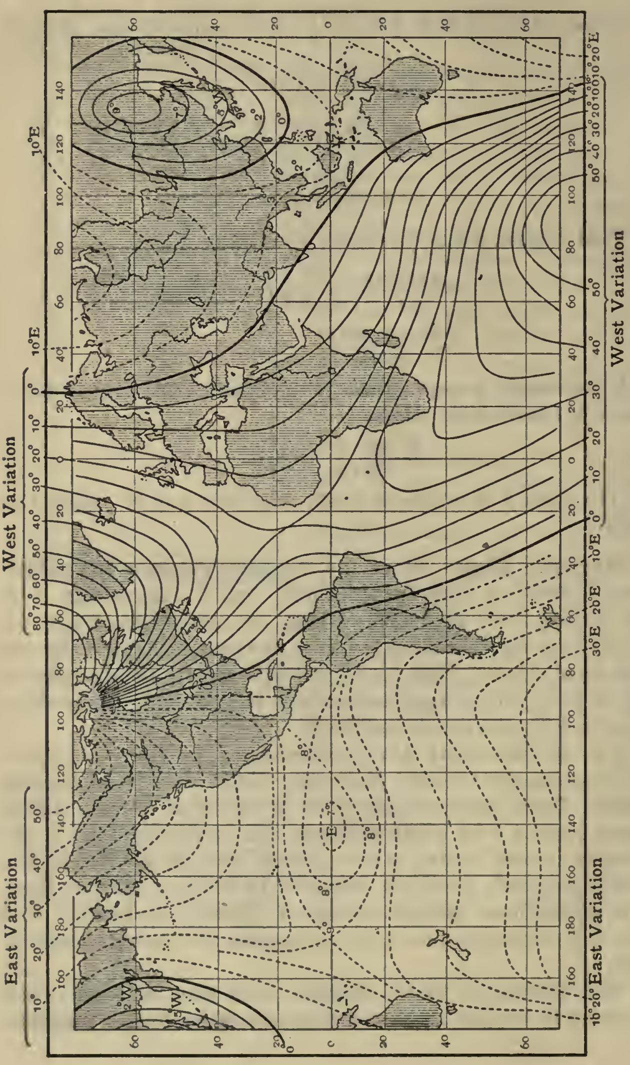

FIG. 80.—Isogonic Lines, or Lines of Equal Declination. (For the Epoch 1905.)

one much stronger than the other, situated relatively to each other as shown in Fig. 81. The *magnetic poles* will be in the positions marked S*m* and N*m*, where the dip needle will be vertical. Fig. 82 represents a plan of the northern hemisphere. Imagine A_1Ng to be the *agonic* line passing through America, and that a compass needle is conveyed round the magnetic equator in an easterly direction. Its declination will become *westerly*, gradually increasing, and then diminishing as it approaches the agonic line A_2Ng situated near the Caspian Sea. The declination then becomes *easterly* for a short distance, as far as the agonic line A_3Ng, which is the western arm of the Siberian Oval. Inside the *oval* the declination is *westerly*, and the agonic line A_4Ng is the eastern arm of the Siberian Oval. Between A_4 and A_1 there is a wide area in which the declination is *easterly*. The alternation of westerly and easterly declination shown in Fig. 80 is readily explained by the joint magnetic forces due to the unequal poles S and *s*.

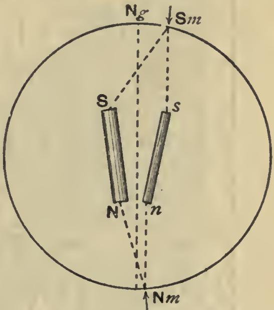

FIG. 81.

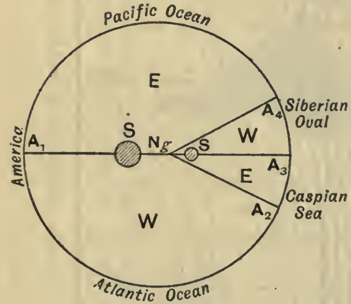

FIG. 82.

Fig. 83 represents the lines of *Equal Dip*, termed **Isoclinic Lines.** The line of *no dip* is termed the **magnetic equator,** and it will be noticed that this by no means coincides with the geographical equator. There are two positions, termed **Magnetic poles**, in which the dip needle points in a vertical direction; Sir John Ross, in 1831, located the North Magnetic Pole in Boothia Felix (Lat. 70° 5′ N., Long. 96° 33′ W.); and the Shackleton antarctic expedition, in 1909, located the South

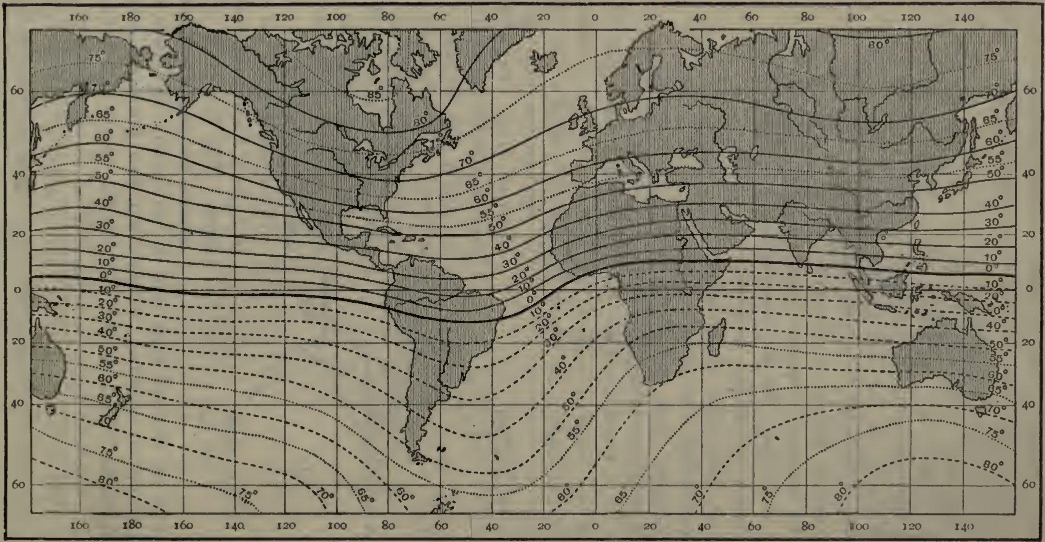

FIG. 83.—Isoclinic Lines, or Lines of Equal Dip. (For the Epoch 1905.)

Magnetic Pole in South Victoria Land in Lat. 72° 25′ S. and Long. 154° E. In each case, if the magnetic poles are defined as the positions where the dip needle assumes a vertical position, there is probably a slow secular change, and a diurnal change by which the position of either pole describes an oval curve several miles in diameter.

It is important to remember that the horizontal intensity diminishes in higher latitudes, although the total intensity as a general rule increases. For example, at St. Helena (Lat. 15° S.) the total force$=0.276$ C.G.S. units, and the Dip$=20°$; hence, since $H/T=\cos i$ (where H and T are the horizontal and total intensities), $H=0.276\times0.94=0.26$ C.G.S. units. At Cape Town (Lat. 34° S.) the total force$=0.345$, and the Dip$=58°$; hence $H=0.345\times0.530=0.183$ C.G.S. units. The horizontal intensity vanishes at the magnetic poles, and has its maximum value at the magnetic equator.

TABLE OF MAGNETIC ELEMENTS, FOR THE YEAR SPECIFIED.

Place.	Year.	Declination.	Inclination.	Horizontal Force, in C.G.S. units.
Kew.	1912	15° 46′.5 W.	66° 56′.5 N.	0.18498
Greenwich.	1912	15° 24′.3 W.	66° 51′.8 N.	0.18528
Potsdam.	1912	8° 45′.9 W.	66° 20′.4 N.	0.18803
Honolulu.	1912	9° 34′.8 E.	39° 38′.4 N.	0.29124
Bombay.	1912	0° 51′.2 E.	23° 56′.1 N.	0.36874
Christchurch (N.Z.).	1911	16° 39′.0 E.	67° 56′.2 S.	0.22494
Mauritius.	1912	9° 25′.5 W.	53° 23′.2 S.	0.23304

Within the area of the British Isles, the magnetic elements have been determined at a large number of stations by Sir Arthur Rücker and Professor Thorpe,[1] the results showing that the isogonal lines extending over this area are extremely irregular in contour. These irregularities are due to masses of magnetic rock (*e.g.* basalt) either protruding through the earth's surface (as in the case of the Malvern Hills), or to underground ridges reaching nearly to the earth's surface. An equally complete survey in other regions would certainly reveal that the

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[1] See Rede Lecture, by Sir A. W. Rücker (*Nature*, Dec. 16 and 23, 1897).

Isogonals and Isoclinals in Figs. 80 and 83 possess many irregularities not shown in these maps.

Secular Changes.—The continuous record of the values of the magnetic elements at any point show that these are undergoing a slight gradual change; such changes are termed **secular.** The earliest record of the declination at London was made in 1580, and was found to be 11° 20′ E.; from that date the easterly declination slowly diminished; the compass pointed true north in 1659, and then gradually moved to the west of true north. In 1823 the declination acquired a maximum value of 24° 30′ W., and afterwards diminished; in 1908 the declination at Greenwich was 15° 53′.5 W., and is diminishing at an approximate rate of 5′ per annum.

The Secular Change in the dip is comparatively small; thus, at London, it was 71° 50′ in 1576, 74° 40′ in 1720, and 66° 54′ N. in 1909. At the present time it is diminishing at an approximate rate of 1′ per annum. Accompanying the diminution of the dip there is a consequent increase in the horizontal intensity (which was 0.1853 C.G.S. units at Greenwich in 1909) which is annually increasing by about 0.00022 C.G.S. unit.

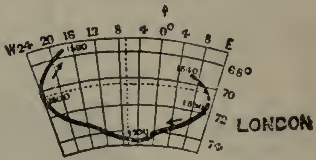

FIG. 84.

The combined Secular Changes in declination and dip are clearly represented by a graphic method (due to Dr. L. A. Bauer), shown in Fig. 84; the curve indicates the apparent path described by the N.-seeking pole of a magnet, freely suspended at its centre of gravity, as seen by an observer situated at the magnet's centre. The horizontal and vertical scales represent changes in declination and dip respectively. It would appear that a complete cycle of changes would require about 480 years, and that the magnetic axis of the earth is rotating round the geographical axis in a direction opposite to that of the earth's rotation.

Annual and Diurnal Changes.—In addition to the secular changes in the magnetic elements there are also smaller changes extending over shorter periods. The annual change in declination at Greenwich amounts to 2′ 25″, with a maximum easterly and westerly value in August and February respectively. Periodic daily (or diurnal) changes in the elements are also observed; thus, the declination has a maximum easterly and westerly value at 8 a.m. and 1 p.m. respectively. There is strong evidence of a relationship between the *range* of the diurnal change and the frequency of sun spots; thus, a year of maximum sunspot frequency exhibits also a maximum range of the diurnal change.

Magnetic Storms.—The magnetic elements occasionally undergo changes which are somewhat violent, but not periodic; these disturbances, called *magnetic storms*, are usually simultaneous with maxima displays of the aurora borealis. It has been suggested that both phenomena may arise from streams of *electrons* projected from the sun and passing through space in the neighbourhood of the earth. This undoubtedly would affect the magnetic elements, but the evidence of cause and effect is not entirely trustworthy.

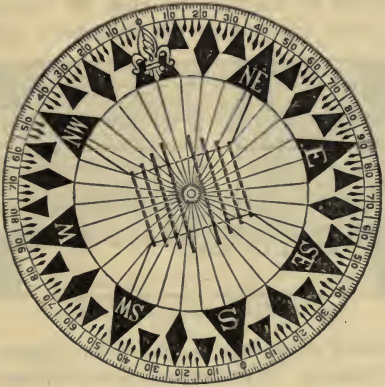

Fig. 85.—Kelvin's Ship's Compass.

The Ship's Compass.—Fig. 85 represents the needle and card, designed by Lord Kelvin, which until recently was the standard pattern adopted by the Admiralty and the mercantile marine. The needle consists of eight separate magnets fastened like the steps of a ladder to two silk threads. At its four corners this arrangement is attached by silk threads to the lower edge of a circular aluminium rim, on the upper edge of which is fastened the paper scale. The rim is also connected by 32 silk threads to a central metal disc containing an inverted sapphire cup which rests on a vertical iridium point. The bowl containing the needle and card is supported on *gimbals* consisting of two concentric rings connected in such a manner that the bowl is carried by two horizontal axes at right angles to one another; this ensures that the bowl is always horizontal.

A more recent type of instrument is the **liquid compass** (Fig. 86), invented by Captain the Hon. W. Chetwynd, R.N.; and this has been adopted by the Admiralty and by foreign navies. The chief advantage is that it has greater steadiness during heavy gunfire. The compass bowl is closed at the top and bottom by glass plates, forming a chamber which is filled completely with a mixture of water and alcohol. A heavy metal rim below the bowl gives stability. The use of glass enables the card to be illuminated from below by a light fixed in the binnacle. The card consists of a mica sheet, on which the scale has been photographed directly, by sensitizing the surface like a photographic negative; after development the film is dried and covered with a thin coat of white paint. The card is supported on a hollow metal float, which also carries two round bar magnets each about 8 cms. long. The *lubber's point* is a horizontal pointer fixed to the bowl; its point is close to the edge of the card, and its length parallel to the ship's keel.

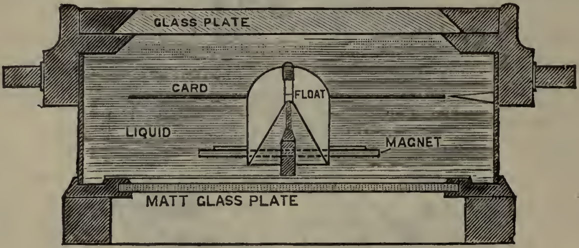

FIG. 86.—Chetwynd's Liquid Compass.

Corrections of the Ship's Compass.—The magnetic disturbances due to the iron of a ship are of two kinds, (i) the

permanent magnetism acquired during the building of the ship, and (ii) the **temporary** magnetism induced by the earth's magnetic field.

(i) **Permanent Magnetisation.**—The direction of this is due partly to magnetisation along the ship from bow to stern, partly to that of the cross-beams and partly to that of vertical portions of the structure. The first of these is usually the most prominent, and its effect on the compass may be compared to that of a long steel magnet placed beneath the compass (Fig. 87 i).

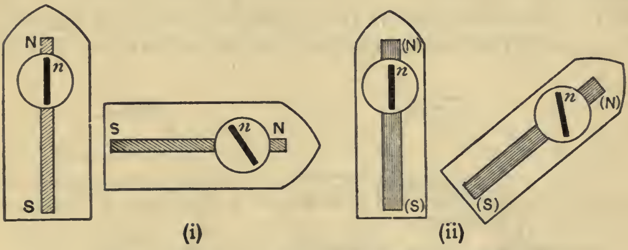

FIG. 87.—Effects on a ship's compass of (i) permanent, and (ii) temporary, magnetisation.

No error is caused when the ship is on a N. and S. course ; and it is a maximum when the course is E. and W. Since, in the process of 'swinging' the ship, the error is a maximum once in each semi-circle it is called the **semi-circular error.** It is corrected by placing in the binnacle below the compass two sets, each varying in number from one to seven, of permanent magnets (M_1 and M_2, Fig. 87A i) horizontal and with their poles pointing in a direction opposite to that of the ship's magnetism.

The error due to the cross-beams is corrected by means of a pair of horizontal magnets (M_3, Fig. 87A i) in the binnacle, and placed athwart-ships.

The vertical magnetisation causes a semi-circular error ; also, when the ship heels over to either side, it gives a horizontal component ; this latter is termed the **heeling error.** Both of these are corrected by means of a third set of magnets (M_4, Fig. 87A ii) suspended vertically in the binnacle and vertically below the centre of the compass-card.

In order to allow for periodical re-correction the magnets are supported on adjustable chains which can be adjusted through the lower doors of the binnacle.

(ii) **Temporary Magnetisation.**—The effect of temporary magnetisation may be compared to that of a horizontal soft-iron bar below the compass (Fig. 87 ii). There is no error when the ship's course is N. and S., or when E. and W.; but the error is a maximum when the course is either N.E., N.W., S.E., or S.W. As the error appears in each quadrant it is termed the **quadrantal error.** This is corrected by a method suggested by Sir George Airy (Astronomer Royal) in 1840, in which two large soft-iron balls (C_1, C_2, Fig. 87A) are fixed one on each side of the compass-bowl.

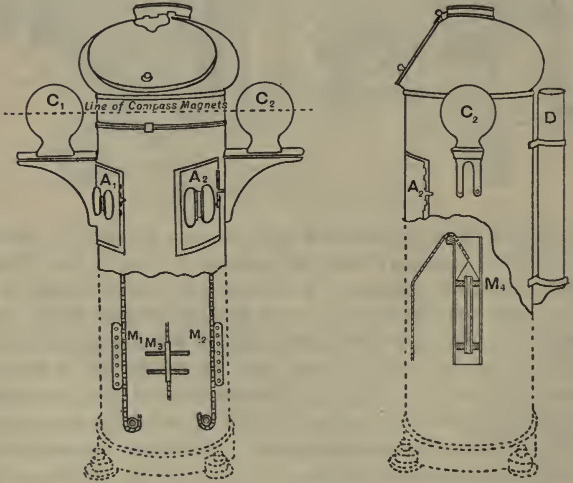

FIG. 87A.—A ship's binnacle, showing corrector magnets, etc.

The earth's vertical component causes an error which is partly semi-circular and partly a heeling error. These are corrected by a vertical bar of soft iron fixed in front of the binnacle with its upper end just above the level of the compass-card. This is known as the **Flinders bar** (D, Fig. 87A ii), since its use was first suggested by Captain Flinders, in 1801. As the binnacle is usually in the fore-part of the ship, and the greater part of the iron structure therefore *behind* the binnacle, the Flinders bar is placed in front of the binnacle.

The adjustment of all the several correctors is carried out in the process of 'swinging the ship,' in which the ship is moored to a buoy and moved successively into the various directions of the compass. The corrections for heeling-error are usually made by means of observations with a special needle suspended like a dip needle. Readings with this are taken on shore and when suspended in the position of the compass, and the necessary correctors are adjusted until the readings are the same.

SUMMARY

The **Magnetic Meridian** is the vertical plane in which the magnetic axis of a freely suspended magnet comes to rest.

Magnetic Declination, or **Variation**, is the angle between the geographical and magnetic meridians.

Magnetic Dip is the angle between the direction of the earth's magnetic lines of force and a horizontal line in the magnetic meridian.

The **Horizontal** and **Vertical Intensities** of the earth's magnetism are the horizontal and vertical components of the *Total Intensity*.

The *Declination* is measured by determining (i) the geographical meridian, and (ii) the magnetic meridian.

The *Horizontal Intensity* is measured by means of two observations: (i) the rate of swing of a magnet suspended horizontally, and (ii) the deflection produced in a magnetometer needle when the same magnet is placed in the *tangent* B *position of Gauss.*

The *Dip* is measured by means of a magnet supported through its centre of gravity and free to move in a vertical plane. Two chief observations are necessary, (i) the determination of the meridian, and (ii) the measurement of the angle of dip. An elaborate series of readings are necessary to avoid possible errors, which are *five* in number.

Isogonic Lines are lines drawn on a map through places at which the *magnetic declination* or *variation* is the same. The isogonic lines drawn through places in which the magnetic declination is zero are usually termed **Agonic Lines.**

Isoclinic Lines are lines drawn on a map through places at which the *magnetic dip* is the same. The line of *no magnetic dip* is termed the *magnetic equator.* The **Magnetic Poles** are places at which the dip-needle is vertical.

Secular Changes.—The gradual and continuous changes observed in the magnetic elements are termed *Secular* changes. A complete cycle of such changes would probably require about 480 years. Shorter cycle changes are called *annual* or *diurnal* according to the periodic time of the cycle.

Violent and aperiodic changes in the magnetic elements are termed *magnetic storms.*

A **ship's compass** is liable to errors owing to the magnetisation of the iron portions of a ship: the magnetisation may be either *permanent* or *temporary.*

QUESTIONS ON CHAPTER VI

1. Give a general account of the distribution of isogonic lines on the earth's surface, describing particularly the lines of no declination.

2. What is meant by (i) declination, (ii) agonic lines, (iii) magnetic equator? Draw a map showing the general position of the agonic lines, and indicate the direction of the declination in the regions into which these lines divide the earth's surface.

3. Given a circular steel plate magnetised along an unknown diameter, explain how to find its magnetic axis, and also the magnetic meridian.

4. A piece of soft iron wire is held in a vertical position with its lower end near the north pole of a compass needle, which is feebly attracted. The wire is then heated bright red by a flame, when it is observed that the attraction ceases. As the wire cools the needle is observed to be repelled. Explain these phenomena.

5. An iron pillar stands vertically in the centre of a room in which the direction of the magnetic meridian is known. Assuming that there is no other iron in the neighbourhood, how would you determine what part of the horizontal magnetic force at a given point in the room, magnetic north or south of the centre of the pillar, is due to the pillar?

6. An electric current is passed through a wire which is parallel to and just below a compass needle, and the needle is thereby deflected. Why would this deflection alter if, other things remaining unchanged, the experiment was made at different places?

7. A uniformly magnetised bar-magnet 10 cms. long, having a moment of 200 C.G.S. units, is placed in a horizontal position with its axis in the magnetic meridian. A small compass needle placed at a

distance of 10 cms. east of the centre of the bar is observed to be in neutral equilibrium. Find the horizontal intensity of the earth's field.

8. If you explore with a magnetic compass the space near the top, middle, and bottom of a cylindrical soft-iron column standing erect, what do you observe? Will it make any difference in the actions if the column is laid prostrate in the magnetic meridian?

9. A magnetic needle is suspended horizontally by a silk fibre, and a bar-magnet is placed a little distance to the east of it. What effects on the needle will be observed as the bar-magnet is rotated round its centre (i) in a horizontal plane, (ii) in the magnetic meridian?

10. Supposing an iron ship to behave like a permanent magnet, with a north pole at the bow and a south pole at the stern, explain how a compass on board will be affected as the ship swings through 360°. Would the effect on the compass be the same in England and at the Equator?

11. A short bar-magnet is placed on a table with its axis perpendicular to the magnetic meridian, and passing through the centre of a compass needle. In London the compass needle is deflected through a certain angle when the centre of the magnet is 25 inches from the centre of the needle. If the experiment be repeated in Bombay the magnet must be moved 5 inches nearer to the needle to produce the same deflection. Use these data to compare the horizontal forces in London and Bombay.

12. A magnet placed due east (magnetic) of a compass needle deflects the needle through 60° from the meridian. If at another station, where the horizontal force of the earth's magnetism is three times as great as at the first, the same magnet be similarly placed with respect to the compass needle, what will be the deflection of the latter?

13. Describe fully how to determine accurately the absolute value of the horizontal component of the earth's intensity at any place on the earth's surface.

14. Given a compass needle and a short bar-magnet, describe two methods which may be employed for comparing the horizontal forces of the earth's magnetism at two places.

15. Describe in detail the method of determining the magnetic dip at any point, explaining the reason for each operation you describe.

16. A small weight is attached to the upper end of a dip needle, whereby the inclination of the needle to the horizontal plane is diminished by one-half. Draw a picture showing the directions of the forces which act upon the needle, and state the conditions of equilibrium.

17. State the means which are adopted, in the determination of the magnetic dip at any place, to eliminate errors due to the possibility

that—(*a*) the magnetic axis of the needle is not in line with its geometrical axis; (*b*) the needle is not balanced exactly at the centre of the vertical circle; (*c*) the centre of gravity of the needle is nearer one end of the needle than the other.

18. A magnetic needle is suspended on a horizontal axis through its centre of gravity. The horizontal axis is slowly turned round in a horizontal plane. Describe and explain the position of the needle for various positions of the axis (i) in England, (ii) at the Magnetic Equator.

19. The vertical components of the earth's magnetic force at two different places are to be compared. How will you do it?

20. A magnetised bar of steel turns about an axis through its centre of gravity and perpendicular to its length, and can move in a plane perpendicular to the magnetic meridian. The magnet is loaded at one end so as to lie horizontal. Explain how such an arrangement can be used to indicate changes in the earth's vertical magnetic force.

21. Explain what observations are necessary for the determination of the total intensity of the earth's magnetic field at any given place.

22. A compass needle makes 40 oscillations per minute at a place where the dip is 60°, and 36 oscillations per minute where the dip is 70. Compare the value of the total magnetic intensity at the two places. ($\cos 60° = 0.5$, $\cos 70° = 0.342$.)

23. A magnet suspended so as to swing horizontally makes 50 vibrations per minute at a place where the dip is 30°, and 40 vibrations where the dip is 45°. Compare the total magnetic intensities at the two places.

24. At Kew the total magnetic intensity is 0.47 C.G.S. units and the dip is 67°: at New York the total intensity is 0.61 and the dip 70°. A magnet swinging horizontally at Kew makes 30 oscillations in a minute. How many would it make in the same time at New York?

25. Explain how to determine the moment of a magnet at a place where you do not know the intensity of the earth's horizontal field.

26. A dip circle is rotated (in azimuth) through an angle θ from the magnetic meridian, and the apparent angle of dip is then $i°$. Prove that the true dip (I) at the same locality is given by the equation

$$\tan I = \tan i \cos \theta.$$

27. The period of vibration of a bar-magnet was found to be 15.3 seconds. It was then placed in the tangent A position of Gauss relatively to a magnetometer needle, with its centre at a distance of 30 cms. from the needle, and the mean deflection was 18°. If the length of the magnet was 11.6 cms. and its moment of inertia 600 C.G.S. units, calculate the value of the horizontal component of the earth's field. ($\tan 18° = 0.325$.)

PART II

STATIC ELECTRICITY

CHAPTER VII

INTRODUCTORY

EXPT. 37.—Rub a stick of sealing-wax with dry flannel, and observe that it has acquired the peculiar property of attracting small fragments of paper, cork, or cotton fibre. By holding the wax near to one end of a wooden lath, balanced on an inverted round-bottomed flask, it may be shown that the force of attraction acts upon all bodies, whatever their mass may be.

DEFINITION.—**When a substance is rubbed with a suitable material, and is then found to possess the property of attracting neighbouring objects, it is said to be electrified (or, to possess a charge of electricity).**

Mutual Action of Electrified Bodies.—EXPT. 38 (i).—Suspend an electrified rod of sealing-wax horizontally, and bring near to one end of it another similarly charged rod of sealing-wax. Notice the **repulsion.**

(ii) Repeat Expt. 38 (i) using, instead of sealing-wax, glass rods which have been rubbed with silk. Notice the **repulsion**.

(iii) Suspend an electrified rod of sealing-wax. Bring near to it a rod of glass, previously rubbed with silk. Notice the **attraction.**[1]

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[1] Considerable care is required in using glass rods, for, without any evident reason, the glass will sometimes become negatively charged when rubbed with silk. If both glass and silk have been dried in a sand-oven the glass will become positively charged, but if the glass has been dried by passing it through a Bunsen flame it will become negatively charged, when rubbed with silk. It has been suggested that this is due to the flame removing the thin film of air condensed on the surface of the glass. If the glass is allowed to become *quite cold* and then warmed again in the sand-oven, it will recover its normal property of becoming positively charged.

Du Fay, who first observed these results, assumed the existence of two kinds of electrification, and applied the terms *vitreous* and *resinous* to the charges generated on the glass and wax respectively. At a later date it was observed that the kind of electrification depended on the substance used as a rubber; *e.g.* glass when rubbed with fur becomes charged with resinous electricity. For this reason, the terms *positive* and *negative*, which were first suggested by Benjamin Franklin, have since been adopted.

With the nomenclature now in use, the accepted rules are that,

(i) *Glass rubbed with silk is* **positively** *charged;*

(ii) *Sealing-wax rubbed with fur (or flannel) is* **negatively** *charged;*

(iii) *Bodies with* **like** *charges* **repel**, *and bodies with* **unlike** *charges* **attract** *each other;*

(iv) *A charged body always attracts an uncharged body.*

Theories of Electrification.—The growth of knowledge concerning static electricity has been accompanied by speculations as to its nature. Early in the eighteenth century du Fay enunciated the idea that there are two kinds of electricity, *vitreous* and *resinous*, which are of the nature of indestructible fluids, the particles of one kind being mutually repellent, but attracting particles of the other kind. According to this theory an uncharged body contained equal quantities of the two fluids, and a charged body contained an excess of one or other of the fluids.

At a later date, 1747, Benjamin Franklin propounded the 'one-fluid' theory, according to which there is only one fluid—called the *positive*. To him we owe the terms 'plus' and 'minus,' or 'positive' and 'negative,' as applied to electric phenomena. In this theory matter takes the place of the negative fluid, and the particles of matter and of the fluid are self-repellent but mutually attracting; also uncharged bodies contain a normal amount of the fluid, such that the attraction of the matter for any of the fluid outside the body just balances repulsion due to the contained fluid; and a body is positively or negatively charged according to whether it contains more than, or less than, the normal amount of the fluid. If the words 'positive' and 'negative' are interchanged Franklin's theory is in close agreement with the remarkable results which have led to the modern **electron** theory.

Electrons.—Recent investigation[1] in the discharge of electricity through rarefied gases (Chap. XXVI.) has demonstrated the existence of particles which are far more minute than the smallest atom and are always associated with a *negative* charge of 4.7×10^{-10} C.G.S. electrostatic unit. These particles have been termed **corpuscles.** It is remarkable that the same charge is associated with a single hydrogen atom in all processes of electrolysis; and, as early as 1881, Dr. Johnstone Stoney suggested that this should be regarded as the natural unit of electricity, and that it should be termed the **electron.** There is reason to believe that the corpuscle and the electron are the same, and that the former is really an 'atom of electricity'; for this reason the terms corpuscle and electron are usually regarded as synonymous. Modern theory suggests that the atom of any form of matter may, under normal conditions, consist of an equal number of *positive* and *negative* electrons; the latter are readily expelled from ordinary matter by slight electrical forces, and travel through vacua with a velocity comparable with that of light. The positive electron has not yet been clearly separated from the atom of matter, and comparatively little information concerning it has been obtained by experimental investigation. The inertia of a negative electron has been proved to be the eighteen-hundredth part of that of the hydrogen atom, and its diameter is approximately 10^{-13} cm., or 10^5 times smaller than the atom of matter.

We may say that a transference of electricity consists in the movement of negative electrons from a point where there is a *gain* of positive electricity to a point where there is a gain of negative; also that a positively charged body is one which has been deprived of some corpuscles, while a negatively charged body contains an excess. Such deductions are strikingly similar to those derived from the 'one-fluid theory' which was originally introduced by Benjamin Franklin.

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[1] See Sir W. Crookes (*Electrician*, vol. xxvi., 1891); Sir J. J. Thomson, *Discharge of Electricity through Gases* (Constable); Sir O. Lodge (*Jour. Soc. Elec. Engineers*, No. 159, Feb., 1903).

All electrical phenomena may be attributed to the movement of negative corpuscles or electrons. Thus a steady current traversing a metal wire consists in the handing-on of electrons from atom to atom of the metal, the atoms oscillating slightly during the process, and thus generating heat. With insulators, the transference is only possible under the influence of extreme forces. In the transference of electricity through electrolytes (p. 454), the negative 'ion' is an atom of matter with more than the normal number of electrons, and the positive 'ion' is an atom with less than the normal number. If the atom has *one* electron too many or too few, the atom is called a *monad*; if it has two too many or too few, it is called a *diad*.

Electroscopes.—Any appliance which serves to detect very small charges of electricity is termed an *Electroscope*. The *Pith-ball Electroscope* consists of a small ball, made from elder-pith, suspended by means of a silk thread. The *Gold-leaf Electroscope* (Fig. 88) is a practical application of the principle that two similarly charged bodies repel one another; it consists of two narrow strips of gold leaf suspended from the lower end of a stout wire, at the top of which a metal disc is fixed; the wire is supported vertically by a plug of insulating material (*e.g.* ebonite or paraffin-wax), and the leaves are protected from air currents by means of a case, the front and back of which are of glass. The sides of the case should be lined on the inside with strips of metal which are earth-connected. Nearly all the leakage which may occur is conveyed along the *surface* of the insulating plug; the insulation is therefore improved by cutting V-shaped grooves on the surface of the plug,

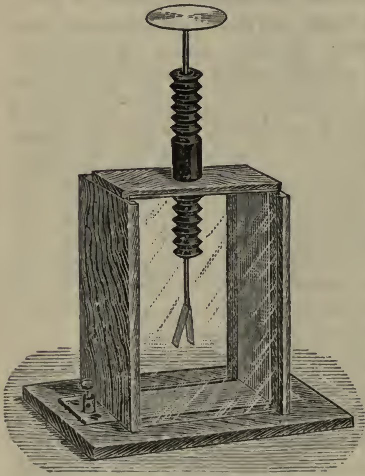

FIG. 88.—A Gold-leaf Electroscope.

whereby the length of path traversed by any leakage is increased. The theory of the Gold-leaf Electroscope is discussed on p. 159.

EXPT. 39.—(i) Hold an electrified rod of sealing-wax near to a pith-ball electroscope. Notice how the ball is attracted towards the wax, and how, after brief contact, it is repelled. The pith-ball acquires a slight negative charge from the wax, causing subsequent repulsion.

(ii) Gently pass the surface of a charged rod of sealing-wax along the edge of the disc of a gold-leaf electroscope. Notice the divergence of the leaves. The instrument is now *negatively* charged. Hold over the disc (while the instrument is still charged) (i) sealing-wax previously rubbed with fur, (ii) glass previously rubbed with silk, (iii) the hand. Notice the *increased* divergence in (i), and the *diminished* divergence in (ii) and (iii). Discharge the electroscope by touching the disc, and repeat the observations after charging the electroscope positively by touching the disc with a feebly charged glass rod.

From these observations the following rules may be deduced:

1. **Divergence is increased by a neighbouring similar charge.**

2. **Divergence is diminished by a neighbouring unlike charge or earth-connected conductor.**

Conditions under which certain substances may be charged or discharged.—A charged rod of sealing-wax or of glass may be discharged by passing it quickly through a flame, or by passing it *gently* through the closed hand; but if the hand is dry, and the movement rapid, the rods may *acquire* a charge. The flame and the hand have **conducted** the charge away, and they are termed **conductors**. A charge cannot be retained by a metal rod if held in the hand, since all metals are conductors; but if the metal is supported on a handle of sealing-wax or glass, which are non-conductors or **insulators**, it may be readily charged.

EXPT. 40.—(i) Fix a brass rod, or a square piece of sheet brass or zinc, on the end of a rod of vulcanite. *Flick* the metal with fur, and observe the charge by means of an electroscope.

(ii) Flick the metal disc of an electroscope with fur. Note the divergence of the leaves, and verify the *negative* charge by means of a charged rod of sealing-wax.

Conductors and Insulators.—In previous experiments, a flame, metals, and the hand have been found to be conductors. Other substances may be tested in the following manner:

Expt. 41.—Charge the gold-leaf electroscope, and touch the disc with a piece of metal wire held in the hand: the leaves collapse immediately. Repeat the observation with a strip of paper, a *dry* glass rod, cotton thread, silk thread (dry and wet), sulphur, paraffin-wax, silk thread dipped in oil, vulcanite, shellac, charcoal, wood, etc.

Substances may be classified, as regards their conducting power, thus:

Conductors.—*Metals*, *the body*, *water*, *charcoal.*

Partial Conductors.—*Paper*, *cotton*, *wood.*

Insulators.—*Glass*, *sealing-wax*, *shellac*, *vulcanite*, *sulphur*, *silk*, *oils.*

The insulating power of glass is often found to be unsatisfactory, especially in the case of soda-glass. This is largely due to the deposition of a film of moisture on the surface, and to chemical changes in the glass itself. The surface is rendered much less hygroscopic by coating it with paraffin-wax or with shellac varnish (prepared by dissolving pure shellac in absolute alcohol); the former is preferable, as the wax may be occasionally scraped so as to expose a fresh surface to the air. Dense flint glass is the most satisfactory type of glass for insulating purposes.

Of all insulators, fused quartz is found to be the most perfect.

Professor Ayrton has designed a glass **insulating stand** which is serviceable in electrostatic experiments. It consists of a conical glass flask, at the bottom of which is fixed a vertical glass tubulure. The end of a glass rod is ground into the tubulure, like the stopper of a glass bottle, and the upper end projects above the neck of the flask and serves as a support for any apparatus. The air of the flask is kept dry by means of strong sulphuric acid contained in the flask, and a split rubber stopper serves to close the neck of the flask when the stand is not in use.

Charging by Induction.—It is useful, at an early stage of the subject, to adopt a more convenient method of imparting charges to insulated conductors. The following experiment will explain how a conductor may be charged by induction.

Expt. 42.—Support, on an insulating stand, a wooden cylinder (AB, Fig. 89) with rounded ends and coated with tin-foil or blacklead. Hold a positively charged glass rod near to the end A. Touch A with a proof-plane, which consists of a small metal disc, *e.g.* a halfpenny, fixed on the end of an insulating handle, and prove that it has acquired a negative charge by bringing it near to the disc of a negatively charged electroscope. Verify the presence of a positive charge on the end B in a similar manner. After momentarily touching the cylinder with the finger, the end A will still retain a negative charge, but there will be no charge at B. Remove the glass rod, and verify that a negative charge is distributed over the surface of the cylinder. The original charges on the ends A and B are *induced* by the charge on the glass rod, and the final charge on the cylinder has been obtained by *induction.*

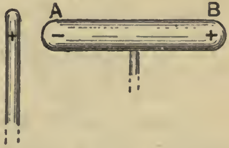

Fig. 89.—Charging a Conductor by Induction.

The cylinder may be charged positively by induction by using a negatively charged rod of sealing-wax instead of the positively charged glass rod.

Both kinds of Electrification are produced simultaneously and in equal quantity.—This statement may be proved by either of the following experiments:

Expt. 43.—(i) Attach a disc of cardboard to the end of a vulcanite rod, and cover one face of the disc with a small piece of fur. Support a small square of glass in a similar manner. After warming the glass, beat it gently with the fur. Keeping the glass and fur in contact, hold them over a charged electroscope, and notice that there is no change in the divergence. Now bring the glass and fur separately over the disc, and observe that one produces an increased divergence and the other a diminished divergence.

(ii) Place a loose-fitting flannel cap, attached to one end of a long silk thread, over the end of a vulcanite rod. Twist the thread round the cap, and pull the thread so as to make the cap rotate round the end of the rod. If the rod is held near a charged electroscope, no change in divergence is observed so long as the cap remains on the rod; but, as soon as the cap is removed, it is easy to detect a negative charge on the rod and a positive charge on the flannel.

Electrification of Gases.—Although solids and liquids can be readily electrified, yet it is extremely difficult to impart a charge to a gas in its normal condition. This is evident in the fact that a well-insulated solid conductor, when charged, does not lose an appreciable part of its charge although completely surrounded by air. The simplest method of charging air is to connect electrically an insulated flame to one terminal of a Wimshurst machine, the other terminal being earth-connected; or, instead of a flame, a pointed needle may be used (see *Action of Points*, p. 218).

Various chemical processes cause electrification in the surrounding air. Thus, Pouillet observed that when carbon burns, the surrounding air was positively charged, while the carbon was negatively charged. Also, air is charged negatively by burning coal-gas. Lavoisier and Laplace found that hydrogen, generated from iron and sulphuric acid, was positively charged. The hydrogen may even be drawn through a plug of glass-wool without losing its charge.

The most efficient method of charging a gas is obtained by the *splashing of liquids*. If water or mercury falls on a metal plate, the surrounding air is negatively while the liquid is positively charged. The effect can be shown by shaking mercury in a bottle, and drawing off the air through a tube. Small amounts of impurities in the water have a remarkable effect; *e.g.* a dilute salt solution will give a positive charge to the air. Lenard has suggested that the effect may be due to a *double electrical layer* at the surface of each drop of liquid. With pure water the positive layer is next the water, and the negative layer next the air. On striking the metal surface the mechanical disturbance may tear this layer asunder, and impart a negative charge to the air.

Elster and Geitel have found *when a platinum wire glows in air* that the wire acquires a negative charge and the surrounding air a positive charge. If hydrogen surrounds the wire the gas is negatively charged, but all other gases give the same results as air.

The Law of Inverse Squares.[1]—This law, which states that **the mutual force exerted between two charged bodies varies**

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[1] A more rigid proof of this Law is given on p. 151.

inversely as the square of the distance, was experimentally verified for the first time by Coulomb, by means of his Torsion Balance (see p. 31), in which the force of repulsion between two small charged spheres is balanced by the torsion in a fine wire. The apparatus closely resembles Fig. 19, but the magnets there shown are replaced by two insulating rods (of shellac), at the end of each of which is fixed a small gilt pith-ball. The torsion-head of the instrument is adjusted until the position of the suspended pith-ball coincides with that occupied by the fixed pith-ball; the latter is charged, inserted into the instrument, and brought into contact with the other pith-ball. The charge is shared and mutual repulsion takes place. If θ is the angular distance between the balls, then θ is also the torsion in the wire. In order to reduce the angular distance, the torsion-head must be rotated in the opposite direction; if the angular distance is reduced to θ_1, by turning the torsion-head through $T_1°$, the total torsion will then be $(T_1+\theta_1)$. If the Law of Inverse Squares is true, then the product *Force* × (*distance*)² should be constant; or

$$\theta \times \theta^2 = (T_1+\theta_1) \times \theta_1^2 = (T_2+\theta_2) \times \theta_2^2 = \ldots.$$

This relationship is only approximately true, even when the angular distance is small, since the true distance (d) between the balls is $2l \sin\frac{\theta}{2}$, where l is the length of the arm carrying the suspended ball. Let B_1 and B_2 (Fig. 90) represent the fixed and suspended balls respectively. The component of F, the force of repulsion, tending to twist the wire is $F\cos\frac{\theta}{2}$, and the moment of this component is $Fl\cos\frac{\theta}{2}$. This moment is balanced by the torsion in the wire. If K is the moment of the couple required to produce a torsion of 1° in the wire,[1] then, when the torsion is $(T_1+\theta_1)$,

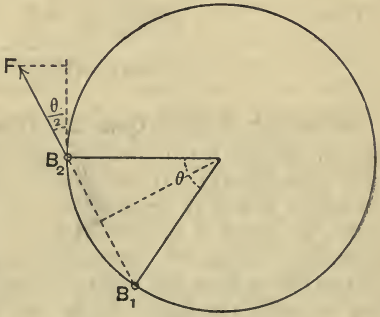

FIG. 90.

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[1] K may be called the *torsional moment* of the wire.

the equivalent couple will be $K(T_1+\theta_1)$. Hence $Fl\cos\frac{\theta_1}{2}=K(T_1+\theta_1)$. By the Law of Inverse Squares, $F\times d^2=$constant. Hence

$$K(T_1+\theta_1) . 4l^2\sin^2\frac{\theta_1}{2}\Big/ l\cos\frac{\theta_1}{2}=\text{constant},$$

or

$$Kl(T_1+\theta_1)4\sin\frac{\theta_1}{2}.\tan\frac{\theta_1}{2}=\text{constant}.$$

If θ_1 *is small*, then

$$2\sin\frac{\theta_1}{2}=\theta_1=2\tan\frac{\theta_1}{2},$$

or

$$Kl(T_1+\theta_1)\,\theta_1{}^2=\text{constant}.$$

The latter expression coincides with that given above.

EXAMPLE.—The balls in a Coulomb torsion balance are of equal size and initially in contact. A charge is then shared between them, and the movable ball comes to rest at an angular distance of 30° from the fixed ball. If the 'torsional moment' of the wire is 2 dynes for 1°, and if the distance of the movable ball from the point of support is 15 cms., find the charge on each ball.

Using the same symbols as in the previous paragraph, the force of repulsion is $q^2\Big/4l^2\sin^2\frac{\theta}{2}$, and the *moment* of this round the point of support is $q^2l\cos\frac{\theta}{2}\Big/4l^2\sin^2\frac{\theta}{2}$.

Hence

$$q^2l\cos\frac{\theta}{2}\Big/4l^2\sin^2\frac{\theta}{2}=K\times\theta;$$

$$\therefore\quad q=2\sin 15^\circ\sqrt{\frac{15\times2\times30}{\cos 15^\circ}}=15.8\text{ units.}$$

Definition of Unit Quantity (Electrostatic).—The definition of the electrostatic unit of quantity is derived from the law of inverse squares. If two small conductors are charged with quantities q and q_1 respectively, and situated at a distance d apart, the force F of attraction or repulsion is given by the equation $F=\pm qq_1/d^2$. If the charges are similar, the force will be one of repulsion and will have a *positive* sign; if the charges are opposite, the force will be one of attraction and have a *negative* sign. The above formula is only true if air is the medium between the two charges. If any other medium is used, of which the specific inductive capacity is K, then $F=\pm qq_1/Kd^2$ (p. 182).

If q, q_1 and d are each equal to unity, the force will be unity. It has been seen that in the C.G.S. system of measurement force is measured in *dynes*, and distance in *centimetres*. Hence **the C.G.S. unit of Quantity (electrostatic) is that quantity which in air exerts a force of one dyne on an equal quantity situated at a distance of one centimetre.**

The close resemblance between the gravitational force of attraction of two masses and the mutual actions between two electrical charges suggests that the charge on a sphere may act on neighbouring charges as though its charge were concentrated at the centre of the sphere (p. 141).

Superficial Density.—If a charge of Q units is distributed uniformly over a surface of A sq. cms., the charge on each sq. cm. is Q/A. This ratio is termed the **Superficial Density** (usually denoted by the symbol σ). The density of charge on the surface of a metal sphere, of radius r, is $Q/4\pi r^2$.

Distribution of Electricity on the Surface of Conductors.—The superficial density of the charge on any conductor varies according to the *form* of the conductor; it is only uniform in the case of a sphere. Since there is no electrical force inside a hollow conductor (p. 142), the density varies in the same way as the thickness of a hollow shell of the same form would vary if adjusted so that there would be no gravitational force at any internal point. In the case of an ellipsoid the density at any point is proportional to the length of the line drawn from the centre and perpendicularly to the tangent at the point; hence, at the ends of any axis the density is proportional to the length of the axis. This was proved by Coulomb both mathematically and experimentally by means of the Torsion Balance.

Fig. 91 represents the distribution of charge on the surface of conductors of various shapes; the superficial density is represented by the distance of the dotted line from the surface of the conductor. The distribution may be investigated by touching the surface tangentially at any point with a proof-plane, which acquires the charge on that portion of the surface; a measure of the charge is obtained by holding the proof-plane at a fixed point above a gold-leaf electroscope and observing the amount of the divergence of the leaves. The proof-plane is then discharged, and another point of the conductor tested in the same way. The observations are more satisfactory if a circular scale is fixed vertically behind the leaves of the electroscope; the instrument might then be calibrated by imparting to it a series of small equal charges and observing the divergence of the leaves after the addition of each charge. The charges acquired by the proof-plane can be more accurately compared if it is made to touch the interior of a hollow conductor placed on the cap of the electroscope, which thus acquires the whole charge (p. 157). After observing the divergence, it is necessary to discharge the instrument before taking another observation.

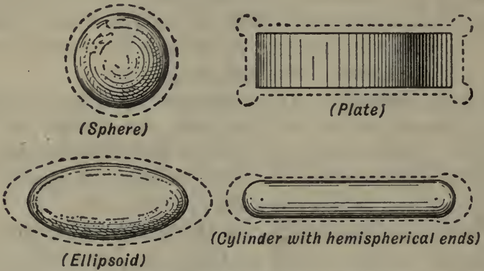
(Sphere) (Plate) (Ellipsoid) (Cylinder with hemispherical ends)

FIG. 91.—Distribution of Charge on Conductors.

A torsion balance, in which a *small* proof-plane is substituted for the fixed ball, may also be used for this purpose. A small permanent charge should be given to the movable ball, and the deflections observed when the charged proof-plane is placed at the zero of the circular scale. In comparing the density at two points, *a* and *b*, the possible errors due to leakage may be obviated by taking three observations at equal time intervals, the 1st and 3rd observations being for the point *a*; the mean of these will give the density at *a*, which is simultaneous with the observed density at *b*.

The main fact to remember is that the more pointed the body the greater is the relative density at the point. A needle may be regarded as an ellipsoid of which one principal axis is very long compared with the other; hence the density at the point will be extremely high. There is always a maximum limit to the density of the charge which can be imparted to a body

(p. 144); beyond this limit any additional charge is given off either by sparks to surrounding bodies, or the air in contact with the surface acquires a portion of the charge and is electrically repelled. This action of a point renders it essential that all electrical apparatus, except in special cases, should be free from points and sharp edges. Further information on the action of points is given on page 218.

SUMMARY

Fundamental Rules.—(i) *Glass rubbed with silk is positively charged.* (ii) *Sealing-wax rubbed with fur or flannel is negatively charged.* (iii) *Bodies with like charges repel, and bodies with unlike charges attract, each other.* (iv) *A charged body always attracts an uncharged body.*

Electrons.—The atom of matter probably consists of an equal number of positive and negative corpuscles, called *electrons*. The negative electron is readily separated from ordinary matter, and its properties have been fully investigated; but the positive electron appears to be associated always with masses of matter comparable with the masses of ordinary atoms. A negatively charged body probably contains an excess of negative electrons, and a substance becomes positively charged when it is deprived of some negative electrons. The inertia of a negative electron is about 1/800 part of that of the hydrogen atom.

The Gold-leaf Electroscope.—(i) *The divergence of the leaves is increased by a similar charge held near to the disc.* (ii) *The divergence is diminished by an unlike charge or earth-connected conductor held near to the disc.*

Fundamental Facts.—Substances may be classified as (i) **conductors,** (ii) **partial conductors**, and (iii) **insulators**.

A charged mass of insulating material is **discharged** most readily if passed through a flame.

Both kinds of electrification are produced in equal quantity.

Gases may be electrified by *chemical action*, and by the *splashing of liquids.*

The Law of Inverse Squares.—*The mutual force between two charged bodies varies inversely as the square of the distance.*

The C.G.S. unit of Quantity (electrostatic) *is that quantity which exerts a force of* 1 *dyne on an equal quantity situated at a distance of* 1 *centimetre.*

The **Superficial Density** of a charge is equal to the *quantity of charge per unit area.* The density is only uniform in the case of a sphere; in other cases the density is greater at points where the curvature is greater, *e.g.* the density is much greater at the apex of a conical conductor than at any point on the curved surface.

QUESTIONS ON CHAPTER VII

1. Describe a method of proving that the force between two small electrified bodies varies inversely as the square of the distance between them.

2. Define unit charge of electricity. Two charged conducting spheres repel each other with a force equal to the weight of a milligram when placed at a certain distance from each other. If the charge on one of the spheres is doubled and the distance between the spheres is also doubled, what is the amount of the repulsion?

3. Two equally charged spheres repel each other when their centres are half a metre apart with a force equal to the weight of 6 milligrams. What is the charge on each, in electrostatic units?

4. Two small spherical pith-balls, each one decigram in weight, are suspended from a point by threads 50 cms. long, and are equally charged so as to repel each other to a distance of 20 cms. Find the charge on each in electrostatic units ($g=980$).

5. Define the surface density of any electrical charge.

If two equal raindrops, charged with equal quantities of electricity of the same kind, were united into one large drop without losing any of their charge, what would be the ratio of the surface densities of the charges before and after union?

6. What is the average surface density of an insulated sphere electrified only by induction?

7. Two small spheres A and B charged with equal quantities of positive electricity are placed at a distance of 10 cms. apart. Where must a third sphere C, charged with twice the quantity on either A or B, be placed in order that B may be in equilibrium, (i) when C is charged positively, (ii) when C is charged negatively?

8. An insulated cubical metal box is charged with electricity. Give a general description of the distribution of the charges on its surfaces, and state how you would prove the accuracy of your statements experimentally.

9. Eight equal metal cubes are placed at a distance from each other and given equal charges. Without being discharged they are then placed on an insulator and built up so as to form a single cube. Compare the original density at a given point on one of the small cubes with the final density at a corresponding point on the large cube. What would the result have been if one of the small cubes had been accidentally discharged?

10. A torsion balance is used to compare the superficial density at two points *a* and *b* of a positively charged conductor. A small positive charge is given to the movable ball, and the torsion-head is adjusted until the ball is opposite the zero of the scale. A proof-plane was brought into contact with the point *a* and then placed inside the balance and, after repelling the ball, fixed at the zero of the scale. The angular deflection was 12°. The proof-plane was removed, discharged, and brought into contact with the point *b*; after proceeding as before, the angular deflection was 15°. What are the relative densities at *a* and *b*?

11. A magnetised knitting-needle, carrying a small insulated pith-ball at one end, is suspended horizontally by a silk fibre; a second pith-ball of the same size is charged and brought into contact with it. Prove that the initial charge is proportional to $\left(\sin\frac{\alpha}{2}\right)^{\frac{3}{2}}$ where α is the angular deflection of the knitting-needle.

12. A short ebonite rod, with a small electrified knob at one end, is mounted so as to turn freely about its centre in a horizontal plane. In a horizontal line with this centre, and at distances from it of a quarter and half a metre respectively, are placed insulated balls that are also charged. The rod makes ten vibrations in a given time, but makes 30 vibrations in the same time if the balls are interchanged. Compare the charges on the two balls.

CHAPTER VIII

THE ELECTRIC FIELD

An Electric Field of Force.—The phenomena of attraction and repulsion observed between charged bodies may be attributed either to the mutual actions of electrical charges at a distance and *quite independently* of the medium between the charges, or to a creation of electric force through the medium which may be supposed to assume new characteristics in the neighbourhood of charged bodies. The latter is the more correct view, and the existence of an *electric field of force* is just as true an expression as that of a magnetic field of force.

The space surrounding a charged body may be mapped out into a series of lines, the direction of which at any point will indicate the direction of the electric force at that point; such lines may be termed **Electric Lines of Force.** *The direction of the force acting along any line of force is arbitrarily chosen as that in which a positively charged body will tend to move.* Lines of force may therefore be supposed to originate from a positively charged conductor and to terminate on a negatively charged conductor; also we may assume that equal charges are present at the opposite ends of each line of force, and that each unit of quantity on a charged conductor is the origin of a definite number of lines of force.

Moreover, all the phenomena of attraction and repulsion may be explained if we attribute to electric lines of force the physical properties of contraction lengthwise and of mutual repulsion sideways (cf. magnetic lines of force).

Experimental Method of Mapping Electrostatic Lines of Force.—The simple appliance shown in Fig. 92 may be used for determining the directions of electric lines of force. It consists of a piece of thin metal wire (5 cms. long) on the ends of which are threaded two small gilt pith-balls; the wire is suspended horizontally by a short silk fibre which is attached to a long silk thread stretched between the ends of two glass rods.

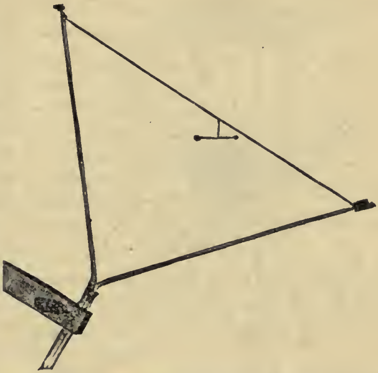

FIG. 92.—Apparatus for Mapping an Electrostatic Field.

EXPT. 44, i.—Connect one terminal of a Wimshurst machine (p. 215) to an insulated sphere, and the other terminal to the nearest gas or water pipe. Hold the pointer in various positions near to the sphere, and observe its direction in each case. The pointer tends to make its length coincide with the direction of the lines of force (Fig. 93).

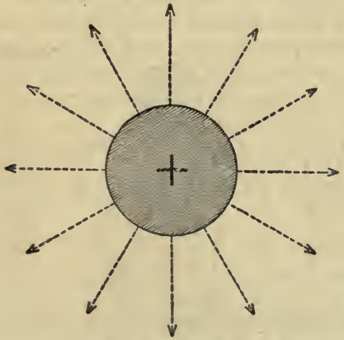

FIG. 93.—Lines of Force due to a Charged Sphere.

EXPT. 44, ii.—Place two insulated metal spheres about 50 cms. apart, and charge them *oppositely* by connecting them to the poles of a Wimshurst machine. Verify the general distribution of the lines of force as shown in Fig. 94.

EXPT. 44, iii.—Connect the two spheres to the *same* terminal of the Wimshurst machine (the other terminal being connected to the nearest gas-pipe). The spheres will now have *similar* charges, and the distribution of the lines of force will resemble Fig. 95.

An interesting method of mapping electrostatic lines of force has been described[1] by Mr. D. Robertson, in which fine sawdust is used (in the same manner that iron filings are used in mapping magnetic fields). Fig. 96 represents the method of conducting the experiment.

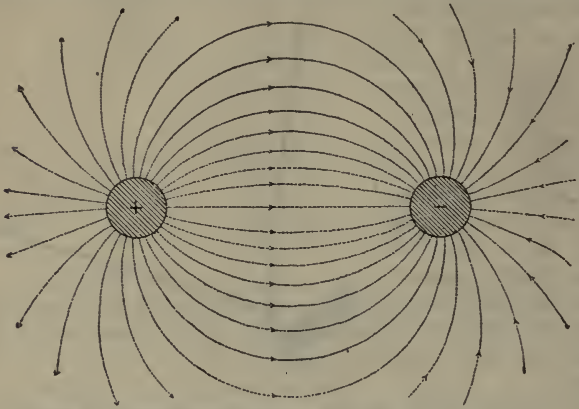

FIG. 94.—Lines of Force due to two oppositely Charged Spheres.

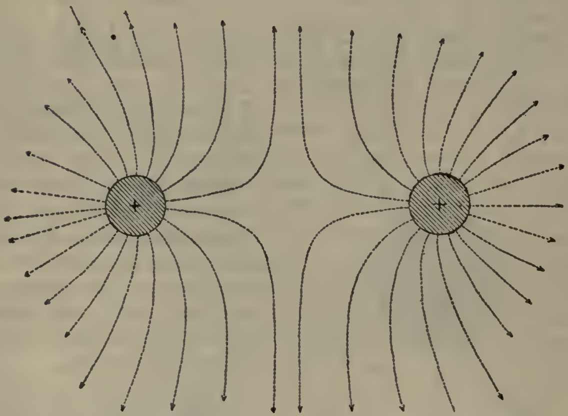

FIG. 95.—Lines of Force due to two similarly Charged Spheres.

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[1] *The Electrical Review*, vol. xlvii. No. 1189 (Sept. 7, 1900).

A is a glass plate, fixed horizontally, to the under surface of which discs of tin-foil are attached. Vertical connecting wires (B, B′) with spring tops touch the discs, and are supported below on metal discs resting on a large sheet of glass. Connection is made to a Wimshurst machine by the wires C and C′. D is a loose wire which serves to short-circuit the machine; it may be raised or lowered by means of a silk thread attached to one end. The sawdust should be fine and fibrous; that obtained from mahogany with a coarse rasp is recommended. The sawdust is sprinkled on the plate, the machine is started, the wire D raised, and the glass plate (A) tapped; D is lowered as soon as the sawdust distributes itself along the lines of force.

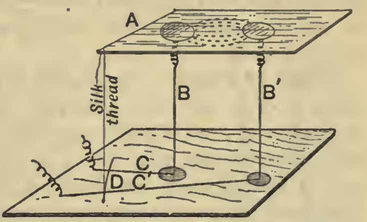

Fig. 96.—Robertson's Method of Mapping an Electrostatic Field.

Diagrams of Electrostatic Fields of Force are drawn by exactly the same method as that adopted for diagrams of magnetic fields of force (p. 51). This method has been used in Fig. 110, which represents the lines of force of the resultant field due to two charges of +30 and −10 units.

Intensity of an Electric Field.—The Intensity at any point is the force acting on a small insulated conductor placed at the point and charged with one unit of positive electrification. From this definition, the intensity at a point distant r cms. from a charge q is q/r^2.

If the intensity is constant over a small area, taken at any point in an electric field and perpendicular to the direction of the intensity, then

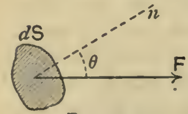

Fig. 97.

Area × Intensity = Total Normal Induction,[1]

or

$$dS \times F = I.$$

If the direction of the intensity (F) is not perpendicular to the small area dS, but is inclined at an angle θ to the normal (Fig. 97), then the component of the intensity acting along the normal is $F \cos \theta$, and

$$I = dS \times F \cos \theta = dS \times N$$

(where N is the normal component of the intensity).

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[1] This is sometimes called the *Total Flux of Force.*

The intensity of a field is not of the same nature as a *Force*; for if q units are placed in a field of intensity F, the force acting on the charge is Fq dynes, or $F = force/q$. Hence the intensity is really *a force divided by a quantity of electricity* (*Dimensions*, p. 518).

Tubes of Force.—Imagine an extremely small portion of the surface of a positively charged conductor, and suppose that a number of lines of force originate from various points on the outer edge of this small area. Such a group of lines may constitute a **tube of force**—the lines of force forming the sides of the tube, and the small conducting areas, where the lines originate and terminate, forming the ends of the tube. The path of such a tube may be graphically represented by a single line of force described along the centre of the tube. The space separating two oppositely charged surfaces may thus be divided up into a number of separate tubes of force with equal and opposite charges at opposite ends of each tube. *If each tube is so drawn that a unit quantity is distributed over each end, it may be termed* **a Unit Tube of Force** (or "*Faraday*" *tube*). The *density* of the charge on any conductor may be also defined as the number of unit tubes of force originating from unit area (p. 129).

The physical properties attributed to lines of force will give to such *tubes* of force a tendency to shorten and to exert what may be compared to a hydrostatic pressure perpendicularly to the sides of the tubes. If two equally and oppositely charged conductors are brought near together, the tubes of force will, owing to their tension, tend to crowd together in the space between the conductors ; if the conductors are actually brought into contact, the opposite ends of each tube will travel towards the point of contact, where the opposite charges will neutralise each other, each tube will thus disappear, and the conductors will be discharged.

If a positively charged conductor is held near to the earth, an induced negative charge will be found on the near portions of the earth's surface, the opposite charges being connected by a system of tubes of force. If any earth-connected conductor (*e.g.* the finger) is held still nearer to the conductor, the tubes shorten, and will now terminate on and near to the finger-tip.

On actually touching the conductor, the tubes disappear and the conductor is discharged.

Expt. 45.—(i) Hold a charged electrophorus disc (p. 162) near to the walls of the room (Fig. 98). Touch with a proof-plane the surface of the wall within the shadow of the disc. Verify by means of an electroscope that the proof-plane is negatively charged.

(ii) Hold a finger near to a positively charged sphere. Touch the finger-tip with a proof-plane, and determine the charge conveyed away on the proof-plane.

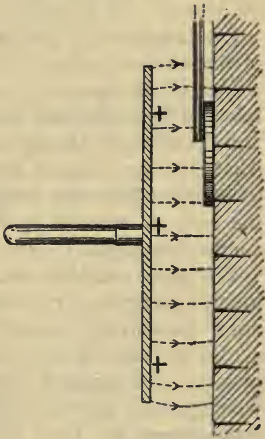

Fig. 98.

Imagine a positively charged conductor to be insulated and situated inside a room, the walls of which are earth-connected. The space between the conductor and the walls is filled with tubes of force, the negatively charged ends of which are distributed over the boundaries of the room. If an *uncharged* insulated sphere is now placed in the field, it will intercept some of the tubes of force (Fig. 99), which thus will be broken into two parts. The broken ends near to *c* will tend to slip down towards the axial line so as to have a minimum length, and here a negative charge will exist; the same number of tubes will emerge from the opposite side of the sphere, and where a positive charge is found. The displacement of these tubes will remove the lateral pressure from those tubes passing just out-

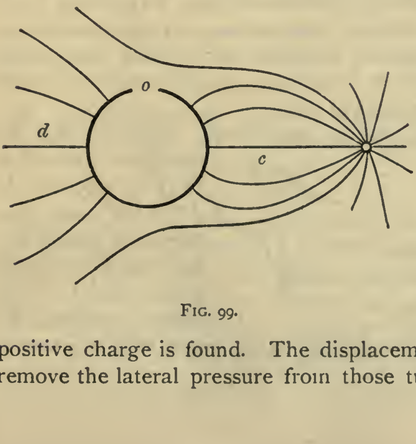

Fig. 99.

side the sphere, and these will be bulged downwards but will not be broken. If the sphere is now touched by the finger, the tubes of force proceeding from *d* to the walls of the room disappear, and the sphere will remain with a negative charge on the side *c*. (Cf. *Charging by Induction*, p. 124.)

Insulating media possess the power of supporting the stress of these tubes of force to different degrees, and in each there is a limit to the amount of stress which can be supported. If the tubes of force become too crowded, the medium breaks down and the opposite charges neutralise each other, accompanied by what is termed a **spark discharge.** Conductors may be regarded as media which are unable to support any electric stress, and the ends of tubes of force may therefore travel freely through conducting media.

We may consider that an insulating medium consists of minute particles of matter insulated from each other, and acted upon inductively by any electric force. A very thin section across any tube of force will then consist of many particles side-by-side, with opposite sides of the section charged oppositely; we might say that the particles are **polarized**, or that an **electric displacement** has been created in each particle. The effect of the charge on either face of the section will be neutralised by the equal charge on the contiguous face of the neighbouring section; the only sections of the dielectric which will exhibit free charges will be where the tube of force terminates, *i.e.* on the surfaces of the charged conductors. In this manner we may regard the insulating medium as the real seat of the electric phenomena, while conductors merely serve as origins or terminations of the tubes of force.

Gauss's Theorem.—Let a small body, charged with $+q$ units, be situated at O (Fig. 100), and let AB be a small portion of a closed surface which completely surrounds O. The induction through AB will be along OF, and inclined at an angle θ to the normal of AB.

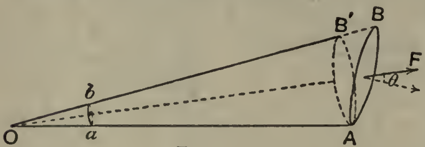

FIG. 100.

If AB′ is a surface drawn perpendicular to OF, then $AB' = AB \cos \theta$. Also, if *ab* is the *solid angle* of the surface AB, then $AB' = ab \times OA^2$.

Hence, the **normal induction** through AB is equal to

$$\begin{aligned} &\text{area } AB \times F \cos \theta \\ &= \text{area } AB' \times q/OA^2 \\ &= ab \times q. \end{aligned}$$

The total normal induction through the whole surface, of which AB is a part, will be $\Sigma(\text{area } ab) \times q = q \times$ surface of a sphere of unit radius $= q \times 4\pi$.

Hence, **the total normal induction over a closed surface is equal to 4π times the charge inside the surface.**

In any Tube of Force the Intensity is inversely as the cross-section taken at right angles to the tube.—Let S and S′ (Fig. 101) represent sections across a tube of force. Since the curved sides of the tube are formed by the lines of force there will be no induction normal to these sides. At the end S, the normal induction is FS *inwards*; at the end S′ it is F′S′ *outwards*. Hence the total normal induction through the surface of the tube is F′S′ – FS. But there is no charge inside the tube; hence

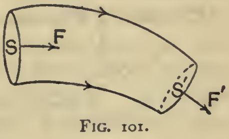

FIG. 101.

$$F'S' - FS = 0,$$
$$\text{or} \quad F'S' = FS.$$

This is quite analogous to the flow of water along a pipe of varying diameter, the velocity of the water being comparable to the intensity of the electric force.

A Charged Sphere acts on External Points as though the Whole Charge were Concentrated at the Centre.—In Fig. 102 A represents a small sphere with its centre at O, and charged with $+q$ units. Consider its action at R, where the normal Intensity is F.

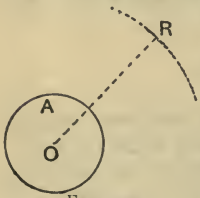

FIG. 102.

Through R describe a sphere surrounding A. The total normal Induction over the surface of the large sphere

$$= F \times \text{area of sphere} = F \times 4\pi OR^2.$$

But, by Gauss's Theorem, the total Induction $= 4\pi q$.

Hence $$F \times 4\pi OR^2 = 4\pi q,$$

or $$F = \frac{q}{OR^2}.$$

In words, **the electrical conditions outside a charged sphere are the same as though the charge were concentrated at the centre.**

The Intensity inside a Hollow Charged Conductor is Zero.—Charges of opposite sign are always found at opposite ends of a line of force; hence, in the case of a hollow conductor charged only on the surface, no line of force can exist within the conductor (as suggested in Fig. 103). The consequent absence of an electric field of force within the conductor results in the internal intensity being zero.

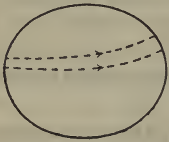

FIG. 103.

Coulomb's Theorem.—Consider the tube of force originating from a small area S (Fig. 104) on the surface of a conductor, which is charged so as to have a surface density σ. Imagine the tube extended a very short distance within the conductor, and that two sections dS and dS' are drawn perpendicularly to the sides of the tube (the former near to the outer surface, and the latter near to the inner surface), thus constituting a cylinder which is very wide in comparison with its length. The normal induction through the sides and through the end dS' is zero; hence the only induction through the boundaries of the cylinder is FdS through the end dS. But, by Gauss's Theorem, the total induction $= 4\pi \times S\sigma = 4\pi\sigma \times dS$ (approximately).

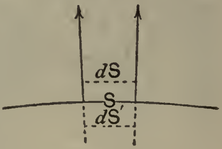

FIG. 104.

Hence $$FdS = 4\pi\sigma \times dS,$$

or $$F = 4\pi\sigma.$$

In words, **the intensity of the field near to the surface of a charged conductor is equal to 4π times the surface density of the charge.**

In the case of a sphere the same result may be obtained more simply. For, since the charge acts as though concentrated at the centre, the intensity, at a distance r, is Q/r^2. But if the point is very near to the surface of a sphere, then r is approximately the radius of the sphere, and $Q = 4\pi r^2\sigma$. Hence, the intensity $= 4\pi r^2\sigma / r^2 = 4\pi\sigma$.

The number of Faraday tubes passing through unit area drawn perpendicularly to the direction of the Force is $\frac{F}{4\pi}$.—Fig. 105 represents a bundle of Faraday tubes originating from a charged sphere. S′ is a cross-section drawn perpendicularly to the direction of F′. By Gauss's Theorem, $F'S' - FS = 0$, or $F'S' = FS$. But, by Coulomb's Theorem, $F = 4\pi\sigma$. Hence

$$F'S' = 4\pi\sigma S.$$

But σS is the quantity of charge on the surface S, and therefore equal to the number (N) of Faraday tubes originating from the area S and passing through S′. Hence $F'S' = 4\pi N$. If $S' = 1$ sq. cm., then $F' = 4\pi N$, or $N = F'/4\pi$.

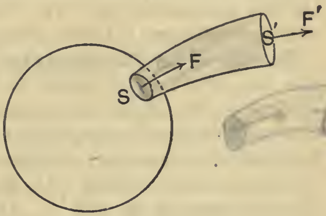

FIG. 105.

Electrostatic Pressure.—Fig. 106 represents a charged conductor; P and Q are two points infinitely near to the outer and inner surfaces respectively, of the small area dS of the conductor.

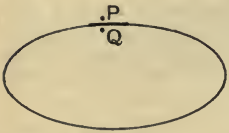

FIG. 106.

The intensity (F) at the point P is due partly to the charge on dS and partly to the charge on the remainder of the conductor; if these components are f and f', then $F = f + f' = 4\pi\sigma$.

At the point Q, the component f due to dS will have changed sign and become $-f$, since its direction is reversed; but the other component f' will be the same as at P. Also the total intensity at Q is zero, since the point is within a closed conductor. Hence

$$-f + f' = 0$$

$$\text{or} \quad f = f' = \frac{F}{2} = 2\pi\sigma.$$

We may regard the area dS as being disconnected from the conductor and situated in the field due to the charge on the remainder of the conductor, the intensity of this field being f'. The mechanical force acting on dS will be equal to the product of the charge on dS and the intensity of the field in which it is situated, or

$$\text{Force} = \sigma dS \times f' = \sigma dS \times 2\pi\sigma = 2\pi\sigma^2 dS.$$

In order to find the Force per unit area, put $dS=1$, then

$$\text{Force}^1 \text{ (per unit area)}=\sigma f'=2\pi\sigma^2=\frac{F^2}{8\pi}.$$

When the air surrounding a charged conductor is at normal pressure (76 cms. of mercury) the maximum value of F is about 100; if the intensity is increased above this value any additional charge escapes into the surrounding air. The corresponding maximum value for the *electrostatic pressure* is $10^4/8\pi$ dynes.

Energy of an Electrostatic Field.—Since the force acting on unit area of a charged surface is $F^2/8\pi$ dynes, the work required in order to move unit area in the opposite direction to F and so that the tubes of force are lengthened by an amount dx is equal to $F^2 . dx/8\pi$ ergs, and the increase in the volume of the field is dx cubic centimetre. The work done is stored up in that volume of the field described by the movement of the surface. Hence, *the energy stored up in unit volume of a field, at a part where the intensity is F, is equal to* $\frac{F^2 . dx}{8\pi}\Big/dx=F^2/8\pi$ *ergs.* In the case of a dielectric of specific inductive capacity K the energy per cubic centimetre of the dielectric is $F^2K/8\pi$ ergs.

Example.—An insulated sphere (5 cms. radius) is charged to a potential of 500 electrostatic units. Calculate the electrostatic pressure per sq. cm., and compare this with the normal pressure of the atmosphere.

$$\text{Density } (\sigma)=\frac{Q}{4\pi r^2}.$$

But the capacity C of a sphere is equal to its radius r (p. 169); and $Q=CV$ (p. 168).

Hence
$$\sigma=\frac{Vr}{4\pi r^2}=\frac{V}{4\pi r}=\frac{500}{20\pi}=\frac{25}{\pi}.$$

$$\text{Pressure (per sq. cm.)}=2\pi\sigma^2=\frac{2\pi\times 25^2}{\pi^2}=\frac{1250}{\pi}\text{ dynes.}$$

The normal pressure of the atmosphere is the weight of a column of mercury 76 cms. long and 1 sq. cm. cross-section, and is equal to

$$(76\times 13.6\times 981)=(1.01\times 10^6)\text{ dynes per sq. cm.}$$

$$\frac{\text{Electrostatic Pressure}}{\text{Atmospheric Pressure}}=\frac{1250}{\pi\times 1.01\times 10^6}=\frac{1}{811\times\pi}.$$

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[1] If the conductor is surrounded by a dielectric of *specific inductive capacity* K, the force per unit area is $KF^2/8\pi$.

Tension along a Tube of Force.—From previous section each unit area experiences a force equal to $2\pi\sigma^2$ dynes. But the number of tubes of force originating from unit area is equal to the density of the charge. Hence the tension along each tube is $2\pi\sigma^2/\sigma = 2\pi\sigma = \frac{F}{2}$ dynes. In words, *the tension due to one tube of force is equal to one-half the intensity of the field at that point.*

ELECTRIC POTENTIAL

Introductory.—If two insulated uncharged conductors (A and B) of the same material are brought into contact there will be no electrical re-distribution. If A is positively charged and B uncharged, then, on bringing them into contact, a portion of A's charge will be transferred to B. If A and B are both charged, there are three possible results on bringing them into contact: (i) a portion of A's charge may be transferred to B; (ii) the charges may remain unaltered; or (iii) a portion of B's charge may be transferred to A. **The condition which determines the transference of charges between conductors is termed Electric Potential.** Also, **if a portion of the charge on a positively charged conductor A is transferred to a conductor B, when contact is made, then A is said to have a higher electric potential than B.**

Potential, in electrical phenomena, is comparable to *temperature* in heat phenomena, since the transference of heat from one body to another is determined by any temperature-differences which may exist. Again, potential may be compared with *level* in hydrostatic phenomena, since the flow of liquid from one point to another is the consequence of any difference of level which may exist.

It is impossible to determine the *absolute* potential of any conductor, but *differences of potential* can readily be measured. The potential of the earth is always taken as the *zero* of potential; the earth may be regarded as having a constant potential, since its dimensions are so great that no charges of experimental magnitude would produce an appreciable change of potential.

The tendency of an electric charge on a conductor to be transferred to conductors at a lower potential may also be realised by considering the forces due to the charge, which would act upon a small positively charged sphere which is free to move and is situated in the immediate neighbourhood. If the conductor is positively charged, a force of repulsion would act upon the small sphere, and work would be done by the electric force in moving the sphere to a greater distance; the force would only become zero at an infinite distance. Similarly, work would have to be done by external agencies upon the small sphere in order to convey it from an infinite distance towards the charged conductor, and the amount of work required would be a measure of the potential at the final position of the sphere. In this manner we derive the following definitions:

(i) **The potential at any point near to a positively charged conductor is equal to the work (in ergs) which is required in order to convey a unit of positive electricity up to the point from an infinite distance.** If the charge is negative, the potential is measured by the work necessary to convey the unit charge *from* the point *to* an infinite distance away; in this case the potential is negative.

(ii) **The difference of potential between two points is equal to the work, in ergs, done by the electrical forces when one unit of positive electricity is transferred from the point of higher potential to that of lower potential.** Hence, **the potentials of two points differ by unity when one erg of work is done in transferring one unit quantity of electricity from one point to the other.**

The work done is independent of the path traversed. For, let A and B (Fig. 107) represent two points, the potentials of which are V_1 and V_2; and assume that a charge Q may be conveyed along either of the paths *a* or *b*. It follows, from the law of the conservation of energy, that the amount of work done will be the same, whichever path is traversed. For, suppose that W ergs are done in transferring the charge along the path *a*, and that (W + *w*) ergs are done if the path *b* is traversed. If the charge traverses the path *b*, and is conveyed back to A along the path *a*, then (W + *w*) ergs are done by the electric forces, and W ergs are done against the electric forces; hence the

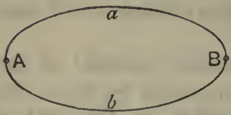


FIG. 107.

charge returns to A with an increase of energy represented by *w* ergs, while no permanent electrical change has taken place. If this were possible any quantity of energy might thus be accumulated, which is contrary to the principle of the conservation of energy. Hence, the work done is independent of the path traversed.

The Potential Slope.—If P and Q are two points, having potentials V_P and V_Q, and so near together that the intensity F is practically constant along the line joining the points, and if *d* is the distance between the points, the work W done by the electric forces in moving a unit of positive electricity from P to Q is equal to F*d*. But, by definition, W is also equal to $V_P - V_Q$. Hence $Fd = V_P - V_Q$, or $F = (V_P - V_Q)/d$. Expressed in words, **the intensity is equal to the rate at which the potential diminishes per unit distance.** If the rate of fall of potential in any field is uniform, then the intensity is uniform at all points. Moreover, if the intensity is uniform, then, since $FS = F'S'$ (by Gauss's theorem), the tubes of force of the field will be of constant cross-section, and the lines of force constituting the tubes will be strictly parallel to each other. The ratio $(V_P - V_Q)/d$ may be termed the *potential slope.*

If dV is the small potential difference between two points separated by a small distance ds, then $F = dV/ds$; or, electrostatic force may be defined as *the differential coefficient of potential with respect to space.*

The Potential at any point, distant r cms. from a charge $+q$, is $\frac{q}{r}$.—Let a small conductor, charged with $+q$ units, be situated at O (Fig. 108), and let r, r_1, etc., be points distant r, r_1, etc., cms. from O.

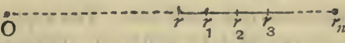


FIG. 108.

The force acting on one unit of positive electricity at $r = q/r^2$.
" " " " " " $r_1 = q/r_1^2$.

The *average force*, between r and $r_1 = q/rr_1$.*

The *work* done in moving one unit, from

$$r \text{ to } r_1 = \frac{q}{rr_1}(r_1 - r) = \frac{q}{r} - \frac{q}{r_1}.$$

---

*Cf. p. 54.

The *work* done in moving one unit, from

$$r_1 \text{ to } r_2 = \frac{q}{r_1 r_2}(r_2 - r_1) = \frac{q}{r_1} - \frac{q}{r_2}.$$

$$\text{,, \quad ,, \quad ,, } r_{n-1} \text{ to } r_n = \frac{q}{r_{n-1} r_n}(r_n - r_{n-1}) = \frac{q}{r_{n-1}} - \frac{q}{r_n}.$$

The *total work* done in moving one unit, from r to r_n,

$$= \frac{q}{r} - \frac{q}{r_n}.$$

But the total work is also equal to the potential difference $(V - V_n)$ between the points r and r_n. Hence

$$V - V_n = \frac{q}{r} - \frac{q}{r_n} = q\left(\frac{1}{r} - \frac{1}{r_n}\right).$$

If r_n is at an infinite distance, $\frac{1}{r_n} = 0$, and $V_n = 0$, therefore

$$V = \frac{q}{r}.$$

Equipotential Surfaces.—No work is required in order to move a charged conductor perpendicularly to the lines of force at the point where it is situated; hence, **an equipotential surface is at all points perpendicular to the direction of the lines of force passing through it.**

In the case of a single charge situated at a point, the equipotential surfaces will be spheres described round the point as centre; for, on any surface, the ratio of q/r is constant, and therefore r is constant. The equipotential surfaces surrounding a charge of 4.35 units are represented in Fig. 42 (p. 56), which applies equally to magnetic and to electric fields of force.

The surface itself of any charged insulated conductor must also be an equipotential surface. For, if there is any potential difference between two points on the surface there will be a transference of electricity until the potential is the same at both points. It follows, therefore, that the lines of force, where they originate, or terminate at the surface of any conductor, must be perpendicular to the surface.

EXPT. 46.—Charge the insulated cylinder (Fig. 109). Connect the disc of a proof-plane to the disc of a gold-leaf electroscope. Holding the proof-plane by its insulating handle, bring it into contact with the cylinder. Move the proof-plane to various parts of the surface, and note that the divergence of the leaves is constant.

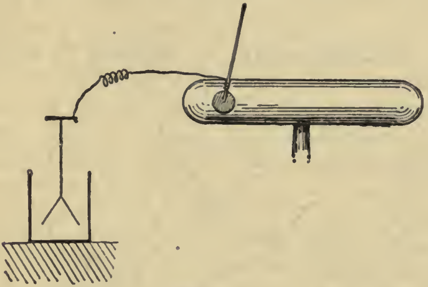


FIG. 109.

The potential V at any point, due to several charges q_1, q_2, etc., situated at distances r_1, r_2, etc., from the point, will be the algebraic sum of the potentials due to each charge, or

$$V=\frac{q_1}{r_1}+\frac{q_2}{r_2}+\dots$$

The equipotential surfaces due to two charges, q and q_1, are obtained by drawing the series of surfaces due to each charge separately; a series of points are found which have the same *resultant* potential, and a line drawn through such points traces out an equipotential surface. The surfaces due to two equal and opposite charges will be identical with the surfaces represented in Fig. 43 (p. 57). In Fig. 110 the continuous lines indicate the lines of force, and the dotted lines indicate the surfaces of equal potential, due to two charges (+30 and −10) situated at points 10 cms. apart. (The diagram is drawn to a scale of $\frac{3}{10}$.)

The Potential inside a charged hollow Conductor is uniform and equal to that of the Conductor.—Imagine an equipotential surface drawn inside the conductor and close to its inner surface, the potential of points on the surface not being the same as that of the conductor itself. Work would be done in conveying a unit charge from one surface to the other. A definite intensity would therefore exist between the surfaces (since $Fd = V_1 - V_2$). But it has been shown (p. 142) that the intensity inside a charged hollow conductor is zero. Hence $V_1 - V_2 = 0$; and the same applies to all points inside the conductor. *Differences* of potential can only be created inside a hollow conductor by introducing within it another insulated charged conductor. The same result may be expressed thus: **No lines of force can exist within a hollow conductor unless this also surrounds other charged conductors.**

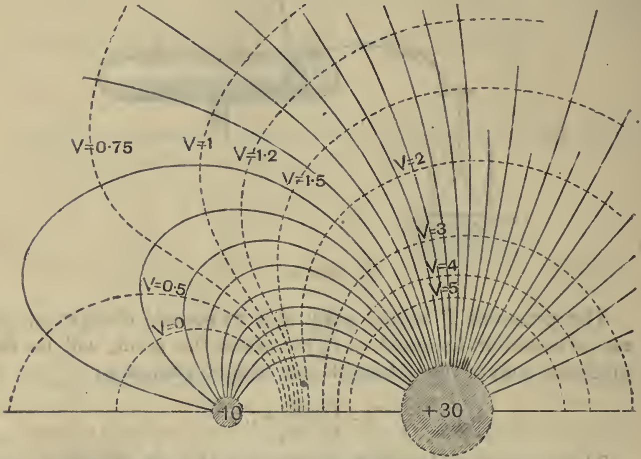


FIG. 110.—Lines of Force and of Equipotential due to two Charges of +30 and −10 units.

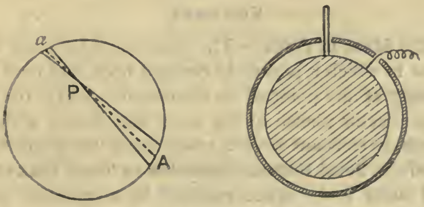
The Charge on a Conductor is distributed entirely on the Outer Surface.—This result follows from the fact that since there are no lines of force within a hollow conductor there can be no charge on an inner surface. It was first proved experimentally by Biot, who found that the charge on a sphere was entirely removed by covering the surface with two thin hemispherical brass shells, each supported on an insulating handle.

EXPT. 47.—Charge an insulated tin by means of an electrophorus. Touch the outside with a proof-plane, and verify the charge with a gold-leaf electroscope. Discharge the proof plane, touch the *inside* of the tin, and verify the absence of any charge.

Faraday obtained the same result by means of a *butterfly net*, which could be turned inside-out by means of a silk thread. The charge was always found on the outside.

Verification of the Law of Inverse Squares.—It may be proved theoretically that, if electric forces obey the Inverse Square law, the electric intensity within a charged conductor must be zero; and it was proved experimentally by Cavendish, about 1785, that the law cannot differ from that of the Inverse Square by more than $\frac{1}{50}$th part.

In Fig. 111 (i) let P be any point within a conductor charged uniformly with +ve electricity. With P as common apex two very narrow cones are drawn which terminate on the surface of the conductor. Since each cone is very narrow we may assume that all points of each base are equidistant from P. If q and Q are the respective charges on the areas at a and A, the intensity at P, due to the charge q, is $q/(\mathrm{P}a)^2$ and in the direction PA; also the intensity at P, due to the charge Q is $\mathrm{Q}/(\mathrm{PA})^2$, and in the direction P$a$. The ratio of these forces is $q(\mathrm{PA})^2/\mathrm{Q}(\mathrm{P}a)^2$. But $q/\mathrm{Q}=(\text{area at } a)/(\text{area at A})=(\mathrm{P}a)^2/(\mathrm{PA})^2$. Hence the ratio is equal to unity, and the electric intensity at P is zero. The same result follows for each pair of cones into which the entire surface may be divided; and the resultant intensity at P due to the whole charge must be zero. If, in the law, the power of the distance is less than 2, the electric intensity at P will be directed from the centre outwards; on the other hand, if the power is greater than 2, the electric intensity at P will be directed from P towards the centre.



(i) FIG. 111. (ii)

In his experimental verification of the inverse square law, Cavendish used a form of apparatus indicated in Fig. 111 (ii). A large conducting sphere, supported by an insulating handle, was surrounded by two concentric hemispheres of conducting material. A short wire, with silk thread attached, passed loosely through one of the hemispheres so as to make electric contact between the inner and outer spherical surfaces. An electric charge was given to the hemispheres, the wire withdrawn by means of the silk thread, and the hemispheres removed. Assuming that the outer sphere was charged +ly; if the power of the distance were less than 2, the inner sphere would have been left with a −ve charge, and if the power were greater than 2, a +ve charge would have remained. On testing the sphere by means of a pair of pith-balls, it was found that no appreciable charge remained on its surface.

In 1870 Maxwell repeated the above experiment, using a quadrant electrometer instead of pith-balls. The greater sensitiveness of the electrometer demonstrated that, in the law of electric force, the extreme limits of the power of the distance are $2 \pm \frac{1}{21000}$.

SUMMARY

Electrostatic Lines of Force.—The direction of the force is arbitrarily chosen as that in which a positively charged body will tend to move.

The **Intensity** at any point of an electrostatic field is the force acting on a small conductor placed at the point and having a charge of +1. If the intensity is constant over a small area, the product *intensity* × *area* is termed the **Total Normal Induction** or **Total Flux of Force.**

A **tube of force** is the space enclosed by a group of lines of force which constitute the sides of the tube of force. A *unit tube of force* has unit quantity of electricity distributed over each end.

Gauss's Theorem.—*The total normal induction over a closed surface is equal to* 4π *times the charge inside the surface*, or $I = 4\pi q$.

The intensity at external points due to a charged sphere is the same as though the charge were concentrated at the centre. The intensity *inside* a hollow charged conductor is zero. The intensity *near* to the surface of a hollow conductor is equal to 4π times the surface density of the charge (**Coulomb's Theorem**).

The number of unit tubes of force passing through unit area drawn perpendicularly to the direction of the force is $F/4\pi$.

The **Electrostatic Pressure,** or mechanical force per unit area of a charged surface, surrounded by air, is equal to $2\pi\sigma^2$ or to $F^2/8\pi$.

The *tension* along any tube of force is equal to $F/2$.

The **Electrostatic Potential** at any point *is equal to the work, in ergs, required in order to convey a unit of positive electricity from an infinite distance up to the point.*

The *difference of potential* between two points is equal to the work done by the electrical forces in transferring a unit of positive electricity from the point of higher potential to the point of lower potential. *The work done is independent of the path traversed.*

The fall of potential per unit distance is equal to the intensity of the field.

The potential at any point, distant r cms. from a charge $+q$, is $+\frac{q}{r}$.

The potential inside a charged hollow conductor is uniform and equal to that of the conductor.

The charge on a conductor is distributed entirely on the outer surface.

QUESTIONS ON CHAPTER VIII

1. Two small insulated conductors unequally and oppositely electrified, are near each other. Show by a diagram the general course of the lines of force in the field, and give general reasons for the shape which you give to any one of the curved lines you draw.

2. Draw carefully the lines of force due to charges 1 and -2 placed at a distance of 1 cm. apart.

3. Calculate from the data of Question 2, (*a*) the position of the point at which the force is zero; (*b*) the position of the points on the line joining the centres of the charged conductors at which the potential is zero.

4. Draw the lines of force and the equipotential surfaces due to charges $+100$ and $+200$ units, contained on two small spheres placed 1 metre apart. Calculate the position of the point at which the force is zero, and also the position of the points on the line joining the centres of the spheres at which the potential is zero.

5. Inside a hollow spherical conductor are placed two small spheres, insulated from each other and from the outer vessel, the one charged with positive and the other with a smaller charge of negative electricity. Draw a picture showing the distribution of the lines of electric force outside the conductor.

6. An insulated uncharged metal sphere is placed at a certain distance from another precisely similar but charged sphere. If the two spheres be connected by a fine wire, will (1) the quantity of electricity which passes, (2) the energy of the discharge, be affected by the distance between the spheres at the moment when the discharge takes place?

7. Explain the term **electric potential.** If 100 units of work must be done in order to move an electric charge equal to 4 from a place where the potential is -10 to another place where the potential is V, what is the value of V?

8. Find the strength of the electric field at a point just outside an isolated sphere of 10 cms. radius charged with 10 electrostatic units.

9. An insulated conductor, placed at some distance from other conductors, receives a charge of positive electricity. Is its potential altered when another conductor, uninsulated and uncharged, is brought near to it? Give reasons for your answer.

10. A hollow metal vessel is insulated and charged to potential V, and the following operations are successively performed: (1) an insulated metal ball is lowered into the jar without touching it, (2) the ball is momentarily earthed, (3) the jar is momentarily earthed, and (4) the ball is removed to a distance. State the changes of potential of the jar and the ball at each stage.

11. Charges of $+5$ units each are placed at the corners of a square the side of which is 10 cms. Find (i) the potential at the point of intersection of the diagonals, and (ii) how much work is required to convey a charge of $+5$ units from the middle of either side to the intersection of the diagonals.

12. Charges 1, 2, 3, and -4 units are placed at the corners of a square taken in order. If the length of each side is 2 cms., find the potential at the middle point of the side joining 1 and 2.

13. The charge and the potential of an isolated sphere are each numerically equal to 10. Draw as correctly as you can the equipotential surfaces for potentials 2, 4, 6, 8.

14. An egg-shaped conductor having been electrified, its middle part is touched by a proof-plane, which is then held near the knob of a gold-leaf electroscope. The knob having been momentarily connected to earth the proof-plane is removed, and, after touching an end of the conductor, is replaced in its position near the electroscope. Describe and explain the effect on the leaves, and state what would happen had the conductor been touched first at the end and then at the middle.

15. How much energy is expended in carrying a charge of 50 units of electricity from a place where the potential is 20 to another where it is 30? What is meant by saying that the potential of a conductor is 20?

16. An insulated ice-pail and an insulated brass ball are both charged with positive electricity at a distance from each other, the pail to a high potential, the ball to a low potential.

The ball is then brought near to the pail and lowered into it, without touching it, until the bottom is reached. After contact the ball is entirely removed. Describe the changes in the potential both of ball and pail (1) before contact, (2) on contact, (3) after removal.

17. A positively charged conductor is brought near an insulated uncharged brass ball. Is the potential of the ball altered thereby? Would this alteration (if any) be modified by the ball having a needle sticking out of its surface, and would any such modification depend on the position of the needle?

18. A brass ball charged with positive electricity is supported inside a hollow metal sphere. How will the distribution of the induced charges of the inside and outside of the sphere be modified when the ball is moved from the centre of the sphere to a point close to the surface? Illustrate your answer by drawing the lines of force in the two cases.

19. An electrified body is brought into the neighbourhood of (*a*) an insulated conductor, (*b*) an earth connected conductor. Describe exactly the effect on the potentials of the electrified body and of the unelectrified conductors in each case.

20. Give what is in your opinion the best proof that the force between point charges of electricity varies inversely as the square of the distance between them.

21. What is the maximum potential, in electrostatic units, to which spherical conductors of 1 cm. and 0.1 cm. radius can be charged when surrounded by air at normal pressure?

22. A small pith ball weighing one decigramme suspended by a silk fibre and charged with positive electricity is repelled when a charged glass rod is brought near it. If the direction of the electric field of the glass rod near the ball is horizontal and its magnitude equal to 20 C.G.S. electrostatic units, when the deflection of the fibre is 45°, what is the charge on the ball?

CHAPTER IX

ELECTROSTATIC INDUCTION

The Total Induced Charges are equal to the Inducing Charge.—The equality of the induced and the inducing charges was first experimentally proved by Faraday by means of a metal ice-pail; the following experiment is therefore frequently referred to as *Faraday's Ice-pail Experiment.*

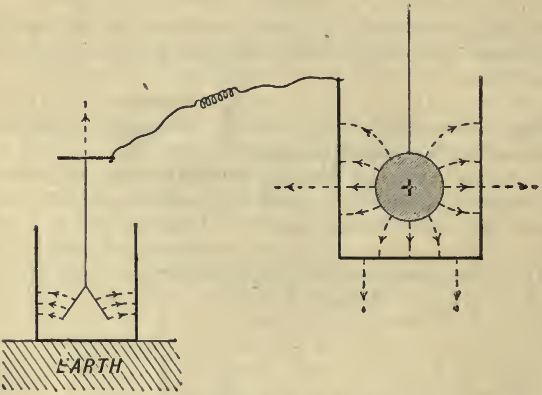


FIG. 112.—Faraday's Ice-pail Experiment.

EXPT. 48.—Introduce a positively charged sphere, supported by an insulating handle, into an insulated hollow vessel (Fig. 112) connected to the disc of an electroscope, and note the divergence of the leaves. Momentarily touch the vessel with the finger; the leaves collapse. Allow the sphere to touch the inside of the vessel; the leaves still show no divergence. Remove the sphere, and observe that it is completely discharged. Repeat the experiment, but omit to touch the vessel with the finger, and note that the divergence remains constant and that the sphere is completely discharged.

Practically all the tubes of force originating from the charged sphere are intercepted by the vessel; a negative charge is induced on the inside of the vessel, and a positive charge on the outside (including the electroscope). When the vessel is touched momentarily, the tubes proceeding from the outside of the vessel vanish. When the sphere touches the inside of the vessel, the fact that the leaves remain undiverged proves that **the negative charge induced on the inside of the vessel exactly neutralises the positive charge on the sphere.** In the second part of the experiment, the divergence of the leaves due to the induced positive charge on the outside of the vessel remains the same even when the sphere's charge has been entirely transferred to the vessel by making contact, which proves that **the induced positive charge is equal to the inducing positive charge on the sphere.**

A charge is transferred completely from one conductor to another if the latter is hollow and capable of containing the former; only a portion of the charge is transferred if contact is made between the outer surfaces.

Charging a Conductor by Induction.—The method of charging a conductor by induction has previously been described (p. 124). The observed results may be explained either by the characteristic properties of lines of force, or by the electrical effects produced by potential differences in any field of force. Thus, the conductor AB (Fig. 113) intercepts *some* of the lines of force proceeding from the positively charged glass rod,

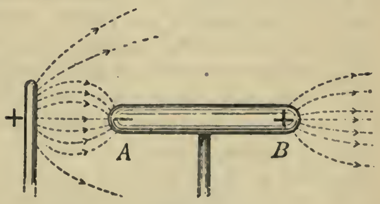


FIG. 113.

creating a negative charge at A, and a positive charge at B, while many of the lines of force proceed to the walls of the room without interruption. On touching AB with the finger, the lines of force between B and the walls of the room vanish (Fig. 114), and many lines of force which previously passed direct to the walls of the room now find their equivalent negative charge by traversing the shorter distance to the end A of the earth-connected conductor. We consequently find that rather more lines of force now terminate on the end A than was the case before connecting to earth, and the negative charge at A is therefore slightly increased. When the charged glass rod is removed to a distance, the lines of force terminating at A are distributed uniformly over the conductor (Fig. 115).

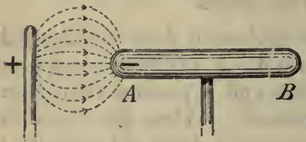


FIG. 114.

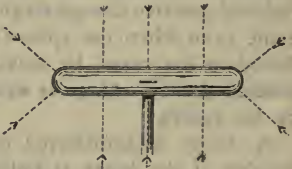


FIG. 115.

On the other hand, we may regard AB as being in a field of positive potential, and that A is in a region of higher potential than B, with the result that positive electricity flows from A to B until the potential of the conductor is uniform. When AB is touched its potential is reduced to *zero*, so that no lines of force pass from the conductor to the walls of the room. The zero potential of the end A is the resultant of its positive potential due to the charged glass rod and of its negative potential due to its own negative charge. When the glass rod is removed, the negative charge accumulated at the end A spreads over the entire surface and gives to the cylinder a negative potential.

EXPT. 49.—Charge an insulated conductor positively by induction, by using a negatively charged sealing-wax rod instead of a positively charged glass rod (Fig. 116). Prove that the charges are, at each stage, of opposite sign to those obtained when a glass rod was used.

As before, the phenomena observed may be explained by considering either the lines of force or the potential differences.

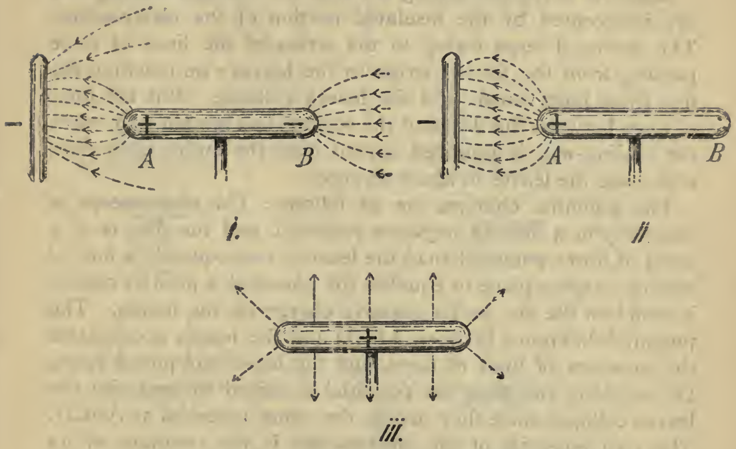


FIG. 116.—The Process of Charging a Conductor Positively by Induction.

Theory of the Gold-Leaf Electroscope.—The theory of the process of charging a gold-leaf electroscope by induction is identical with that described in the previous section. Fig. 117 represents the electroscope under different electrical conditions; ABCD represents the tin-foil strips and base, which are earth-connected, and therefore at zero potential. The observations can be explained in terms of potential with the aid of the potential diagrams drawn to the right of each figure. The vertical dotted line is the line of zero potential; positive and negative potentials are represented by horizontal lines drawn to the right and left respectively of the zero line. The thick continuous lines represent the changes in potential along the line of the electroscope's axis. Strictly, the lines showing changes of potential through the intervening air spaces should be hyperbolic curves instead of straight lines (cf. Fig. 42, p. 56).

EXPT. 50.—Hold a negatively charged rod of sealing-wax over the disc of an electroscope (Fig. 117, i). Momentarily touch the disc

(Fig. 117, ii), and remove the sealing-wax to a distance (Fig. 117, iii). *The instrument is now positively charged.*

Lines of force proceeding towards the charged sealing-wax are intercepted by the insulated portion of the electroscope. The leaves diverge owing to the stress of the lines of force passing from the tin-foil strips to the leaves; on touching the disc these lines vanish, and the leaves collapse. But the lines of force between the disc and the sealing-wax remain, and, when the sealing-wax is removed, spread over the entire conductor, and cause the leaves to again diverge.

The potential changes are as follows: The electroscope is originally in a field of negative potential, and the disc is at a point of lower potential than the leaves; consequently a flow of electricity takes place to equalise the potential, a positive charge is found on the disc and a negative charge on the leaves. The potential-difference between ABCD and the leaves necessitates the presence of lines of force, and the leaves are pulled apart. On touching the disc, the potential is raised to zero, and the leaves collapse since they are at the same potential as ABCD. The *zero* potential of the electroscope is the resultant of its negative potential due to the wax and its positive potential due to its own positive charge. On removing the sealing-wax, the positive charge is distributed over the entire conductor, giving to it a positive potential, with a consequent divergence of the leaves.

EXPT. 51.—Hold a positively charged glass rod over the disc of a positively charged electroscope (Fig. 117, iv), and note the *increased* divergence. Hold a negatively charged body some distance away from the disc (Fig. 117, v), and note the *diminished* divergence; gradually bring the charged body nearer, and note that the leaves slowly collapse (Fig. 117, vi); bring the charged body still nearer, and note that the leaves again diverge (Fig. 117, vii). Hold the hand over the disc, and note the diminished divergence (Fig. 117, viii).

In Fig. iv the potential of the instrument is *increased* owing to the proximity of the positively charged body. In Fig. v its potential is slightly *reduced*; in Fig. vi it is *zero*; in Fig. vii it is *negative*; in Fig. viii it is slightly *reduced* by the negative charge induced on the lower surface of the hand. **Note that the hand and an oppositely charged body have the same effect.**

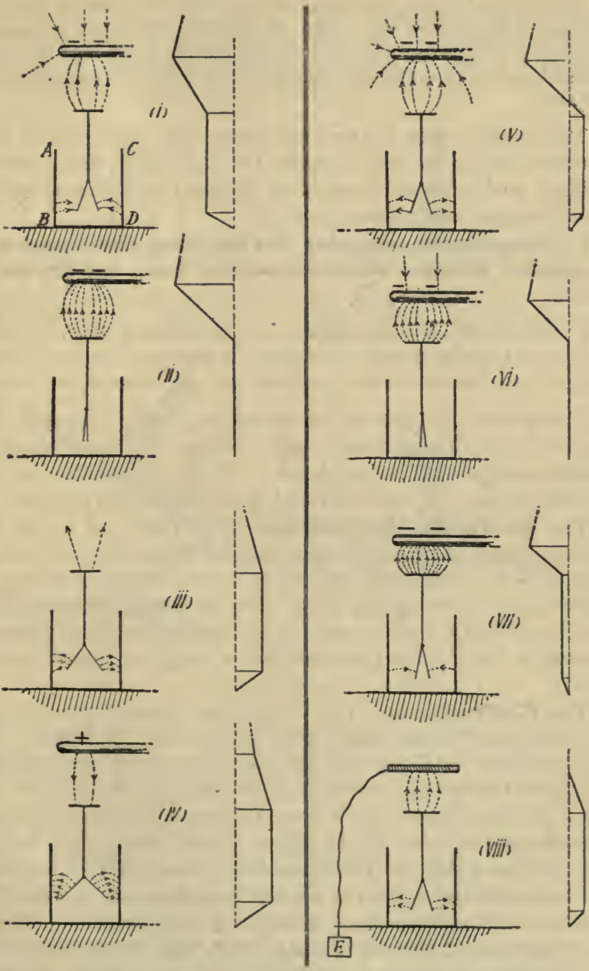


FIG. 117.—A Gold-leaf Electroscope under various electrical conditions.

EXPT. 52.—Repeat Expt. 50, using a positively charged glass rod instead of the sealing-wax. The electroscope will then be charged negatively. Note that the divergence is now increased by a *negatively* charged body, and diminished by a *positively* charged body and also by the hand.

The various steps in this experiment may be explained in a manner similar to that adopted for Expt. 50. The rules for using a gold-leaf electroscope, as deduced from these observations, are previously given (p. 123).

It is important to remember that **the leaves only diverge when a potential difference exists between the leaves and the tin-foil strips.**

EXPT. 53.—Place the electroscope on an insulating stand. Connect the disc to the tin-foil strips by means of a thin wire. Hold a strongly charged body near to the disc, and note that the leaves do not diverge.

Changes in divergence are more accurately observed if a portion of a circular paper scale is fixed to a vertical rod of sealing-wax just behind the leaves. With this modification the instrument may be used for rough quantitative measurements.

The Condensing Electroscope.—The power of a gold-leaf electroscope to detect extremely small differences of potential is increased by connecting the leaves to the insulated plate of a simple plate condenser (p. 170); such an arrangement is known as a *condensing electroscope.* One form of the instrument is shown in Fig. 159, and the method of using it is described on p. 237.

The Electrophorus.—This instrument consists of a shallow circular metal vessel filled with sealing-wax or shellac, and a metal disc of about the same diameter with an insulating handle. The sealing-wax is charged negatively by flicking with fur; the metal disc is held over the surface, and is acted upon inductively (Fig. 118, i). In actual practice the disc is laid on the charged surface. The contact is far from perfect, therefore but few points of the shellac are discharged on touching the disc. Also, the strain of the lines of force passing between the table and the charged surface tends to draw the charge within the shellac, thus preventing its discharge. On touching the disc, the lines of force entering the upper surface vanish (Fig. 118, ii);

when removed to a distance the disc is positively charged (Fig. 118, iv). The *energy* which becomes evident when a spark is obtained between the charged disc and an earth-connected conductor originates from the mechanical work done in stretching the lines of force when the metal disc is raised.

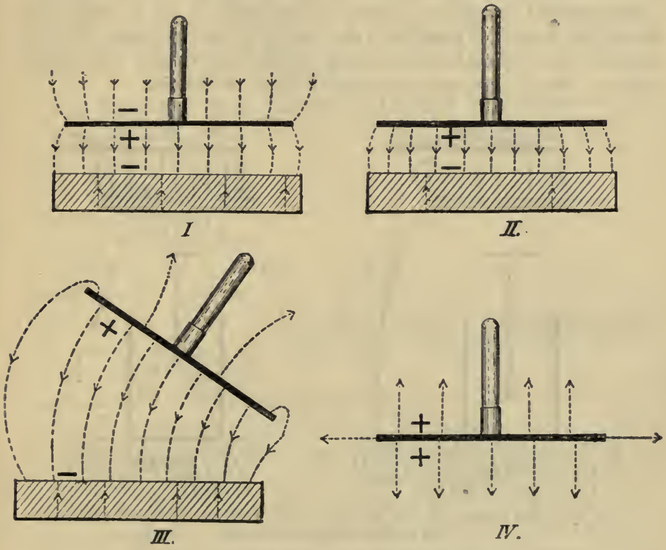


FIG. 118.—The Principle of the Electrophorus.

Electrical Screening.—When a charged insulated conductor is surrounded by an earth-connected conductor, the lines of force are restricted to the space between the two conductors; the earth-connected conductor *screens* all points outside its boundaries from any electric forces which may originate from the space which it surrounds. Also, the space which it surrounds will be screened from all electric forces originating from the space outside its boundaries. The walls of a room are earth-connected conductors, consequently the lines of force due to any experiment performed in a room are restricted to the space enclosed by the walls of the room.

Expt. 54.—(i) Make a cylindrical jacket of iron gauze, consisting of gauze sides and top, large enough to cover completely the electroscope without touching any part of the instrument. Place the jacket over the electroscope, and standing on the table; the gauze is earth-connected, and therefore analogous to the walls of the room. Hold the charged electrophorus disc near to the gauze. The electroscope is unaffected.

(ii) Remove the jacket, invert it, and place it on a stand (non-insulating) as near as possible to the disc of the electroscope. Hold an electrified rod of sealing-wax inside the jacket (without touching the gauze). Notice that the electroscope is again unaffected.

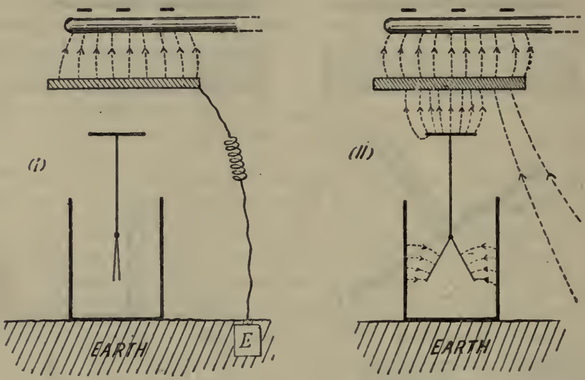


Fig. 119.—An Electroscope is screened by an Earth-connected Metal Disc but not by an Insulated Disc.

(iii) Hold the sealing-wax just over the disc of the electroscope. Observe the divergence. Now place a large sheet of metal, earth-connected by holding it in the hand, between the wax and the disc (*and touching neither*). Observe the collapse of the leaves. *The metal plate is screening the electroscope* (Fig. 119, i).

(iv) Replace the earth-connected metal sheet by an insulated sheet. The electroscope is no longer screened (Fig. 119, ii).

We have already learned that the easiest method of completely discharging a charged body is to pass it through a flame (p. 123). If the body is a conductor, it is not even necessary to bring the flame into contact with it, the discharge taking place when the flame is still several cms. distant. The conductor will, however, retain its charge if it is electrically screened from the flame by the method shown in Fig. 119 (i).

EXPT. 54.—(v) Hold a burning taper or match near to the disc of a charged electroscope. Notice how rapidly the instrument is discharged. Recharge the electroscope, and hold an earth-connected metal plate between the disc and the flame; notice that the instrument is not being discharged so long as it is screened, but that the discharge takes place as soon as the screen is removed.

The Multiplication of a feeble Charge by Induction.—Let A and B (Fig. 120) represent two flat metal plates separated from a conducting table by a thin layer of insulating material. If a small positive charge is given to A it is possible, with the aid of a carrier plate of the same size as A or B and fitted with an insulating handle, to charge A and B to very high positive and negative potentials respectively. Hold the carrier plate C just above A but not touching it, and touch C momentarily with the finger; it now has a slight *negative* charge. Move C to the position C_2 and allow it to touch plate B to which its charge is now transferred. Move C to the position C_3 and momentarily touch it with the finger; it now acquires a slight *positive* charge, which is transferred to plate A on moving C into the position C_4 and allowing it to touch A. The charge on A is now slightly *greater* than it was initially, and B has acquired a *negative* charge. By repeating the process several times considerable charges may be imparted to the plates. Compare this process with the action of Kelvin's *water-dropping apparatus* (p. 213).

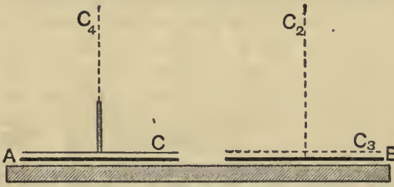


FIG. 120.

SUMMARY

Faraday's Ice-pail Experiment is a simple method of proving experimentally that *the total induced charges are equal to the inducing charge.*

The process of *charging a conductor by induction* may be described either by considering the lines of force of the electrostatic field or by considering the re-distribution of electricity due to potential-differences in the field. The process of charging a *gold-leaf electroscope* and its use in determining neighbouring electrical charges is explained in the same way.

The leaves of a gold-leaf electroscope only diverge when a potential-difference is set up between the leaves and the tin-foil strips.

Electrical Screening.—An earth-connected conductor screens all points outside its boundaries from any electric forces which may originate from the space enclosed by it; it similarly screens any internal point from any electric forces originating in the space outside.

QUESTIONS ON CHAPTER IX

1. Explain how a gold-leaf electroscope may be applied to the comparison of potential differences by observing the successive angles of separation of the leaves produced by communicating a series of small equal charges to a large conductor connected with the instrument.

2. A deep metal can is placed on the top of a gold-leaf electroscope, and a positively charged conducting body is lowered into the can without contact. Draw a figure showing all the charges existing when the body is in the can, and state what happens to each when (i) the body touches the inside of the can, (ii) the can is subsequently touched by the observer.

3. A gold-leaf electroscope is placed upon an insulated metal stand. State and explain the indication of the leaves when the stand receives a positive charge. How would the indication be modified if the leaves were earth-connected?

4. Can the leaves of an electroscope be made to diverge when they are kept at zero potential? If so, describe how, and explain why.

5. Devise an experiment to show that the potential of a conductor is in general not dependent only on the nature and amount of the electrification upon it.

6. Two insulated conducting spheres A and B are placed near but not in contact with each other, B being connected by a fine wire with the cap of a gold-leaf electroscope. State and explain the behaviour of the leaves of the electroscope when (i) A receives a positive charge,

(ii) the wire is removed, (iii) the electroscope is momentarily touched, (iv) the wire connection is restored.

7. Two equal insulated conducting spheres A and B are placed at some distance apart and connected by a long thin wire. They are then charged. An un-insulated metal sphere C is brought near to A. After this the wire and the sphere C are removed in turn. Will the charges on A and B now be equal? Give reasons for your answer.

8. A metal cup is placed on the plate of a gold-leaf electroscope which is then charged. Separate drops of water are allowed to fall into it from a metal saucepan held in the hand. Water is afterwards poured from the saucepan in a continuous stream. Describe the effect on the electroscope in each case.

9. Two similar deep metal jars are placed on the caps of two similar electroscopes at some distance apart and the caps are connected by a fine wire; a positively electrified ball is lowered into one of the jars without contact. Explain the effect as to potential and divergence on both sets of leaves, and also that which occurs on breaking the wire connexion by means of a silk thread and then removing the ball without allowing it to touch the jar.

10. Explain why an initial repulsion is followed by an attraction when a small conductor with a considerable charge is brought gradually near to a large conductor with a small charge, the charges being of the same sign.

11. An electroscope is lowered into the inside of an insulated metal vessel without touching it. How are the leaves affected (i) when a positive charge is given to the vessel; (ii) when the tin-foil strips on the side of the electroscope are momentarily earth-connected? Give reasons for the effects observed.

12. Describe the electrophorus and the mode of using it, explaining carefully, in accordance with the principle of the conservation of energy, how it is that you are able by the aid of this apparatus to obtain an indefinitely great number of electric discharges, while the initial charges remain undiminished.

13. A cake of shellac is excited with negative electricity. Explain how to obtain equal charges of positive and negative electricity from it without discharging it.

CHAPTER X

CAPACITY. CONDENSERS. SPECIFIC INDUCTIVE CAPACITY

Introductory.—When a charge is given to two insulated conductors in contact with each other, it is so distributed that the *potentials* of the conductors are the same; but if the *quantity* of electricity on each conductor is determined it will be found that the total charge is not necessarily divided equally between them.

EXPT. 55.—Place two metal spheres[1] (of different sizes) in contact on an insulating stand, each sphere being attached to an insulating handle. Place a hollow can on the top of the electroscope disc. Charge the spheres slightly. Lower the larger sphere into the can, and allow it to touch the inner surface. Note the divergence. Discharge the electroscope and repeat with the smaller sphere. Note that the divergence is less.

The charge required to raise a conductor to a given potential evidently depends upon its dimensions, and the *ratio between the charge and the potential to which the conductor is thereby raised is termed the* **Capacity** *of the conductor*, or

$$Capacity = \frac{Quantity}{Potential}.$$

The capacity of a conductor may therefore be defined as the quantity of electricity required to raise the potential of the conductor by one unit.

---

[1] Instead of spheres, it may be convenient to use small glass bottles coated on the outside with tin-foil, and with a rod of vulcanite fixed to the cork.

The relationship between the dimensions and the capacity of a conductor may be shown in a simple manner by means of a tin-foil roller-blind (Fig. 121) which is insulated on glass rods and may be rolled up or down by means of a silk thread.

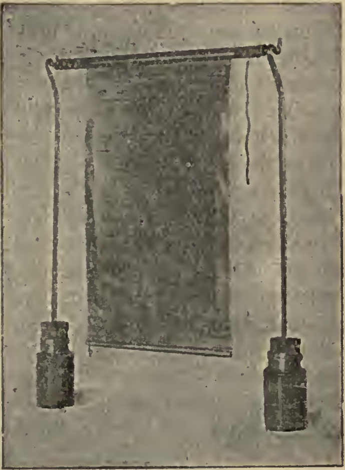


FIG. 121.—To illustrate Expt. 56.

EXPT. 56.—Connect one of the lower corners of the blind to an electroscope by means of a wire. Charge the blind, and observe the divergence. Slowly roll up the blind, and observe the changes in the divergence. The *capacity* depends upon the area of surface of the tin-foil which is exposed, while the *quantity* of charge remains the same, hence when the blind is rolled up the potential should increase.

Capacity of a Sphere.—It has already been shown (p. 141) that the charge on an insulated sphere acts at any external point as though the charge were concentrated at the centre. If a sphere, of radius r cms., is charged with Q units, then the potential (v) at any point distant d cms. from the sphere's centre is given by the equation $v = Q/d$. But if $d = r$, then v will be the potential of points on the surface of the sphere. Hence

$$\text{the Potential of the Sphere (V)} = \frac{Q}{r}$$

$$\text{or} \quad r = \frac{Q}{V}.$$

But, by definition,

$$C = \frac{Q}{V}.$$

Hence, **the capacity of a sphere is numerically equal to its radius.**

CONDENSERS

The Principle of a Condenser.—The capacity of any conductor depends not only upon its dimensions, but also upon the presence of neighbouring conductors. For example, if an insulated conductor connected to a gold-leaf electroscope is positively charged, the potential is appreciably diminished if the hand is held near to the sphere (cf. Expt. 51), a result which is due to the negative charge induced on the surface of the hand. This change in the potential indicates that the capacity of the sphere is temporarily increased by the neighbouring earth-connected conductor. This constitutes a **Condenser**, which may be defined as **any arrangement by which the capacity of a conductor is artificially increased.** The simplest type of condenser consists of two neighbouring conductors, one insulated and the other earth-connected. The **capacity of a condenser is the quantity of charge required to raise the potential of the insulated conductor by unity.** Subsequent experiments will show that the capacity of a simple condenser depends upon:

(i) **the area of the two conductors,**
(ii) **their distance apart,** and
(iii) **the medium** (or **dielectric**) **separating them.**

An experimental proof that the capacity of a condenser is directly proportional to the effective area of the plates is scarcely necessary, for the plates may be regarded as being made up of a number of smaller pairs of plates, each with the same thickness of dielectric. The capacity of the large condenser will be equal to the sum of the capacities of the constituent smaller condensers.

Fig. 122 represents a form of air condenser which may be used to demonstrate the influence of *area* and *thickness of dielectric* on the capacity. *AB* is a vertical board on the surface of which a sheet of tin-foil is pasted. *C* is a sheet of tin-foil supported by means of a glass rod arranged so that *C* may be rolled up or down; the glass rod is supported on two vertical rods of vulcanite. *C* is lowered by the silk thread *D* and is raised by an elastic thread which winds round one end of the rod and is fixed below to one of the vulcanite rods. The thickness of the air space between the sheets of foil is observed by means of the scale S.

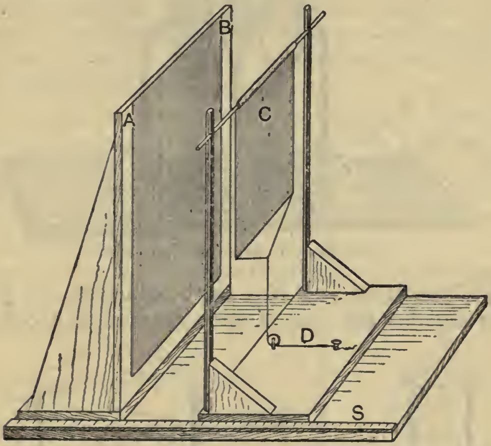


FIG. 122.—Apparatus to demonstrate the Principle of a Condenser.

EXPT. 57.—Wind up C until only about $\frac{1}{4}$ of its area is exposed. Connect C to a gold-leaf electroscope, and give to it a small charge. Note the thickness (t) of the dielectric, also the length of C which is exposed and may be taken as a measure of the effective area (A) of the condenser. Calculate the ratio A/t. Carefully note the divergence in the electroscope. Increase the area (A), separate the plates until the divergence is the same as before, and again calculate the ratio A/t. Repeat the observations several times. In each position of C the *quantity* and the *potential*, and therefore the *capacity*, are the same; if the numerical value of A/t is constant, we see that the capacity is proportional to the ratio A/t. But the capacity is directly proportional to the area, hence *the capacity varies inversely as the thickness of the dielectric.*

The Field of Force of a Simple Air Condenser.—Fig. 123 represents a convenient form of condenser for investigating the field of force and the influence of different dielectrics on the capacity. A and B are two brass plates supported on vulcanite rods; C is an insulated plate which serves to investigate the potential at various points of the field. In Fig. 124 (i) the plate A is charged positively, and B is earth-connected. If the plates are near together the lines of force are straight except near to

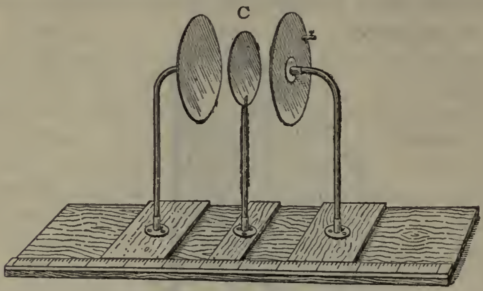


FIG. 123.—A Simple Air Condenser.

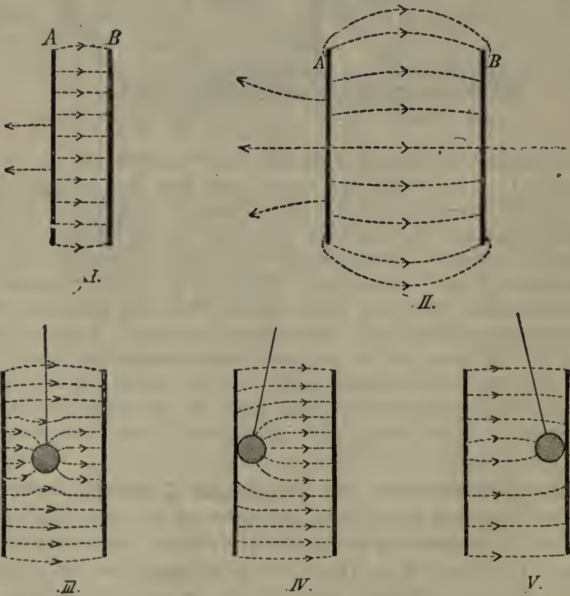


FIG. 124 (I.-II.).—The Field of Force of a Simple Air Condenser. (III.-V.).—To illustrate Expt. 58.

the edges, where they are slightly bent (due to the side pressure which the lines of force exert). A few lines of force may proceed from the outer face of A to the walls of the room which constitute, with A, the plates of a condenser (of which the plates are very far apart). In Fig. 124 (ii), A and B have been separated to a greater distance; the lines of force at the edges of the plates are more curved and more lines proceed from A to the walls of the room.

Expt. 58.—Suspend a pith-ball, by means of a silk thread, between A and B (Fig. 124, iii). If nearer to A than to B, it will be attracted by A, and the lines of force joining the pith-ball to A will vanish (Fig. 124, iv). The ball is now attracted by B, when more lines of force vanish (Fig. 124, v). This process will continue until the condenser is discharged.

Fig. 125 (i) represents the potential diagram of the condenser when the plates are close together, with A positively charged

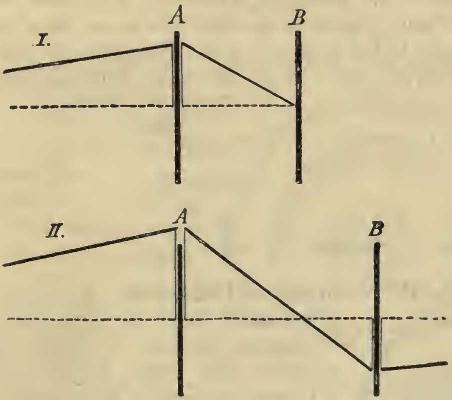


Fig. 125.—Potential Diagram of a Plate Condenser.

and B at zero potential. If B is now disconnected from the earth and removed to a greater distance from A, its negative charge will now give to it a negative potential (Fig. 125, ii). At some intermediate point the potential will be zero. If the plates have equal and opposite charges the point of zero potential will be mid-way between them.

Expt. 59.—Charge the condenser, the plates being about 8 cms. apart. Connect the plate C (Fig. 123) to an electroscope, and place it between A and B. Observe that the divergence is greater when C is nearer to A, and verify that it has positive potential by holding near to it a positively charged body. Separate A and B until about 16 cms. apart, and place C in various positions between them. The leaves diverge when C is near A, and gradually collapse when moved towards B; when moved still nearer to B, the leaves again diverge; verify that the potential of C is now *negative.*

The Capacity of a Simple Air Condenser.—Let the plates be at potentials V and zero respectively; let A represent the area of each plate, and d their distance apart. If the plates are very near together we may assume that the tubes of force are straight and uniformly distributed; the tubes will therefore be of uniform cross-section. Since, in any tube of force, Fs is constant (p. 141), therefore in the present case the intensity (F) is the same at all points of the field. The work required to convey one unit of positive electricity from one plate to the other will be equal to the product of the *Intensity* and the *distance*, and this will be equal to the *potential difference* between the plates, or

$$V = F \times d.$$

But, by Coulomb's law, $F = 4\pi\sigma = \dfrac{4\pi Q}{A}.$

Hence

$$V = \frac{4\pi d \times Q}{A}.$$

Therefore

$$\textbf{Capacity } (C) = \frac{Q}{V} = \frac{A}{4\pi d}.$$

The Capacity of a Spherical Condenser.—Fig. 126 represents a spherical condenser, consisting of two concentric spheres of radii a cms. and b cms. The inner sphere is insulated, the outer sphere is earth-connected, and the dielectric is air.

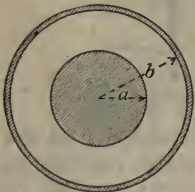


Fig. 126.

Suppose a charge $+Q$ to be given to the inner sphere. All the tubes of force will terminate on the inner surface of the outer sphere, where $-Q$ units will be distributed. The potential at all points within the outer sphere, due to its charge $-Q$ only, is uniform and equal to $-Q/b$. The potential of the inner sphere due to its own charge $(+Q)$ is $+Q/a$. Hence **the resultant potential of the inner sphere** $= \frac{Q}{a} - \frac{Q}{b}$

$$\text{or} \qquad V = Q\left(\frac{1}{a} - \frac{1}{b}\right) = Q\left(\frac{b-a}{ab}\right).$$

Hence, the **Capacity** $= \frac{Q}{V} = \frac{ab}{b-a}$.

If a and b differ by a very small amount, then $b=(a+x)$, where x is a very small length. Therefore $C = a(a+x)/x = \frac{a^2}{x} + a = \frac{a^2}{x}$. $\left(\text{Since } a\right.$ may be neglected in comparison to $\left.\frac{a^2}{x}\right)$. Hence

$$C = \frac{a^2}{x} = \frac{4\pi a^2}{4\pi x} = \frac{\text{surface of inner sphere}}{4\pi \times \text{thickness of dielectric}}.$$

This result is identical with that for a simple air condenser.

If b is great compared with a, then $C = \frac{ab}{b-a} = \frac{a}{1-\frac{a}{b}} = a$ approximately,

or, the capacity is equal approximately to that of a single sphere.

Successive Condensers.—In Fig. 127, A represents an insulated sphere, surrounded by concentric conductors B and C. Two condensers are in this way formed, viz. the outer surface of A with the inner surface of B, and the outer surface of B with the inner surface of C. Let a charge Q be given to A, and let the potentials of A, B, and C, be V_1, V_2, and V_3 respectively. If C_1 and C_2 are the capacities of the condensers AB and BC respectively, then

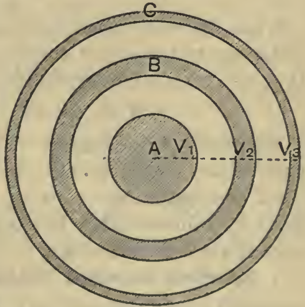


FIG. 127.

$$Q = C_1(V_1 - V_2) = C_2(V_2 - V_3).$$

Eliminate V_2; thus $C_1(V_1 - V_2) = C_2(V_2 - V_3)$,

$$\text{or} \qquad V_2 = (C_1V_1 + C_2V_3)/(C_2 + C_1).$$

Hence,

$$Q = C_1\left(V_1 - \frac{C_1V_1 + C_2V_3}{C_1 + C_2}\right) = \frac{C_1C_2}{C_1 + C_2}(V_1 - V_3).$$

Therefore,

$$\textbf{the combined capacity} = \frac{C_1C_2}{C_1+C_2}.$$

(Compare this expression for that of Leyden jars arranged **in cascade** (p. 188).)

EXAMPLE.—A sphere (diameter 6 cms.) is charged with 105 units and then enclosed in another sphere, insulated and uncharged, of internal diameter 7 cms. and thickness 0.25 cms. The outer sphere is then put to earth. Find the potential of the inner sphere before and after the outer sphere is earth-connected.

Before the outer sphere is touched, its outer surface and the surrounding earth-connected surfaces (supposed to be distant) constitute a condenser, in which the potential (V_3) of the one surface is zero. The capacity (C_2) of this condenser = outer radius of sphere = 3.75 cms.

Also $C_1 = r_ar_b/(r_b - r_a) = (3 \times 3.5)/(3.5 - 3) = 21.$

Since $$Q = \frac{C_1C_2}{C_1+C_2}(V_1 - V_3),$$

$$105 = \frac{35}{11} \times V_1, \text{ or } V_1 = \frac{105 \times 11}{35} = 33 \text{ (electrostatic units).}$$

When the outer sphere is touched, the charge on its outer surface disappears and a simple spherical condenser remains. The formula $Q = C_1(V_1 - V_2)$ is now applicable. Since $C_1 = 21$, and $V_2 = 0$, then

$$V_1 = Q/C = 105/21 = 5 \text{ (electrostatic units).}$$

Standard Condensers.—The capacity of any condenser is determined experimentally by comparison with that of a *standard* condenser, the capacity of which is found by calculation from its dimensions. Three principal types of standard condenser are here described.

1. **Spherical Standard Condenser.**—Lord Kelvin designed a standard condenser similar in principle to that used by Faraday, and represented in Fig. 131. Owing to the difficulty of turning perfectly true spherical surfaces and of fixing them exactly concentrically to each other, other forms of standard are preferred.

2. **Kelvin's Guard-ring Condenser.**—In a simple plate condenser of small dimensions the tubes of force near to the edges of the plates are not straight (pp. 172 and 174), and the electrification is not uniform; the expression for the capacity, $A/4\pi t$, is therefore only approximately true. The error is obviated by sur-

rounding the plate with a flat circular ring R (Fig. 128) in the same plane as the plate; if the plate and guard ring are in electrical contact, and if both are charged to the same potential, the tubes of force originating from the plate are perfectly straight. With this arrangement the value of A in the above expression is equal to the mean area of the plate, and of the opening in the guard ring. The plate and guard ring form the lid of a flat circular metal box, which is supported on an insulating stem, and the plate is supported from the base of the box on an insulating stem. A wire connected to the plate passes through the side of the box. The upper plate P is supported by a glass stem fixed to a micrometer screw, and its distance from R may be varied by rotating a fixed nut round the screw. The method of using the condenser is as follows: The small plate and guard ring are electrically connected and charged, while the plate P is earth-connected. The guard ring is then disconnected and connected to earth. If V is the potential to which the small plate is raised, the quantity of electricity Q upon the plate is equal to $VA/4\pi t$.

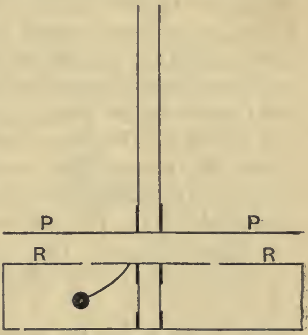


FIG. 128.—Kelvin's Guard-ring Condenser.

3. **Kelvin's Cylindrical Condenser.**—A condenser of variable capacity, and constructed so that the capacity can easily be calculated, is often required. This is fulfilled in the cylindrical condenser (Fig. 129), which consists of three co-axial cylinders,

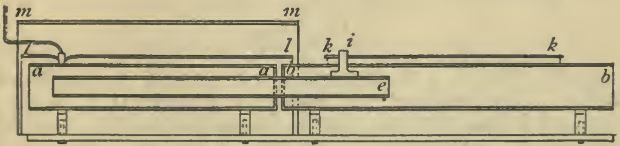


FIG. 129.—Kelvin's Standard Cylindrical Condenser.

a, *b*, and *e*. The two former are of the same diameter, and about 30 cms. long; they are insulated on vulcanite supports. The inner cylinder is also supported on four vulcanite feet resting on the inner surface of *b*, and is loaded at one end so as to rest stably; it can be moved to and fro so as to vary the length enclosed within *a*, and this length can be observed by the arm *i*, which moves along a slot in the tube *b*, and in contact with a scale *k*. The parts marked *l* and *m* are metal covers to screen the tube *a* from external disturbances. In use, the cylinder *a* is insulated and charged, while the others are earth-connected. By p. 190, the capacity per unit length is

$$C = \frac{1}{2 \log_e \frac{r_2}{r_1}}$$

where r_1 and r_2 are the external radius of tube *e* and the internal radius of tube *a* respectively.

Comparison of Capacities, by Electrostatic Methods.—1. FARADAY'S METHOD. This method enables the capacities of two condensers to be compared by means of a quadrant electrometer (p. 205). One of the condensers, of which the capacity is C_1, is charged to a potential V_1 which is measured by connecting the plates to opposite pairs of quadrants of the electrometer. The insulated plates of the two condensers are then momentarily connected, and the common potential V_2 is measured; if one of the condensers has air as the dielectric this should be used as the first condenser so as to minimise errors due to *electric absorption* (p. 181). Since the total quantity of electricity remains the same, then, if c is the capacity of the electrometer,

$$V_1(C_1+c)=V_2(C_1+C_2+c). \quad \text{.............. ....(1)}$$

As a general rule c is negligibly small, hence

$$\frac{C_1}{C_2}=\frac{V_2}{V_1-V_2}.$$

But if c cannot be disregarded, it is determined in the following manner: The electrometer is charged to a potential v_1, which is measured by the deflection, and its charge is then shared with the condenser C_1, and the common potential v_2 is measured. Then,

$$v_1 c = v_2(C_1+c),$$

$$\text{or} \qquad c=\frac{v_2 C_1}{v_1-v_2}.$$

By substituting this value of c in equation (1),

$$\frac{C_1}{C_2} = \frac{V_2}{V_1 - V_2} \cdot \frac{v_1 - v_2}{v_1}.$$

This method may be termed 'Faraday's' since it so closely resembles his fundamental experiment (p. 180) on specific inductive capacity.

2. Kelvin's Null Method. This method requires three condensers of known capacity, one of them being variable. The four condensers of capacities C_1, C_2, C_3, and C_4, are arranged in series (Fig. 130); C_1 is the condenser of which the capacity is to be measured, and C_4 is a standard variable condenser. The positive and negative terminals of a battery B are connected to the point A and to earth respectively; the points D and D′ are connected to opposite quadrants of an electrometer Q. The external plates of C_1 and C_4 are earth-connected. As a general rule the electrometer needle is deflected owing to the P.D.[1] between the points D and D′; but the capacity C_4 may be adjusted so that there is no deflection, when D and D′ must have the same potential. If Q_1 and Q_2 represent the quantities of electricity on the inner plates (*i.e.* the plates nearest to A) of C_2 and C_3 respectively, then

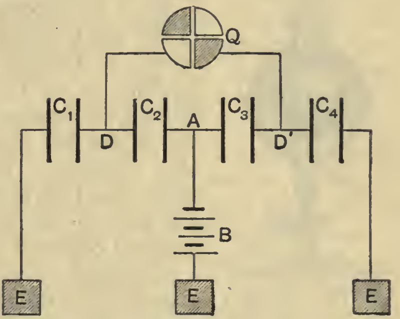


Fig. 130.—Kelvin's Null Method of comparing Capacities.

$$Q_1 = C_2(V - v) = C_1(v - 0),$$
$$Q_2 = C_3(V - v) = C_4(v - 0)$$

where V and v represent the potentials of A and of the points DD′.

Hence $$C_2/C_3 = C_1/C_4,$$

or $$C_1 = \frac{C_2}{C_3} \cdot C_4.$$

This method is analogous to the Wheatstone Bridge method of measuring resistance (p. 307).

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[1] The letters P.D. may be used as an abbreviation for *potential-difference.*

Faraday's Experiments on Dielectrics.—Faraday conducted a series of experiments to determine how the capacity of a spherical condenser is altered when air, as the dielectric, is replaced by other insulating media. He used two condensers of exactly the same dimensions and similar to Fig. 131. A brass sphere A is suspended concentrically with an outer brass spherical shell B, by means of a thin rod and knob *a*, which is supported by a long shellac plug *l*; the tap R allows the space between the spheres to be filled with any gas, and the pressure to be varied. Let the two condensers be denoted by the symbols X and Y. In the first experiment the dielectric in both condensers was air, and the outer shells were both earth-connected. A charge was given to the inner sphere of X, and its potential observed by touching its knob with a proof-plane or carrier ball, and obtaining a measure of the charge removed by means of a torsion-balance. The knobs of X and Y were then connected together, and the potential again measured. The charge obtained on the proof-plane was found to be one-half of that previously obtained, showing that the original charge was equally divided between the condensers, the dimensions of X and Y being the same, their *capacities* were equal. The condensers were discharged, and the air in Y was replaced by another dielectric (*e.g.* shellac or sulphur), and the previous observations were repeated. It was found, after connecting the knobs of X and Y, that *the mutual potential was considerably less than one-half the original potential of* X, *showing that the capacity of* Y *was greater than that of* X.

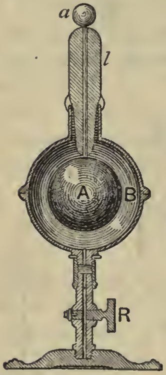

FIG. 131.—Faraday's Spherical Condenser.

If C_1 is the capacity of X, and V_1 is the potential to which its inner sphere is raised, then the charge imparted to it is C_1V_1. If C_2 is the capacity of Y, and V_2 is the mutual potential after connecting X and Y, then the charge on X is C_1V_2, and the charge on Y is C_2V_2. Hence

$$C_1V_1 = C_1V_2 + C_2V_2,$$

or

$$\frac{C_2}{C_1} = \frac{V_1 - V_2}{V_2}.$$

The ratio between the capacity of a condenser when any dielectric other than air is used and its capacity when air is used was termed by Faraday the **Specific Inductive Capacity** (S.I.C.) of the dielectric. This is usually denoted by the symbol K.

Hence
$$K=\frac{C_2}{C_1}=\frac{V_1-V_2}{V_2}.$$

Residual Charge.—If a condenser having a solid or liquid dielectric is charged in the usual manner, and then left to itself, it is found that the P.D. between the plates gradually gets less; the change is more rapid at first, and finally becomes imperceptible. The capacity of the condenser, therefore, appears to increase. This takes place even when the insulation is perfect, and it may be attributed to a change produced in the dielectric. Similarly, if the condenser is discharged after the charge has been retained for some time the P.D. is at first zero; but after a short time a P.D. of the same sign as before can be detected, which increases rapidly at first then more gradually and finally becomes constant. In this manner it is possible to obtain one or more subsequent discharges of diminishing magnitude; such subsequent discharges are due to what is known as the **residual charge.** The phenomenon indicates that the dielectric does not instantaneously recover from a strained condition, and in this sense it resembles the slow recovery of an imperfectly elastic body after being deformed, as observed in silk or glass fibres when used as suspensions for galvanometer needles. In both phenomena it is observed that jarring, vibration, etc., aids the return to a normal condition.

The apparent uncertainty of the capacity of a condenser is avoided if the observations are taken immediately after it has been charged, for it has been observed that the charge imparted to a condenser is practically constant, providing that the period of charging does not exceed about half a second. This charge may be termed the *instantaneous charge*, and **the capacity of a condenser may be strictly defined as the amount of instantaneous charge required to produce unit difference of potential between its plates.**

The phenomenon of residual charge is particularly evident in condensers with paraffin-paper as the dielectric, it is less evident with quartz than with glass, and is quite absent with Iceland spar.

Relation between Intensity of Field (F) and Specific Inductive Capacity (K).—It has been shown (p. 174) that the potential (V) of the insulated plate of a simple air condenser is given by the equation $V=(F \times d)$, where F = the Intensity of the Field, and d = distance apart of the plates. Let A and B be two condensers consisting of infinite planes, each with plates at a distance d cms. apart, A having air as a dielectric, and B having a dielectric of S.I.C. = K. If equal charges are imparted to both A and B, the potential difference in B will only be $1/K$ of that in A, since the capacity of B is K times that of A. Hence $V_B = V_A/K$. But $V_A = F_A \times d$, and $V_B = F_B \times d$.

Therefore $F_B \times d = (F_A \times d)/K$, or $F_B = F_A/K$.

In words, **the Intensity of the Field in a dielectric of** *S.I.C. = K* **is** *1/K* **of that of a field in which air is the dielectric, the charges and dimensions being the same.**

Also, since $F_A = 4\pi\sigma$ (p. 142), then $F_B = \frac{4\pi\sigma}{K}$.

And, since $F = \frac{V_1 - V_2}{d}$ (p. 147), the potential-slope is proportional to the Intensity of the field. Hence, **the potential slope in any dielectric is inversely proportional to its Specific Inductive Capacity.**

The Force of Attraction between the Plates of a Condenser (separated by a dielectric of S.I.C. = K).—The force on each unit area $= \frac{\sigma F_B}{2}$ (p. 145) $= \frac{4\pi\sigma^2}{2K}$ dynes per sq. cm.

Hence, **if the charges are given,** *the force of attraction varies* **inversely** *as K.*

Also, $\frac{\sigma F_B}{2} = \frac{K F_B^2}{8\pi} = \frac{KV^2}{8\pi d^2}$ dynes per sq. cm.

Hence, **if the potentials are given,** *the force of attraction varies* **directly** *as K.*

Parallel Plate Condenser with Compound Dielectric.—Let AD (Fig. 132) represent a plate condenser with its plates at a distance d cms. apart, and let BC represent a slab of thickness t and S.I.C. = K. Let V be the potential difference between D and A.

The work done in conveying a unit charge across the air-gaps AB and CD $= F(d-t)$ ergs; also the work done in conveying the unit charge through the slab

$$BC = \left(\frac{F}{K} \times t\right) \text{ ergs.}$$

Hence, **total work in conveying unit charge from A to D**

$$= F\left\{(d-t) + \frac{t}{K}\right\} \text{ ergs,}$$

and this is equal to the potential difference between A and D.

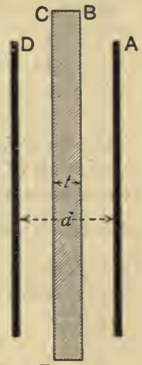

FIG. 132.

Therefore $V = F\left\{(d-t) + \frac{t}{K}\right\}$.

But, by Coulomb's Law, $F = 4\pi\sigma$.

Therefore $V = 4\pi\sigma\left\{(d-t) + \frac{t}{K}\right\} = \frac{4\pi Q}{A}\left\{(d-t) + \frac{t}{K}\right\}$.

Hence $\quad C = \dfrac{Q}{V} = \dfrac{A}{4\pi\left\{(d-t) + \frac{t}{K}\right\}}$.

This formula may be used in order to make an approximate determination of the S.I.C. of a solid dielectric.

EXPT. 60.—Connect the insulated plate of a simple air-condenser (Fig. 123) to a gold-leaf electroscope. Earth-connect the other plate, and give a suitable charge to the condenser. Note the divergence of the leaves and the distance (d_1) apart of the plates. Insert between the plates a slab of paraffin-wax[1] (of thickness t, and of larger area than the condenser plates), and determine the distance (d_2) between the two plates when the divergence is the same as before. Remove the paraffin-wax, and re-determine the distance d_1, taking the mean of the two observations. Then, if C_1 and C_2 are the capacities with air alone and with the compound dielectric,

$$C_1 = A/4\pi d_1,$$

$$C_2 = A\Big/4\pi\left\{d_2 - t\left(1 - \frac{1}{K}\right)\right\}.$$

---

[1] It may happen that the surface of the wax will acquire a slight charge either by touching the plates or by rubbing the fingers. This should be tested for and, if present, removed by passing the slab quickly through a Bunsen flame.

But since the divergence was the same in both cases, $C_1 = C_2$.

Hence
$$d_1 = d_2 - t\left(1 - \frac{1}{K}\right),$$
or
$$t - (d_2 - d_1) = \frac{t}{K},$$
or
$$K = \frac{t}{t - (d_2 - d_1)}.$$

The same formula may be derived by the following reasoning: a layer of air of thickness t cms. is replaced by a layer of material which is equivalent to t/K cms. of air, hence the total thickness of the air dielectric is reduced by an amount $\left(t - \frac{t}{K}\right)$ cms.; this is the distance through which one of the plates must be moved so that the initial and final capacities are the same.

Hence
$$t - \frac{t}{K} = d_2 - d_1,$$
or
$$K\{t - (d_2 - d_1)\} = t.$$

Accurate Measurement of Specific Inductive Capacity.—1. BOLTZMANN'S METHOD. In 1874 Boltzmann measured the S.I.C. of several solids by the following method. One pair of quadrants of an electrometer, the negative terminal of a battery, and one plate of a plate-condenser were all connected to earth. The following observations were then made.—(i) the positive pole of the battery was connected to the insulated pair of quadrants and the deflection noted; this gave a measure of the potential V_1 of the battery terminal; (ii) the battery terminal was disconnected from the quadrants, which were then discharged, and the insulated plate of condenser, with air as dielectric, was connected momentarily with the battery terminal; (iii) the condenser was disconnected from the battery, and joined to the electrometer, the common potential V_2 being indicated by the deflection; (iv) the observations were repeated with the air replaced by a solid dielectric. If C_1 and C_2 are the capacities of the condenser with air and with the solid dielectric, if c is the capacity of the electrometer, and if V_3 is the common potential in the fourth observation, then
$$C_1V_1 = V_2(C_1 + c)$$
and
$$C_2V_1 = V_3(C_2 + c).$$

Hence
$$K = \frac{C_2}{C_1} = \frac{V_3}{V_1 - V_3} \cdot \frac{V_1 - V_2}{V_2}.$$

In his experiments Boltzmann really replaced only a *portion* of the air between the plates by the solid dielectric: in this case the above formula becomes more complicated.

2. HOPKINSON'S METHOD. Dr. John Hopkinson[1] used the method of comparison of capacities of two condensers (p. 178). The apparatus (Fig. 133) consisted of a standard sliding condenser S, a guard-ring condenser DP, a battery B of 48 Daniell cells, and a quadrant electrometer Q. The middle point of the battery, the outer tube of S, the plate P, and one pair of quadrants of Q were all earth-connected. The terminals of the battery were joined to the inner tube of S, and to the plate D and guard-ring R, respectively, which thus acquired equal and opposite potentials. The wires from the battery were then disconnected, and the connections shown by dotted lines were made; the direction of deflection was observed, and the capacity of S was altered until no deflection resulted on connecting up to Q, showing that the capacities of S and of DP were equal. The solid dielectric was then inserted, and the observations repeated, the capacity of S being adjusted until it was again equal to that of the guard-ring condenser. The S.I.C. of the dielectric was calculated from the two values of the capacity of S.

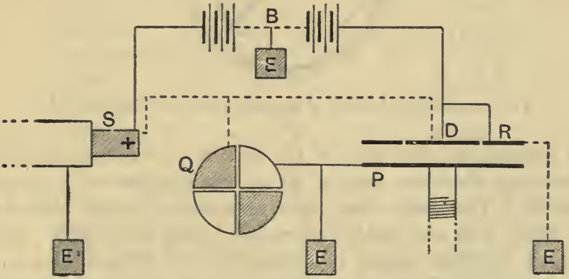

FIG. 133.—Hopkinson's Method of Measuring Specific Inductive Capacity.

Specific Inductive Capacity of Liquids.—1. SILOW'S METHOD. The apparatus (Fig. 134) consists of a glass cylindrical vessel coated inside with four vertical strips of tin-foil, the opposite strips being joined together like the quadrants of an electrometer. A platinum needle was suspended inside the vessel, the needle consisting of a horizontal arm terminating in pieces of metal curved so as to be parallel to the sides of the vessel.

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[1] Gray's *Absolute Measurements*, vol. i. (Macmillan).

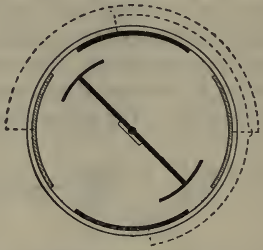

FIG. 134.—Silow's Method of Measuring the S.I.C. of a Liquid.

The needle and one pair of the foil strips were always earth-connected ; the other pair of strips was charged to a constant potential. The deflections were observed (i) when the vessel was filled with air, and (ii) when filled with the liquid. It was assumed that the angles of deflection were proportional to the S.I.C. of the liquid, for, *if the potentials are constant, the force of attraction between the arm and the charged strips is directly proportional to the S.I.C. of the dielectric* (p. 182). It would be more correct to suspend the needle from a torsion-head, as in Coulomb's torsion balance, and to observe the angles through which the torsion-head would have to be turned in order to bring the needle back to its position of equilibrium.

2. HOPKINSON'S METHOD. The apparatus consists of two co-axial cylinders closed at the lower end so as to form an annular vessel to contain the liquid under examination ; another co-axial cylinder was suspended within the liquid from an ebonite support. The outer vessel and the suspended cylinder formed the earth-connected and insulated coatings of a condenser. The capacity of the condenser, (i) when the dielectric was air, (ii) when the vessel was filled with the liquid, was determined by comparison with a standard condenser.

SPECIFIC INDUCTIVE CAPACITY.

The following Table gives the Specific Inductive Capacity of various dielectrics, assuming that the S.I.C. of air is unity.

SUBSTANCE.	S.I.C.
Glass:	
Double extra dense flint ($\Delta=4.5$),	9.90
Dense flint ($\Delta=3.66$),	7.38
Plate,	8.45
Ebonite,	3.15–3.48
Gutta-Percha,	3.3–4.9
Mica,	6.33–8.0
Paraffin,	2.29
Shellac,	2.95–3.73
Sulphur,	2.24
Petroleum,	2.02–2.19
Turpentine,	2.15–2.28
Distilled Water,	76.0
Hydrogen (at N.T.P.),	0.9997
Carbon Dioxide (at N.T.P.),	1.0008

Leyden Jars.—The Leyden jar (Fig. 135) is the most frequent type of condenser used in elementary experiments. It consists of a glass jar coated inside and outside with tin-foil to within 2 or 3 inches of its edge. A metal knob is supported vertically by a rod which is connected to the inner coating of the jar. The Leyden jar may be regarded as a plate condenser, with glass as dielectric. Hence its capacity is given by the formula (p. 174) $C=KA/4\pi t$; where K=S.I.C. of glass, A=area of either coating, and t=thickness of the glass.

The function of the dielectric in a condenser may be clearly shown by means

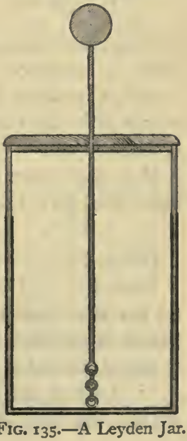

FIG. 135.—A Leyden Jar.

of a Leyden jar, the coatings of which can be readily separated (Fig. 136).

EXPT. 61.—Charge the jar. Lift out the inner coating by means of an insulating rod, and place it on the table. Lift out the glass jar. Replace the jar and also, *by means of the insulating rod*, the inner coating. A spark may be obtained. Recharge the jar, and separate the parts as before. Hold the glass jar in the left hand and insert the right hand inside the jar so as to touch the sides. Notice the numerous slight discharges obtained. Replace the parts and verify that the jar is now discharged.

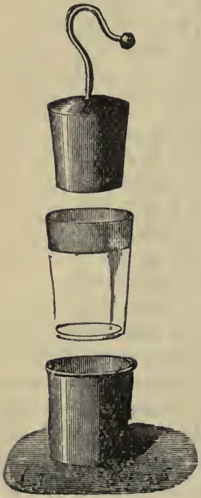

FIG. 136.

It is evident that the energy of the charge is represented by the *strained* condition of the dielectric, and that the charges are not contained by the metal coatings.

Charging Leyden Jars in-Parallel and in-Series.—(i) IN-PARALLEL. If the inner coatings of several Leyden jars (of capacities c_1, c_2, ...) are joined together by a conducting wire, and the outer coatings similarly connected, the jars are said to be *connected in-parallel* (Fig. 137, i). If a charge of Q units is imparted to the jars, whereby the potential of the inner coatings is raised to V, then $Q = CV$, where C is the total capacity of the system of jars.

If q_1, q_2, ... are the charges in the individual jars, then

$$q_1 + q_2 + \ldots = c_1V + c_2V + \ldots.$$

But $q_1 + q_2 + \ldots = Q = CV$.

Hence $\qquad C = c_1 + c_2 + \ldots,$

or, **the total capacity of the jars, connected in-parallel, is equal to the sum of the capacities of the individual jars.**

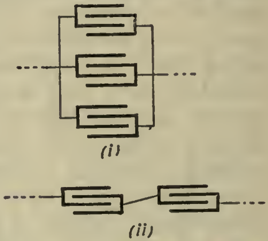

FIG. 137.—Leyden Jars connected in-Parallel and in-Series.

(ii) IN-SERIES (or, *in cascade*). In this method all the jars, except the last, are insulated. The inner coating of the 2nd jar is connected to the outer coating of the 1st, the inner coating of the 3rd with the outer coating of the 2nd, and so on, (Fig. 137, ii). If q_1 is the charge given to the inner coating of the 1st jar, a charge $-q_1$ will be induced on its outer coating, and $+q_1$ on the inner coating of the 2nd jar, etc.; hence the charges in each jar will be the same. If v_1, v_2, ... are the potential differences in successive jars, then

$$v_1+v_2+\ldots=\frac{q_1}{c_1}+\frac{q_1}{c_2}+\ldots=q_1\left(\frac{1}{c_1}+\frac{1}{c_2}+\ldots\right).$$

If the *total capacity* of the jars in series is C, and if the potential difference between the 1st and last coatings is V, then

$$V=\frac{q_1}{C}.$$

But $$V=v_1+v_2+\ldots=q_1\left(\frac{1}{c_1}+\frac{1}{c_2}+\ldots\right).$$

Hence $$\frac{1}{C}=\frac{1}{c_1}+\frac{1}{c_2}+\ldots$$

or, **the reciprocal of the total capacity is equal to the sum of the reciprocals of the capacities of the individual jars.** It is evident that the capacity of any system of condensers joined in-series is always less than that of any condenser in the system.

Submarine Cables as Condensers.—A submarine cable constitutes a condenser, in which the inner copper core represents the insulated coating, the layer of insulating material is the dielectric, and the surrounding water is the earthed coating. To a less extent, an aerial telegraph line is also a condenser, the wire and the surface of the earth forming the two coatings, while the intervening air is the dielectric.

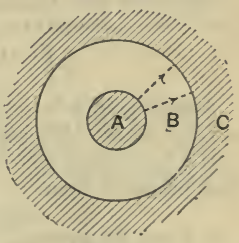

FIG. 138.

CAPACITY OF A SUBMARINE CABLE.—Fig. 138 represents a cable, of which A is the insulated conductor, B is the dielectric, and C is the surrounding water. Let the radii of the inner and outer conductors be r_1 and r_2.

The tubes of force radiate outwards from the centre of A. Near to the surface of A, $F=4\pi\sigma/K$. If V is the potential of A, then

$V-0=\Sigma F\,.\,dx$, and the charge (Q) on unit length of A is $2\pi r_1\sigma$. At a distance x from the axis, $F=\frac{4\pi\sigma}{K}\times\frac{r_1}{x}$, (since the tubes of force radiate outwards from the centre of A, the cross-sections of the tubes vary as their distance from the axis, and therefore the intensity varies inversely as this distance) (p. 182). Hence $F=\frac{2Q}{Kx}$. Therefore

$$V=\frac{2Q}{K}\int_{r_1}^{r_2}\frac{dx}{x}=\frac{2Q}{K}\log_e\frac{r_2}{r_1}.$$

The Capacity per unit length $=\frac{Q}{V}=K/2\log_e\frac{r_2}{r_1}$ electrostatic units. Here $e=2.7183$, the base of Napierian logarithms. If expressed in ordinary logarithms the formula becomes $C=K\Big/\left(2\log_{10}\frac{r_2}{r_1}\times 2.3026\right)$.

The Energy of a Charged Condenser.—We can imagine that a *large* plate condenser is charged by the transference of successive and infinitely small charges of 1 unit each from the earth-connected plate to the insulated plate. Let each such charge raise the potential by an amount v.

The transference of the 1st unit will require no work, and will raise the potential of the insulated plate to v.

The 2nd unit will require v units of work, and will raise the potential to $2v$.

The 3rd unit will require $2v$ units of work, and will raise the potential to $3v$.

The nth unit will require $(n-1)v$ units of work, and will raise the potential to nv.

The total work required

$$=v\{1+2+\ldots+(n-1)\}$$
$$=v\left\{\frac{1+(n-1)}{2}\right\}(n-1)=\frac{n\times v(n-1)}{2}$$

Assuming that n is an odd number, then $v(n-1)/2$ is the *average potential during the process of charging.*

Hence, if Q is the number of units conveyed, and V is the final potential, then

$$\text{work required}=Q\times V/2.$$

But **the energy stored up in the condenser is equal to the work done in the process of charging**; hence

$$\textbf{energy}=\frac{QV}{2}=\frac{Q^2}{2C}=\frac{CV^2}{2}.$$

In the case of a plate condenser with a dielectric of which the S.I.C. is equal to K, then $C=KA/4\pi d$; hence

$$\text{energy}=\frac{QV}{2}=\frac{2\pi d}{KA}\times Q^2=\frac{KA}{8\pi d}\times V^2;$$

in any case, *for a given quantity* Q *the energy is proportional to* d/KA, *and for a given potential-difference* V *the energy is proportional to* KA/d.

The energy of a charged condenser may be regarded as being distributed through the dielectric, which is in a state of strain; this energy becomes evident in the spark which accompanies the discharge of the condenser. If the plates are connected by a length of dry cotton (or other poor conductor) the discharge takes place more slowly: each tube of force moves outwards towards the cotton, the opposite ends of each tube travel along the cotton in opposite directions, and the tube finally vanishes. The energy represented by each tube of force reappears as heat in the cotton thread.

It can be proved, if the plates of a charged condenser are connected by a conductor of *low resistance*, that the discharge is **oscillatory,** the charge surging to-and-fro along the conductor several times and rapidly diminishing in magnitude, the whole process being complete in a very short interval of time. The discharge through a low resistance may be compared to the vibrations of a steel spring clamped at one end, and vibrating in air, while the discharge through a high resistance is comparable to the behaviour of a bent spring immersed in a dense viscous liquid.

EXAMPLE.—If the force of attraction between two large parallel plates charged with equal and opposite charges $\pm Q$ is independent of the distance between them, find the energy of the charge, and the difference of potential between them in terms of F and Q, when they are at a distance of 1 cm. apart.

Let S be the area of either plate, F the force of attraction, and V the difference of potential and d the distance between the plates. The intensity of the field between the plates is uniform, since the force of attraction is independent of the distance and the tubes of force are therefore straight; and the intensity $=4\pi\sigma=V/d$.

Hence (p. 143) $F=2\pi\sigma^2S=2\pi\sigma Q=2\pi QV/4\pi d$.

But $d=1$;

$\therefore\ V=2F/Q$.

Also $\text{Energy}=\frac{1}{2}QV=\frac{1}{2}Q\times\frac{2F}{Q}=F.$

Loss of Energy when a Charged and an Uncharged Condenser are Connected.—If a condenser of capacity C_1 is charged with Q_1 units, the energy of the charge is $Q_1^2/2C_1$. Let this condenser be joined by an insulated wire to an uncharged condenser of capacity C_2. The quantity Q_1 will now be distributed over a condenser of capacity (C_1+C_2), and the energy will be $Q_1^2/2(C_1+C_2)$. The energy lost is equal to $\frac{Q_1^2}{2}\left\{\frac{1}{C_1}-\frac{1}{C_1+C_2}\right\}$, and is represented by the spark which passes when the jars are connected.

If two charged condensers are joined together it can be proved that there is always a loss of energy *except when the initial potentials are the same.* Let Q_1, C_1, V_1, and Q_2, C_2, V_2 be the charges, capacities, and potentials of two charged condensers. The total energy $=\frac{1}{2}\frac{Q_1^2}{C_1}+\frac{1}{2}\frac{Q_2^2}{C_2}$. After connecting the condensers, the total energy $=(Q_1+Q_2)^2/2(C_1+C_2)$.

$$\text{The loss of energy}=\frac{1}{2}\left\{\frac{Q_1^2}{C_1}+\frac{Q_2^2}{C_2}-\frac{(Q_1+Q_2)^2}{(C_1+C_2)}\right\}$$

$$=\frac{1}{2}\left\{\frac{Q_1^2C_2(C_1+C_2)+Q_2^2C_1(C_1+C_2)-(Q_1^2+2Q_1Q_2+Q_2^2)C_1C_2}{C_1C_2(C_1+C_2)}\right\}$$

$$=\frac{1}{2}\left\{\frac{Q_1^2C_2^2+Q_2^2C_2^2-2Q_1Q_2C_1C_2}{C_1C_2(C_1+C_2)}\right\}=\frac{1}{2}\frac{(Q_1C_2-Q_2C_1)^2}{C_1C_2(C_1+C_2)}.$$

This is only equal to zero when $Q_1C_2=Q_2C_1$, or when

$$\frac{Q_1}{C_1}=\frac{Q_2}{C_2}, \quad \text{or} \quad V_1=V_2.$$

Condensers of large capacity.—We may distinguish between two classes of condensers, one having sufficient thickness of dielectric to resist the high P.D. used in static experiments, the other having large capacity and suitable for the low P.D. available when the condenser is charged by means of a voltaic battery. In the case of *static* condensers the plates are so far apart, and the capacity therefore so small, that the charge stored in such condensers is usually far too small to measure by means of a galvanometer (p. 281). The condensers of large capacity and suitable for experiments with voltaic batteries and galvanometers are constructed in the manner shown in Fig. 139; a number of sheets of tin-foil are separated from each other by double sheets of thin paraffined paper or mica, the sheets of dielectric projecting 4 or 5 cms. all round the sheets of foil. After the condenser is thus built up (and if paraffined paper is used), it is pressed between hot metal plates to remove the surplus paraffin wax. The even-numbered and odd-numbered sheets of foil are joined to separate terminals.

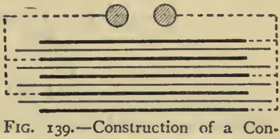

FIG. 139.—Construction of a Condenser of large capacity.

Since the capacity of such condensers is usually expressed in terms of the **Farad**, which is the *practical* unit of capacity in the electromagnetic system, it may be well to explain briefly the relationship between this unit and the unit of capacity in the electrostatic system. The method of measuring the capacity of such condensers is described on p. 331.

The Farad and Micro-Farad.—A condenser has a capacity of one farad when a P.D. of one volt between the plates charges each with one coulomb of electricity.

The *volt* and the *coulomb* are the practical units of P.D. and of quantity in the electromagnetic system, and are respectively equal to 10^8 and 10^{-1} C.G.S. units (p. 293). Hence, since $C = Q/V$,

$$1 \text{ Farad} = \frac{10^{-1}}{10^8} = 10^{-9} \text{ C.G.S. units.}$$

The relationship between the electromagnetic and the electrostatic systems of units is explained in a subsequent chapter (p. 520), where it is stated that

1 electromagnetic unit of quantity $= v$ electrostatic units,

and 1 ,, ,, potential $= \frac{1}{v}$,, ,,

v being equal to 3×10^{10}, which is approximately the velocity of light in vacuo in cms. per sec. Hence, since $C = Q/V$,

$$\text{the electromagnetic unit of capacity} = v \Big/ \frac{1}{v}$$

$$= v^2 \text{ electrostatic units of capacity.}$$

$\therefore$ 1 *Farad* $= 10^{-9} \times (3 \times 10^{10})^2 = 9 \times 10^{11}$ *electrostatic units of capacity.*

The *Farad* is rather a large unit of capacity for ordinary purposes, hence one-millionth of a farad, termed the *microfarad*, is more commonly used.

$\therefore$ 1 *Microfarad* $= 9 \times 10^5$ *electrostatic units of capacity.*

Thus, for a plate condenser with air as dielectric, if A is the area of either set of plates and t is the distance between them

$$C=\frac{A}{4\pi t} \text{ electrostatic units}$$
$$=\frac{A}{1.131\times 10^7\times t} \text{ microfarads.}$$

EXAMPLE.—1. A Leyden jar, of 10 cms. diameter, is coated with tinfoil inside and outside to a height of 10 cms.; the glass is 2 mms. thick and its S.I.C. $=7$. Find its capacity in microfarads.

The area of the base of the jar is 25π sq. cms., and the area of foil on the outside is 100π sq. cms.; hence total area $=125\pi$ sq. cms. Also $t=0.2$. Hence

$$C=\frac{125\pi\times 7}{1.131\times 10^7\times 0.2}=1.21\times 10^{-3} \text{ microfarads.}$$

2. Of how many plates 100 cms. square, separated by 1 millimetre of air, and with the two external plates of the same polarity, must a condenser be composed to have a capacity of not less than 0.33 microfarad?

If there are n plates, each of area a sq. cms., the *total surface* is $2na$ sq. cms.; but the two external surfaces are ineffective, hence the *total effective surface* is $(2na-2a)$ sq. cms. Hence

$$A=\frac{2a(n-1)}{2}=a(n-1) \text{ sq. cms.}$$

$$\text{Since } A=C\times(1.131\times 10^7\times t),$$
$$10^4(n-1)=0.33\times 1.131\times 10^7\times 0.1,$$
$$\text{or} \quad n=37.32.$$

Hence 38 sheets are theoretically required, but as the total number of sheets must really be an *odd* number, the number actually required is 39.

3. A cable consisting of a wire 3 millimetres in diameter and insulated with 3 millimetres of gutta-percha (S.I.C. $=4.26$) is placed in water. Calculate the capacity of 1 kilometre length of the cable.

The capacity of a cable per unit length (p. 189) is equal to

$$\frac{K}{2\log_{10}\frac{r_2}{r_1}\times 2.3026} \text{ electrostatic units.}$$

Hence, the capacity of a length l cms. is

$$\frac{Kl}{2\log_{10}\frac{r_2}{r_1}\times 2.3026\times 9\times 10^5}=\frac{2.413\ Kl}{10^7\times\log_{10}\frac{r_2}{r_1}} \text{ microfarads.}$$

In this case $r_2=0.45$, $r_1=0.15$, $l=10^5$, $K=4.26$.

Hence, $$C=\frac{2.413\times 4.26}{10^2\times\log_{10}3}=0.215 \text{ microfarad.}$$

SUMMARY

The **Capacity** of a conductor is *the quantity of electricity required to raise its potential by one unit.* The capacity of a sphere is numerically equal to its radius in centimetres.

A **Condenser** is *any arrangement by which the capacity of a conductor is artificially raised.* The simplest form consists of two neighbouring conductors, one insulated and the other earth-connected. The **Capacity** of a condenser is *the quantity of electricity required to raise the potential of the insulated conductor by one unit.*

Capacity of a Simple Air Condenser.—If A is the area of either plate, in sq. cms., and d their distance apart then

$$C=\frac{A}{4\pi d} \text{ electrostatic units.}$$

Capacity of a Spherical Condenser.—If the radii of the inner sphere and of the inner surface of the outer sphere are a cms. and b cms. then

$$C=\frac{ab}{b-a} \text{ electrostatic units.}$$

Standard Condensers.—The simplest types of these are (i) the *spherical*, (ii) Kelvin's *guard-ring*, and (iii) Kelvin's *cylindrical* condenser.

Comparison of Capacities (electrostatic methods). (i) Faraday's method ; (ii) Thomson's Null Method.

Specific Inductive Capacity (S.I.C.). The S.I.C. of a dielectric is defined as *the ratio of the capacity of a condenser when the dielectric is used to the capacity when air is used.*

Residual Charge.—The P.D. between the coatings of a charged condenser, with solid or liquid dielectric, does not remain constant even when the insulation is perfect. *The capacity of a condenser is therefore more accurately defined as the amount of* **instantaneous** *charge required to produce unit P.D. between its plates.*

Influence of S.I.C. on the Intensity of a Field.—The intensity of a field in a dielectric of S.I.C. $=K$ is $1/K$ of that of a field in which air is the dielectric, the charges and dimensions being the same.

If the charges are given, the electrical force between two charged conductors varies *inversely* as K.

If the potentials are given, the electrical force varies *directly* as K.

Plate Condenser with Compound Dielectric.—If d is the distance between the plates and t is the thickness of a layer of dielectric of S.I.C. = K, then

$$C = \frac{A}{4\pi\left\{(d-t)+\frac{t}{K}\right\}} \text{ electrostatic units.}$$

Measurement of S.I.C.—(i) Boltzmann's and (ii) Hopkinson's methods for solid dielectrics; and (i) Silow's and (ii) Hopkinson's methods for liquid dielectrics.

Capacity of Leyden Jars in-Parallel and in-Series.

$$\text{(i) } \textit{In parallel}: \quad C = c_1 + c_2 + \ldots.$$

$$\text{(ii) } \textit{In series}: \quad \frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_2} + \ldots.$$

Capacity of a Cable.—If r_1 and r_2 are the radii of the wire and of the solid dielectric, the capacity *per unit length* is

$$\frac{K}{2\log_{10}\frac{r_2}{r_1} \times 2.3026}.$$

Energy Stored up in a Charged Condenser.—If Q and V represent the quantity of electricity and the P.D., the energy of the charge is

$$\frac{QV}{2} = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2.$$

If the dielectric of a plate condenser has S.I.C. = K, the energy is

$$\frac{QV}{2} = \frac{2\pi d}{KA} \times Q^2 = \frac{KA}{8\pi d} \times V^2.$$

The Farad and Microfarad.—*A condenser has a capacity of* 1 *farad when a P.D. of one volt charges each plate with one coulomb of electricity.*

1 Farad $= 9 \times 10^{11}$ electrostatic units.

1 Microfarad $= 9 \times 10^{5}$,, ,,

QUESTIONS ON CHAPTER X

1. A metal sphere of 5 cms. diameter has a charge of 50 units of positive electricity. At four points, whose distances from the centre of the sphere are 20, 30, 40, and 50 cms., are charges respectively of +20, −15, +20, and −50 units. What is the potential of the sphere?

2. Two insulated spheres having radii of 3 and 1 centimetres respectively are placed a long way apart, a charge of 15 units is given to the larger sphere, what charge must be given to the smaller in order that the larger sphere may neither gain nor lose charge when the two spheres are connected by an insulated wire?

3. Equal quantities of positive electricity are communicated to two insulated metallic spheres whose radii are as five to one. What are their relative electrical potentials? The spheres are then put in conducting communication by means of a long thin wire, which is afterwards removed. What are now the relative electrical surface densities of the spheres? State in each case which sphere has the greater potential or surface density.

4. Define what is meant by the capacity of a condenser, and apply your definition to find the capacity of a system formed of two large parallel plates.

5. At what distance should the plates, 3 cms. in diameter, of an air-condenser be placed in order to have the same capacity as a sphere 100 cms. diameter.

6. A brass ball of diameter 16 is electrified with 136 units and is then put momentarily into contact, by a long fine wire, first with an insulated uncharged brass ball of diameter 18, and then with an insulated uncharged circular brass disc of diameter 12 placed at a distance of $\frac{1}{8}$ from, and coaxially with, a similar disc which is uninsulated. Show that its potential drops on these contacts by amounts that are as 5 to 4.

7. A Leyden jar standing on an insulating stool is electrified by a machine, while its outer coating is touched by the knob of an exactly similar Leyden jar held in the hand. The first jar being now disconnected from the machine, it is taken in the hand either (1) by its outer coating, and presented with its knob to that of the second jar, or (2) by its knob and presented with its outer coating to the knob of the second jar. Does a spark pass in either case? Explain the action.

8. A Leyden jar A, of capacity 3, is insulated, and the outer coating is connected by a wire with the inner coating of another Leyden jar B, of capacity 2, the outer coating of which is uninsulated. If the inner coating of A be charged so that the potential is V, what is the potential of the inner coating of B?

9. An insulated sphere of 2 cm. radius is connected by a long thin wire with another insulated sphere, the radius of which is 6 cm., and which is surrounded by a third sphere of 8 cm. radius concentric with

it. The wire which connects the first and second spheres passes through a small hole in the third so as not to touch it. All the spheres are conductors. Calculate the capacity of the two connected spheres.

10. The inner coating of one spherical Leyden jar, whose surfaces have radii 12 and 14 respectively, is charged with 25 units of positive electricity, and the inner coating of another, with surfaces of radii 8 and 12, is charged with 5 positive units, the outer coatings of both being earth connected. Their inner coatings are then momentarily joined by a fine wire; in which direction will electricity pass, the dielectric in both jars being air, and the distance between the jars considerable? Give full reasons for your answer.

11. The inner coating of an insulated Leyden jar is electrified to the potential V, the diameters of the coatings being b and B, and their thicknesses small. Describe the electric condition of the coatings of the jar, and determine how much their potentials are altered, if at all, when the outer coating is put to earth.

12. Within a very thin hollow spherical conductor is placed a sphere of the same substance, but insulated from it and concentric with it; their diameters are respectively 16 and 10 centimetres. The potential of the inner sphere is 9, and the charge on the exterior surface of the spherical conductor is 24. What changes of potential would occur if the inner were to touch the outer sphere?

13. Within a spherical vessel of brass 1 cm. thick, the external diameter of which is 14 cm., a brass ball 8 cm. in diameter is hung by a silk thread so that the centres of the two spheres coincide. If the ball is charged with 36 units of positive electricity, and if the potential of the vessel is 7, what is the potential of the ball?

14. Describe a method of comparing the capacities of two small condensers. A condenser is made of 2 concentric spheres (radii $= a$ and b, b being the radius of the outer sphere). If the outer sphere is connected to earth and the inner maintained at potential V, find the charge on either of the spheres.

15. Given that a frictional machine, turned steadily, generates the same quantity of electricity at every revolution, show how to compare the capacities of two Leyden jars with moderate accuracy. No auxiliary apparatus but some wire and a pair of knobs on insulating pillars is to be used.

16. Two Leyden jars are exactly alike, except that in one the tinfoil coatings are separated by glass, and in the other by ebonite. A charge of electricity is given to the glass jar, and the potential of its inner coating is measured. The charge is then shared between the two jars,

and the potential falls to 0.6 of its former value. If the specific inductive capacity of ebonite be 2, what is that of glass?

17. What is meant by the specific inductive capacity of a substance? Compare the attraction between two parallel plates maintained at given potentials (i) when the space between the plates is filled with air, (ii) when the space is half filled by air and half by a parallel plate of a dielectric whose S.I.C. is 10.

18. What is electrostatic capacity? What is the capacity of a condenser made of sheet glass 2 mm. thick with tinfoil coatings each 30 cms. square, if the specific inductive capacity of glass is 7.5?

19. The area of one of the coatings of a Leyden jar is 1000 sq. cm., the specific inductive capacity of the glass is 5, and its thickness 3 mm. What is the radius of a conducting sphere which has the same electric capacity as the Leyden jar?

20. Two electrified balls are in presence of each other: in what way is their mutual action modified by the introduction of a thick glass plate between them? Give reasons for your answer.

21. A sphere of radius 40 millimetres (mm.) is surrounded by a concentric sphere of radius 42 mm., the space between the two being filled with air. What is the relation between the capacity of this system and that of another similar system in which the radii of the spheres are 50 and 52 mm. respectively, and the space between them is filled with paraffin of specific inductive capacity 2.5?

22. Paraffin has a higher specific inductive capacity than air. If an electrified ball be suspended in an insulated conducting vessel which it does not touch, state what change, if any, will be produced in the electrical condition of the system by pouring melted paraffin into the space between the two.

23. Two uncharged insulated brass plates, each metallically connected with the cap of a separate electroscope, are placed parallel to each other. One is charged, and then a plate of shellac is inserted between them. What effects are produced on the electroscopes during these operations?

24. Two equal horizontal metal discs A and B are placed symmetrically one over the other and separated by air, A being insulated and B earth-connected. When A is charged the plates attract each other. Will the attraction be the same when the space between them is filled with paraffin? Give reasons.

25. What is meant by charging Leyden jars in cascade? Three Leyden jars whose capacities are 0.5, 1.0, and 1.5 are arranged in cascade. What is the capacity of the combination?

26. A charged isolated sphere is connected by a fine wire to a similar uncharged sphere of double the radius. Neglecting the capacity of the wire, find the loss of energy in terms of the energy of the original charge.

27. Two Leyden jars, A and B, receive charges of +50 and −30 units respectively, their outer coatings being earth-connected, and their knobs at potentials 20 and −10. Determine the energy of discharge of each jar after their knobs have been in contact with each other.

28. Two Leyden jars are charged with quantities of electricity in the ratio 2 : 3. If in the jar which receives the larger charge the tinfoil surface is twice as great and the glass is twice as thick as in the other, compare the quantities of heat produced by discharging them.

29. Two condensers are exactly alike, except that one has air and the other glass for the dielectric. Equal charges are given to the two condensers. In which is the energy of the charge greater?

30. A Leyden jar consists of two concentric spherical surfaces of 5 and 6 cm. diameter respectively, the intervening space being filled with air. The outer sphere is uninsulated, the inner is charged with 20 units of electricity. How much work is done when the inner sphere is put to earth?

31. Two spheres, of diameters 6 and 10 centimetres respectively, placed at a distance from each other, are charged with 8 and 12 units of positive electricity respectively. They are then connected by a fine wire. Does any spark pass? If so, how much energy is dissipated?

32. Two Leyden jars, whose capacities are 1 and 2, receive charges 3 and 4 respectively. Compare their combined electric energies before and after their knobs have been in contact.

33. A condenser A has plates of area 1000, and dielectric of thickness 4; another condenser B has plates, area 800, and the same dielectric of thickness 5. Compare the charges and energy in A and B, when they are connected A to a source of potential 4, and B to a source of potential 5.

34. A and B are two similar condensers, but the dielectric between the plates of A is air, and that between the plates of B is paraffin of inductive capacity 2. Both are insulated and connected in series, the inner coating of A and the outer of B being connected to the terminals of an electric machine. Calculate the ratio of the energies of the charges possessed by the two condensers.

35. Three similar Leyden jars A, B, C, are connected in series, the inner coating of A to the outer of B, the inner of B to the outer of C, the inner of C to the outer of A. The terminals of an electrical

machine are connected to the inner coatings of A and C, the jars are charged and disconnected. Compare the energies of the charges in each.

36. Describe an experiment for comparing the specific inductive capacities of two non-conducting liquids.

37. A condenser consists of 196 circular sheets of tinfoil separated by mica (S.I.C. $=6.6$) 0.4 millimetre thick. What must be the radius of the sheets if the condenser has a capacity of $\frac{1}{3}$ microfarad.

38. If the average radius of the earth is 6370 kilometres, calculate its capacity in microfarads.

39. How can three condensers, each having a capacity of $\frac{1}{2}$ microfarad, be arranged so that the capacity of the combination is $\frac{3}{4}$ microfarad?

40. Determine the capacity of a cable 20 kilometres long, consisting of copper wire 3 millimetres diameter insulated with guttapercha 6 millimetres thick, when immersed in water. (S.I.C. of guttapercha $=4.26$. $\text{Log}_{10}5=0.699$.)

41. How would you compare the capacity of a Leyden jar with that of a standard condenser by some one method? Describe the precautions that must be taken to secure an accurate result.

42. A parallel sided slab of dielectric, 5 cm. thick, and of specific inductive capacity 2, is placed between the plates of an air condenser, consisting of two very large flat plates at a distance of 10 cm. apart. What will be the effect on the difference of potential between the plates of the condenser, of the introduction of the slab, and to what distance apart must the plates be moved to bring the potential back to its original value?

43. State Gauss's theorem, and deduce from it the capacity of a long condenser consisting of two concentric cylinders.

CHAPTER XI

ELECTROMETERS

Attracted Disc Electrometers.—The first instrument for the quantitative measurement of potential-difference was constructed by Sir William Snow Harris, and its principle depended upon the force of attraction between two conductors charged to the potential-difference under observation. The instrument consisted of a circular plate suspended from one end of a balance-beam and counterpoised by weights placed in a pan suspended from the other end of the beam. Immediately under the circular plate was fixed an insulated plate of similar dimensions. When a potential-difference is set up between the plates the upper plate is attracted downwards with a force depending upon the magnitude of the potential-difference, and this force can be determined by adjusting the counterpoise until it is just balanced. Accurate results are impossible with an instrument of such simple construction, since the tubes of force are not uniformly distributed over the surfaces of the plates; but this difficulty has been overcome in **Kelvin's Absolute Electrometer** by surrounding the attracted disc (C, Fig. 140) with a *guard-ring* B, the aperture of which is slightly larger than the area of the disc. The *effective* area of the disc

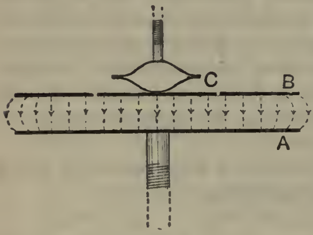

FIG. 140.—The Principle of an Attracted Disc Electrometer.

is the mean of the areas of the disc and the aperture. The guard-ring and disc are electrically connected by a wire joining the guard-ring to the metal supports of C. C is suspended from the lower arms of three fine springs (shaped like carriage springs) distributed uniformly round the upper surface of C. The upper arms of the springs are attached to a glass rod supported from a brass rod on which a fine screw thread is cut.

If V_1 and V_2 are the potentials of C and A respectively, and if S = the effective area of the attracted disc, d = the distance between C and A, σ = the density of charge on either surface, and F = the intensity of the field of force, then
the force (f) *acting on the attracted disc*

$$=\frac{F}{2}\times S\sigma=\frac{4\pi\sigma}{2}\times S\sigma=2\pi\sigma^2 S.$$

Also $$F=\frac{V_1-V_2}{d}=4\pi\sigma\text{ ; hence }\sigma=\frac{V_1-V_2}{4\pi d}.$$

Therefore, $$f=2\pi S\left(\frac{V_1-V_2}{4\pi d}\right)^2=\frac{(V_1-V_2)^2 S}{8\pi d^2}.$$

Hence $$(V_1-V_2)=\sqrt{\frac{8\pi d^2\times f}{S}}.$$

The method of determining the force (f) may be explained thus: The screw support of C is so adjusted that it is in the same plane as B, and a small weight (m grams) is placed on C, which is thus depressed below the level of B. The disc C is now raised by its screw support until it is again in its original plane. The force now acting on C is mg dynes (where $g=981$ cm./sec.^{2}). The weight is then removed. If a field of force is set up between A and C, the plate A is raised until the electrostatic attraction pulls C downwards into its original plane, when the force of attraction must be equal to mg dynes. *The potential difference between* A *and* C *is given by the equation*

$$V_1-V_2=d\sqrt{8\pi mg/S}.$$

In actual practice it is difficult to measure d accurately, whereas it is easy to measure *differences* in the distance if the support of A carries a fine scale and vernier. For this reason the plate C is charged to a constant high potential, V, while the plate A is earth-connected and raised or lowered until C is flush with the guard-ring; the scale attached to A is then read. A is now insulated, connected to the conductor the potential of which is required, moved up or down until C is again flush with the guard-ring, and the scale reading noted. The *difference* between the scale readings gives the difference between the distances separating A and C in the two observations. If V is the potential of C, V_1 and V_2 the successive potentials of A, then $V - V_1 = d_1\sqrt{8\pi mg/S}$, and $V - V_2 = d_2\sqrt{8\pi mg/S}$.

Hence $$V_2 - V_1 = (d_1 - d_2)\sqrt{8\pi mg/S}.$$

Fig. 141 represents the complete instrument. The outer and inner surfaces are for the most part coated with tinfoil, so that the case serves as a Leyden jar. The inner coating is connected to the guard-ring (B) and attracted disc (C). A *replenisher* (p. 213) R has its inductors connected to the inside and outside respectively of the Leyden jar; by this means the potential of the inside may be raised or lowered to any desired extent. The constancy of potential of the jar is observed by the *gauge* (Fig. 142) which is supported on the cover of the instrument (J, Fig. 141): the gauge itself is an *attracted disc electrometer*, in which a square piece of aluminium *p* is supported in an orifice in the plate G by means of a stretched wire *f*. One side of the disc terminates in a long arm *h* which is forked at the end, and a black hair is stretched between the points of the fork. Immediately below *p* is a circular disc connected to the inside of the jar, and the attraction due to this is balanced by the torsion of the wire. The potential of the inside of the jar is regulated so that the hair is situated between two black dots on a white surface fixed behind the hair. The position of

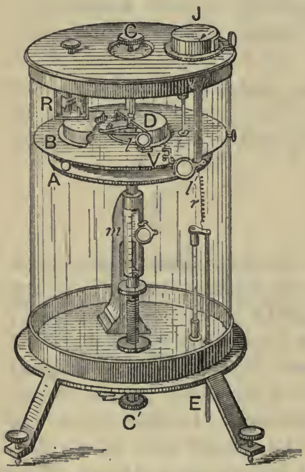

FIG. 141.—Kelvin's Absolute Electrometer.

the plate A (Fig. 141) is observed by means of the vertical scale *m* and a circular graduated plate fixed to the screw. The attracted disc is of aluminium and may be raised or lowered by the milled-head C. A black hair is attached horizontally to the disc; the image of the hair formed by the lens *l* is made to coincide with the two screwpoints at V, which are viewed through the lens *l′*.

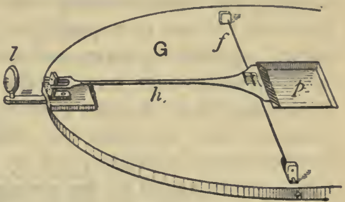

FIG. 142.—The Gauge of an Absolute Electrometer.

EXAMPLE.—The attracted disc of an absolute electrometer was so adjusted that a weight of 0.26 gram was required to pull it down to the sighted position. The weight was removed, the instrument charged, and the lower plate (earth-connected) was adjusted until the attracted disc was in the sighted position. After reading the scale of the lower plate, it was insulated and connected to the positive pole of a water-battery, the negative pole of which was earth-connected. It was found necessary to raise the lower plate through a distance 0.205 cm. in order to bring the attracted disc into the sighted position. If the area of the disc was 16.51 sq. cms., find the potential-difference of the battery terminals.

In the formula $V_1 - V_0 = (d_1 - d_0)\sqrt{8\pi mg}/S$,

$V_0 = 0$, $d_1 - d_0 = 0.205$ cm., $m = 0.26$ gm., $g = 981$, $S = 16.51$.

Hence $V_1 = 0.205\sqrt{8\pi \times 0.26 \times 981/16.51}$

$= 0.205 \times 19.704 = 4.039$ electrostatic units.[1]

This mode of using the instrument is termed *heterostatic*, since an auxiliary charge is used which is independent of those under examination. If only the latter are used the method is termed *idiostatic*, but there is the disadvantage that the absolute distance between the plates would have to be measured.

The Quadrant Electrometer.—This instrument, designed by Lord Kelvin, is used for the accurate measurement of potential-differences. It consists, in its simplest form, of a flat metal needle suspended, by means of a fine vertical fibre, horizontally and symmetrically over an insulated metal disc which has been cut into four quadrants. Opposite quadrants are connected together by wires, and the needle is so adjusted as to lie symmetrically over one of the diameters dividing the disc. If the needle is charged positively, also one pair of quadrants positively and the other pair negatively, the needle is repelled by the former and attracted by the latter, until the electrical forces are balanced by the torsion in the suspending fibre.

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[1] Since 1 electrostatic unit of P.D. $= 3 \times 10^{10}$ electromagnetic units (p. 521), and since the *volt* $= 10^8$ electromagnetic units (p. 293), the potential-difference of the battery terminals $= \frac{4.039 \times 3 \times 10^{10}}{10^8} = 1211.7$ volts.

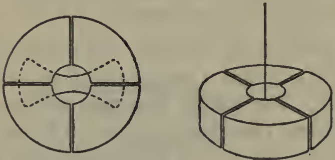

FIG. 143.—The Quadrants and Needle of a Quadrant Electrometer.

In the later forms of the instrument the four quadrants are made from a flat cylindrical brass box (Fig. 143), and each quadrant is supported by means of glass rods. The needle is a flat aluminium sheet of *dumb-bell* outline, and is suspended inside the quadrants. The needle is attached to the lower end of a rigid wire, to which a mirror is attached; a platinum wire hangs below the needle and dips into a quantity of pure sulphuric acid which forms the inner coating of a Leyden jar (Fig. 144); tin-foil on the outside of the vessel forms the outer coating of the jar. The needle and its attachments were formerly suspended by two silk fibres, constituting a *bifilar suspension*; but it has recently been found that a single quartz fibre is a more satisfactory form of support. The Leyden jar (and needle) are charged through the *charging rod* (Fig. 144), and its potential is regulated by the *gauge* and a replenisher. Each pair of quadrants has an external electrode. If V, V_1, and V_2 are the potentials of the needle and the two pair of quadrants, and c is a constant depending upon the dimensions of the instrument, it can be proved that the deflection θ of the needle is given by the equation[1]

$$\theta = c(V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right). \quad \text{......................}(1)$$

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[1] See J. J. Thomson's *Elements of Electricity and Magnetism*, p. 98 (Camb. Univ. Press).

In practice, the potential of the needle is much higher than that of either pair of quadrants, so that V may be regarded as great in comparison to $(V_1+V_2)/2$. Hence

$$\theta=c(V_1-V_2)V \text{ approximately. } \dots\dots\dots\dots\dots\dots (2)$$

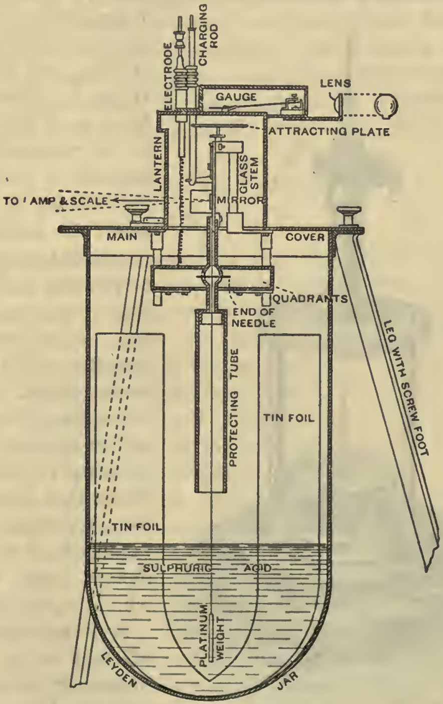

FIG. 144.—Kelvin's Quadrant Electrometer.

The potential difference (V_1-V_2) is determined by previously connecting opposite pairs of quadrants to two points of known potential difference (V_3-V_4) (*e.g.* the terminals of a standard battery), and the deflection θ_1 observed, when $\theta_1=c(V_3-V_4)V$.

Hence $$\theta/\theta_1=(V_1-V_2)/(V_3-V_4),$$

or $$(V_1-V_2)=(V_3-V_4)\theta/\theta_1.$$

It is evident, from equation (2) above, that the instrument is applicable to the measurement of small potential differences only when the needle is charged to a high potential, as otherwise the deflection obtained would be inconveniently small. This method, in which a constant and independent potential is given to the needle, is known as the *heterostatic* method of using the instrument. When considerable differences of potential are to be measured the independent charging of the needle is unnecessary; the needle is then connected to one pair of quadrants, and the *electrodes* are connected to the conductors, the potential difference between which it is desired to measure; this is known as the *idiostatic* method of using the instrument. With this arrangement the instrument may be regarded as an air condenser, one plate of which is movable.

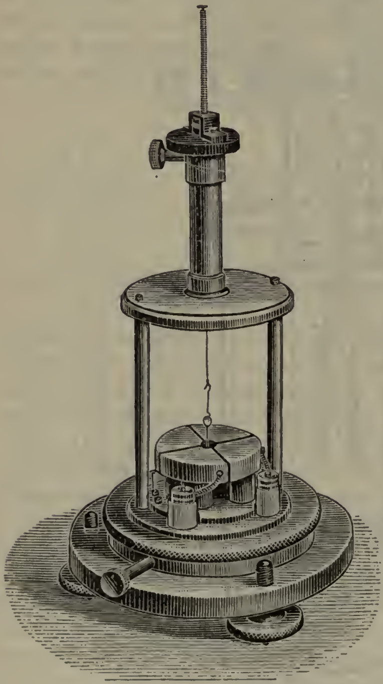

FIG. 145.—The Dolezalek Electrometer.

A simple and highly efficient type of quadrant electrometer has recently been designed by Dr. F. Dolezalek (Fig. 145). The quadrants are insulated on rods of amber, and two adjacent quadrants can be rotated so as to allow ready access to the needle. The needle consists of paper thinly coated on one side with metal, and is suspended by means of a quartz fibre, which affords high insulation. In some of the instruments a charging key enables the needle to be momentarily connected to a source of E.M.F.; in other cases, the quartz fibre is rendered conductive by previous immersion in a solution of calcium chloride or other hygroscopic substance. The lightness and dimensions of the needle render the instrument practically 'dead-beat.'

By connecting the needle of a quadrant electrometer to one pair of quadrants the instrument may be used for the measurement of **rapidly alternating potentials.** The formula to be used in this case is derived from equation I (p. 206) in the following manner: if the needle is connected to the quadrants which have an instantaneous potential V_1, then $V = V_1$, hence

$$\theta = c(V_1 - V_2)\left(V_1 - \frac{V_1 + V_2}{2}\right)$$
$$= c(V_1 - V_2)^2.$$

The deflection remains constant since the value of $(V_1 - V_2)^2$ is the same whether $(V_1 - V_2)$ is positive or negative.

Electrostatic Voltmeter.—For the measurement of high differences of potential, of 1000 volts or more, Lord Kelvin has designed the *electrostatic voltmeter* (Fig. 146) which, in principle, closely resembles the quadrant electrometer used idiostatically and may be regarded as an air condenser. It consists of a vertical aluminium plate A pivoted on horizontal knife edges; a small weight *w* is attached to the lower end of A and serves to control its position, and a pointer *p* moves over a circular scale. The two fixed plates Q and Q′ are insulated and fixed parallel to each other on opposite sides of A. When the movable and fixed plates are charged to different potentials the former moves so as to increase the electrostatic capacity of the condenser until the couple due to the electrical forces is balanced by that due to the weight *w*; the former couple is proportional to the *square* of the potential difference between the plates (p. 182). Different grades of sensibility are obtained by using different controlling weights.

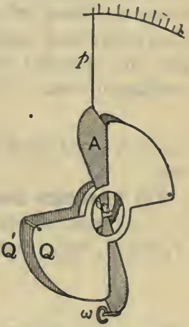

FIG. 146.—Kelvin's Electrostatic Voltmeter.

It is important to notice that this instrument may be used for measuring rapidly alternating potentials, for the deflection will remain unaltered when the potential-difference is reversed. Another advantage is that no current passes through the instrument, and no power is therefore wasted.

For measuring differences of potential varying from 50 to 800 volts Lord Kelvin has designed the **multicellular voltmeter,** which consists of a number of vanes similar to that in Fig. 146 attached horizontally at equal distances on a vertical spindle of aluminium wire suspended from a torsion head by means of a fine platinum wire. A vertical row of fixed quadrants are adjusted so that each vane moves between two quadrants. The deflection of the vanes is observed by means of an aluminium pointer fixed to the top of the spindle and moving over a horizontal circular scale.

SUMMARY

The Attracted Disc Electrometer.—If d is the distance between the attracting discs, at potentials V_1 and V_2, when the attracted disc is drawn into the 'sighted' position, and if mg is the force necessary to pull the attracted disc into the same position when both plates are uncharged, then

$$V_1 - V_2 = d\sqrt{8\pi mg/S},$$

where S is the mean area of the attracted disc and the gap in the guard-ring.

The Quadrant Electrometer.—When the suspended vane is charged to a constant potential V, and opposite pairs of quadrants to potentials V_1 and V_2 respectively, the deflection θ is given by the equation

$$\theta = c(V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right)$$

where c is a constant depending upon the dimensions of the instrument.

The Electrostatic Voltmeter resembles in principle the quadrant electrometer. The movable vane is pivoted so as to move in a vertical plane. The instrument is used for measuring potential differences of 1000 volts or more.

For smaller differences of potential the **multicellular voltmeter** may be used. Both these instruments possess the great advantage of being available for alternating current circuits.

QUESTIONS ON CHAPTER XI

1. An insulated plate 10 cms. in diameter is charged with electricity and supported horizontally at a distance of .1 mm. below a similar plate suspended from a balance and connected to earth. If the attraction is balanced by the weight of one decigram, find the charge on the plate ($g = 980$ C.G.S.).

2. An uninsulated metal plate, of area 150 sq. cms. and surrounded by a guard-ring, is suspended from the end of one arm of a balance, at the opposite end of which is a counterpoise. Below this plate, at a distance of 2 cms., is a large insulated plate. When the latter plate is charged it is found that 38 milligrams have to be added to the counterpoise in order to keep the suspended plate flush with the guard-ring. Find the potential of the lower disc, and state the units in which the answer is expressed ($g = 980$).

3. Explain what is meant by difference of potential, and describe some method by which it can be measured in absolute units.

4. Describe some form of quadrant electrometer, and explain how it may be used to compare the E.M.F.'s of two batteries.

5. A is a gold leaf electroscope, B a quadrant electrometer. For measuring small potential differences B is found to be more sensitive than A; does it necessarily follow that it will be more sensitive for small charges? Give reasons for your answer.

6. Describe the attracted disc electrometer, and explain how it can be used to measure differences of potential.

CHAPTER XII

ELECTRICAL MACHINES. ACTION OF POINTS

Types of Electrical Machine.—Any mechanical appliance designed to produce electrical effects on a large scale may be termed an *Electrical Machine.* The types depending on **Friction** (*e.g.* the *Glass-cylinder Machine*) have been entirely replaced by machines depending on **Induction**, of which the electrophorus (p. 162) may be regarded as the simplest type.

The Glass-cylinder Machine.—This consists of a glass cylinder mounted on a horizontal axis which can be rotated by means of a handle. When rotated the glass is positively electrified by rubbing against a silk pad fixed on an insulating support on one side of the cylinder. On the opposite side of the cylinder a horizontal metal comb, the teeth of which project towards the cylinder, is supported on an insulating stand. The teeth of the comb become negatively charged by induction from the positively charged glass passing near to them; a stream of negatively charged air leaves the points of the comb and neutralises the charge on the glass, while the comb itself remains positively charged. Meanwhile the uncharged glass is again charged on passing the silk pad. In order to obtain a continuous supply of positive electricity on the comb it is necessary to connect the silk pad to earth. A supply of negative electricity may be obtained from the silk pad by disconnecting it from earth and earth-connecting the metal comb. If both the comb and the silk pad are insulated and joined by a wire, a continuous transfer of electricity takes place along the wire, constituting a weak *electric current.*

MACHINES DEPENDING UPON INDUCTION

Kelvin's Water-dropping Apparatus.—The water-dropping apparatus devised by Lord Kelvin is perhaps the simplest arrangement which can be devised in order to explain the principle upon which all induction machines are based. A, B, C, D (Fig. 147) are four insulated metal cylinders, of which C and D contain metal cones with narrow openings at the apex. A and D, also B and C, are connected together by wires. F is a T-tube connected to a water-tap; the arms of the T-piece are bent vertically downwards and fixed so that the ends are within the cylinders A and B. *Suppose* A *and* D *to have a very slight positive charge.* A drop of water collecting on the jet within A will, since it is earth-connected, acquire a slight negative charge which it will carry downwards to C when it leaves the jet; C *and* B *thus acquire a slight negative charge.* The drops falling through B will now acquire a positive charge which will be carried downwards and imparted to D. The charges on A and B will thus mutually augment each other. The charges acquired may be observed by connecting AB and CD to two separate gold-leaf electroscopes. A and B may be termed **Inductors**, also C and D may be termed **Collectors.**

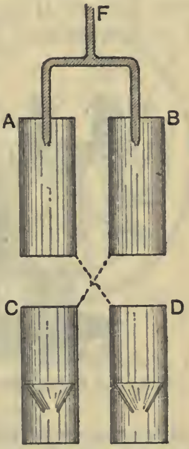

FIG. 147—Kelvin's Water-dropping Apparatus.

The energy represented by the charges acquired by the cylinders is derived from the kinetic energy of the falling drops of water. Owing to the repulsion between the charges on a falling drop and on the cylinder which it is approaching, the velocity of the drop is slightly less than it would be if the cylinders were uncharged, and *the loss of kinetic energy is a measure of the energy of the charge imparted to the cylinder.*

Kelvin's Mouse Mill (or **Replenisher**).—Figs. 148 and 149 represent a general view and a horizontal section of this simple form of induction machine, devised by Lord Kelvin for use with his absolute electrometer. R is a vertical rod of vulcanite which can be rotated by a milled head attached to its upper end; R carries two horizontal vulcanite arms on the ends of which are mounted two curved metal carriers C and D, their radii being inclined at an angle to the arm supporting them. A and B are cylindrically shaped pieces of metal which are insulated and serve as *inductors.* The springs *s* and *s*′ are fixed to A and B, and project inwards through holes in the plates. S and S′ are connected together by a wire M and insulated from other parts of the apparatus. *Suppose that* A *has a small positive charge, and that the carriers are rotated in the direction shown.* C and D become connected through S and S′, and will acquire negative and positive charges respectively. Contact is broken while still under the influence of A, and the carrier C will subsequently touch the spring *s*′ and give up its negative charge to B (since it is practically surrounded by the inductor B). On touching S′, the carrier C will acquire a positive charge which is subsequently given up to A. In this manner opposite charges accumulate on A and B. By rotating in the opposite direction the charges are gradually diminished. *The energy represented by the accumulated charges is derived from the mechanical work done in separating the oppositely charged carriers and inductors.*

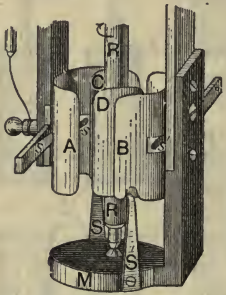

FIG. 148.—Kelvin's Mouse Mill.

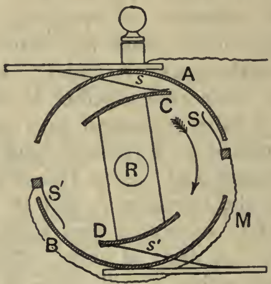

FIG. 149.

Toepler's Influence Machine (sometimes termed the **Voss**

Machine).—This machine, introduced in 1865, consists of two discs of glass, one of which is fixed and the other rotates in front of it. The theory of the machine is best explained by Fig. 150, in which the inductors and carriers are represented as portions of two concentric cylinders, the outer one of which corresponds to the fixed plate. The two inductors A and B are fixed to the back of the stationary plate; the carriers *a*, *b*, *c*, etc. are fixed to the front of the rotating plate. C is an insulated metal arm terminating in the metal brushes which touch opposite pairs of carriers as they pass. D and E are collecting combs connected to the discharging knobs at F. Each inductor carries a metal brush (n_1, n_2) which touches each carrier as it passes.

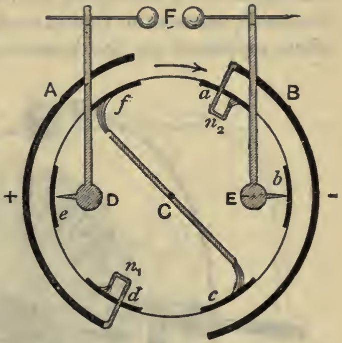

Fig. 150.—Diagram of the Toepler Machine.

Imagine that one of the sectors, d, has a slight positive charge. On passing the brush n_1, a portion of its charge will be acquired by the inductor A and will act inductively on the carriers *f* and *c*, which acquire a slight negative and positive charge respectively. The carrier *f* will subsequently give up part of its negative charge to B, and the carrier *c* will at the same time augment the positive charge on A. In this manner the charges on the inductors rapidly increase, thus increasing the charges induced on the carriers. Finally, since the carriers will still retain a portion of their charges after passing the brushes n_1 and n_2, the metal combs (D and E) will acquire positive and negative charges, and a series of sparks will pass between the discharging knobs.

As a rule the machine is self-starting; but if it is not so, a piece of charged sealing-wax may be held for a moment opposite the sector *c*.

The Wimshurst Influence Machine.—This machine (Fig. 151) consists of two varnished glass-plates, placed as close together as possible, and geared so as to rotate in opposite directions. An even number of metal sectors is fastened on the outer surface of each plate, and these sectors serve both as *inductors* and *carriers*. A conductor with wire brushes at the ends is fixed diagonally across the face of each plate. Two Leyden jars are often supported on the base-board of the machine with their knobs connected to the collecting-combs by movable wires. The action of the machine may be explained by Fig. 152, in which the two plates are represented as two concentric cylinders of glass rotating in opposite directions. The neutralising brushes are represented by n_1n_2 and n_3n_4. In order to start the machine it is sufficient if one of the sectors has a slightly different potential from that of the others ; as a rule this is the case, and the machine is then *self-starting*. Imagine that one

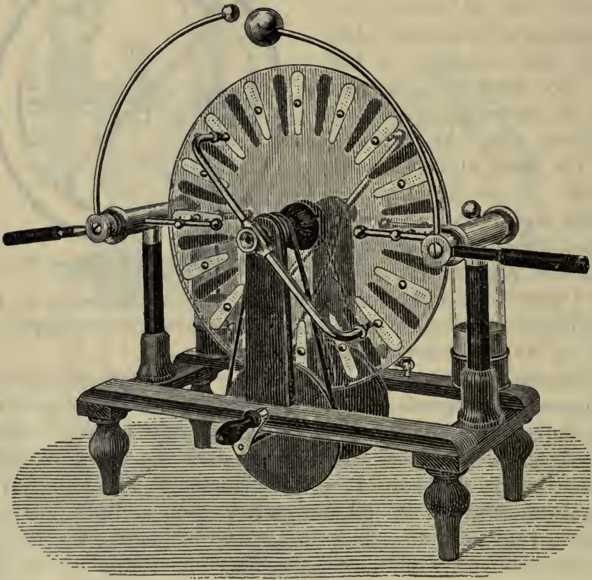

FIG. 151.—The Wimshurst Machine.

of the back sectors at the top of the diagram has a slight positive charge. When it comes opposite the brush n_1 it will act inductively on the sector touching n_1, giving to it a slight negative charge, and simultaneously giving a positive charge to the sector touching n_2. These sectors, with their induced charges, leave the brushes and rotate into positions opposite the brushes n_3 and n_4; the sectors touching n_3 and n_4 will now receive induced positive and negative charges respectively, which they will retain after leaving the brushes. Thus, after one or two revolutions, all sectors approaching the left-hand comb will have positive charges, and all sectors approaching the right-hand comb will have negative charges. The sectors will be neutralised by the combs, the knobs connected to which will acquire positive and negative charges respectively.

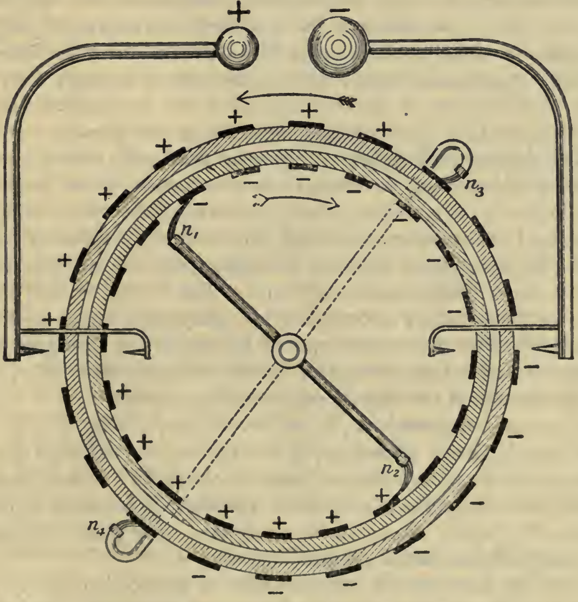

FIG. 152.—Diagram of the Wimshurst Machine.

The theory of the Wimshurst Machine, as explained above, has been verified by constructing a machine with plates of thin flexible insulating material instead of glass. If the collecting combs are removed, and the plates rotated, the discs will be bent together at the top and bottom, and bulge apart at the sides.

The quantity of electricity[1] *generated in a given time is proportional to the speed of revolution, but the potential difference between the terminals is independent of the speed.* An influence machine may be regarded as a source of electric current in which both the potential difference and the internal resistance are very high; with a machine of average size, the P.D. may amount to 50,000 volts, and the internal resistance to 10^9 ohms (the latter varying inversely as the speed).

The Action of Points.—It has been previously stated (p. 129) that the density of the charge on the surface of an insulated conductor is only uniform when the curvature is the same at all points of the surface (*i.e.* when the surface is spherical), and that the density is greatest on those parts of the conductor which have the greatest curvature. The cause of this effect may be more clearly understood by considering the case of two insulated spheres (of radii r_1 and r_2) which are insulated and connected by a thin wire. If V is the common potential, σ_1 and σ_2 the *density* of the charge on the spheres, then

$$\sigma_1 = Q_1/4\pi r_1^2 = C_1V/4\pi r_1^2 = r_1V/4\pi r_1^2 = V/4\pi r_1\,;$$

also $\sigma_2 = V/4\pi r_2$. Hence $\sigma_1/\sigma_2 = r_2/r_1$, or, *the densities of the charges are inversely proportional to the radii.* Consequently, if the smaller sphere is reduced in size until it becomes a mere point, the density on its surface becomes very great compared with that on the larger sphere.

The air particles in contact with a surface on which the density of charge is great acquire a portion of the charge and are electrically repelled. This action continues until the conductor is discharged.

EXPT. 62.—Attach a sewing-needle to one terminal of a Wimshurst machine by means of soft wax, taking care that the needle and terminal are in metallic contact. Connect the other terminal to earth. On turning the machine hold a candle flame near to the point and observe how it is blown aside. Transfer the needle to the other terminal and repeat the experiment. Allow the current of air to impinge on an insulated metal plate, and verify that the charge acquired is the same as that of the terminal to which the needle is attached.

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[1] Kohlrausch, with a Holtz Machine, obtained only 1 c.c. of mixed gases in 40 hours from the electrolysis of water.

If the pointed conductor is free to move, the mutual repulsion between the point and the air in contact with it will cause the point to move in the opposite direction. This explains the principle of *Hamilton's mill.* The effect is greater when the discharge is *positive* than when *negative.*

Lightning Conductors.—If a pointed conductor is charged *by induction* the same phenomena observed in Expt. 62 will take place.

EXPT. 63.—Hold a needle in the hand with its point towards the terminal of a Wimshurst machine. Hold a metal plate between the point and the terminal and verify that the plate is now charged *oppositely* to the terminal used.

This explains the action of *lightning conductors.* In certain atmospheric conditions clouds become charged to a high potential, and a disruptive discharge may take place either between oppositely charged clouds or between a cloud and the nearest point of the earth (where an induced opposite charge will be distributed). By fixing an *earth-connected* point above the highest part of a building the charge on the cloud will be partially or completely neutralised by the current of charged air proceeding from the point. A flame, owing to its conductivity, will serve as a lightning protector, but it is not so efficient as a metal point. Further details concerning lightning conductors are given on p. 228.

Character of the Discharge from an Electrical Machine.—

(i) SPARK DISCHARGE.—The intensity and the frequency of the spark discharge depends largely upon the capacity of the discharging terminals. If no Leyden jars are connected to the terminals the spark is frequent and not very luminous; but if the capacity is raised by connecting a Leyden jar to each terminal, the time required to produce the potential-difference necessary to break down the insulating power of the dielectric is increased, hence the sparks are less frequent, but at each discharge a much greater quantity of electricity passes between the terminals, and the sparks are therefore far more brilliant (Fig. 153).

The length of spark depends upon (i) **the P.D. between the terminals,** (ii) **the dielectric,** (iii) **the shape of the terminals, and** (iv) **the pressure** (**if the dielectric is a gas**) ; but it seems to be independent of the metal of which the terminals consist. The maximum *strain* which any dielectric is capable of bearing may be expressed by the fall of potential per unit length of the dielectric, and is generally known as the **dielectric strength.** Lord Kelvin found that the dielectric strength becomes less as the length of spark increases. The following table, based upon the results obtained by Baille,[1] gives the P.D. in *volts* required to spark through air, at 760 mms. and 15°-20°C., between parallel plates and between spheres of different size.

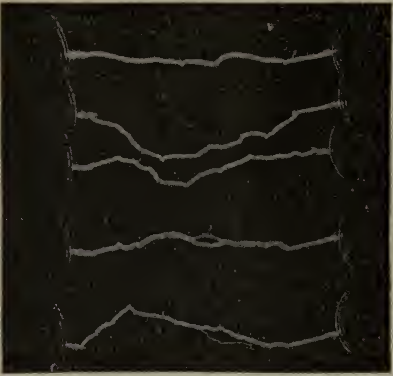

FIG. 153.—The Spark Discharge.

Spark Length in cms.	Flat plates.	Dielectric Strength.	Spheres 6 cms. diam.	Spheres 1 cm. diam.
0.05	2682	53640	2688	2754
0.10	4410	44100	4440	4530
0.20	7626	38130	7680	7860
0.30	10603	35340	10830	10470
0.40	13431	33580	13500	12750
0.50	16341	32682	16530	16410
0.60	19146	31910	19560	19200
0.70	22230	31757	22620	22590
0.80	25458	31820	26400	26010
0.90	28420	31577	29220	28770
1.00	31647	31647	33870	31620

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[1] *Annales de Chimie et de Physique*, xxv., 1882.

For spark lengths greater than 0.2 cm. the relationship between the P.D. and the spark length in air may be represented by the equation

$$V = 1500 + 30{,}000\,d$$

where V is the P.D. in volts and d is the distance in centimetres.

If the P.D. is only applied *for a short time* the air can withstand a much higher P.D. without a spark passing, in fact in all cases there is an appreciable lapse of time, known as the 'lag,' between the application of a P.D. and the passage of a spark. Also the passage of the first spark facilitates the passage of subsequent sparks, and this effect remains for several minutes. It has also been observed that the illumination of the *negative* terminal by ultra-violet light facilitates the passage of a spark. Experimental evidence has been obtained which proves that a spark is preceded by an extremely small current, which indicates that the air acquires slight conductivity.

Sir J. J. Thomson[1] has explained how the 'lag' may be attributed to the presence of a few *negatively charged ions* (p. 463) in the air between the plates; and these, under the influence of the strong field, acquire a high velocity and by violent collision with other molecules give rise to further ionisation of the air. In this manner the air gradually acquires conductivity, followed by the passage of a spark. The 'lag' would of course be diminished by any agency which accelerated the ionisation of the air, and it is known that this property is possessed by ultra-violet light.

With a constant spark length the P.D. required to cause the passage of a spark diminishes when the *pressure* on a gaseous dielectric is diminished; at first the fall in the P.D. is in direct proportion to the fall of pressure, but after acquiring a minimum value, at a pressure termed the *critical pressure*, it finally increases with great rapidity. The 'critical pressure' for air between parallel plates is about 3 mms. of mercury. Paschen has proved that *if the product of the pressure of the gas and the length of spark is kept constant the* P.D. *required to produce the spark is also constant*: this is known as **Paschen's Law.** The minimum P.D. below which no spark is obtained, however short the path, has been found to be 350 volts.

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[1] *Phil. Mag.*, p. 278, 1900.

The relative *dielectric strengths* of different media, approximately expressed in kilo-volts per centimetre of spark length, are given in the following table.

TABLE OF DIELECTRIC STRENGTHS

Substance.	Dielectric Strength.
Air, - - -	23·8
Hydrogen, - -	22.2
Paraffined paper,	360.0
Turpentine, - -	94.0
Paraffin oil, - -	87.0

(ii) BRUSH DISCHARGE.—This phenomenon can only be clearly observed with the aid of a large machine, and is obtained when the discharging knobs are well polished and too far apart to discharge by sparks. In appearance, the brush discharge consists of a short bright line which originates from the positive terminal and ends in a radiating brush of comparatively slight luminosity.

(iii) GLOW DISCHARGE.—This effect is observed when the terminal is sharply pointed, *e.g.* when a needle is attached to one terminal, the other terminal being earth-connected. In air at normal pressure the effect can be observed at a distance only when a large machine is used, but the *glow* discharge is very characteristic of a discharge through a rarefied gas.

(The discharge of electricity through gases is more fully discussed in Chapter XXV.).

Mechanical, Chemical, and other effects of an Electric Discharge.—MECHANICAL EFFECTS.—A sheet of glass supported between two insulated metal terminals may be punctured when the discharge from a charged battery of Leyden jars is passed between the terminals. When the sheet is small the discharge may pass over the surface and round the edge of the glass, but this can be prevented by placing a drop of oil on the glass round one of the terminals, or by fixing a short glass tube to the plate by means of sealing-wax and using a thick copper wire as a terminal which can be enclosed within the tube.

A sheet of cardboard is readily perforated by an electric discharge, and it will be seen that the edges of the perforation on both sides of the card are burred outwards.

CHEMICAL EFFECTS.—If a piece of white filter paper, previously soaked in an emulsion of starch and potassium iodide, is laid on a sheet of glass and fixed just below the terminals of an electric machine, patches of blue coloration will be developed where the discharge strikes the paper, owing to the liberation of iodine from the potassium iodide. The same chemical change is produced when the paper is touched by the terminals of a voltaic battery, but in this case the coloration is only evident round the *positive* terminal. The coloration near *both* terminals of an electric machine is evidence that the discharge is of an oscillatory nature.

When an electrical machine is being used, the characteristic smell of *ozone* is often evident; the conversion of oxygen, O_2, into ozone, O_3, being due to the electric discharge. By means of somewhat elaborate apparatus this method is adopted in the chemical laboratory for the production of ozone.

Fig. 154 represents a form of apparatus for the *electrolysis of* water by means of the discharge derived from an electrical machine. The success of the experiment depends upon the smallness of the platinum surfaces exposed to the water—very thin wire should be used and only the extreme ends should be unprotected by the glass tubes; in any case the volume of mixed gases obtained is extremely small (foot-note, p. 218). One terminal should be joined to an

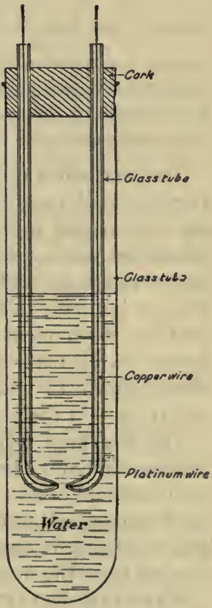

FIG. 154.

insulated knob fixed *near to* the knob of a Leyden jar standing on the table; one terminal of the machine is joined to the Leyden jar and the other to the remaining terminal of the apparatus.

It may also be shown that the discharge is capable of decomposing *ammonia* into its elements nitrogen and hydrogen, and of bringing about the union of nitrogen and oxygen.

HEATING EFFECTS.—Theoretically the total available energy in a discharge may be represented by $\frac{1}{2}QV$ *ergs*, where Q is the quantity of electricity transmitted and V is the initial potential-difference. The energy equivalent to 1 gramme-degree, or *calorie*, of heat is 4.2×10^7 ergs —this is known as *Joule's equivalent*, and is usually denoted by the symbol J. Hence, if the energy represented by the charge is entirely resolved into heat, the number of heat units generated will be $\frac{1}{2}\frac{QV}{J}$. Recent investigation has shown that, in an electric discharge, the heat produced is only a small fraction of the total energy of the charge; thus with a spark 6 millimetres long the heat is no greater than if a wire of resistance 2 ohms had been placed in the gap.

The heat effect of the discharge may be shown by connecting two insulated metal balls by a short length of *very fine wire*; on discharging a Leyden jar battery through the wire it is volatilised with explosive violence. If the discharge is made to pass through a small heap of *gunpowder*, it is simply scattered, but not ignited; this is owing to the discharge being so brief that the mechanical scattering takes place before the powder is heated to its temperature of ignition. But if the discharge is slowed-down by including a poor conductor—*e.g.* a piece of wet string—in the circuit the powder will be ignited.

The discharge from a Leyden jar will ignite *ether*. Also, if the experimenter stands on an insulating stand with one hand on a terminal of an electrical machine, the other terminal being earth-connected, and holds a finger near to a gas burner from which *coal-gas* is escaping, the gas will be ignited by the spark passing from the finger to the burner.

MAGNETIC EFFECTS.—A steel sewing-needle may be magnetised by placing it inside an insulated open spiral of thick guttapercha-covered copper wire through which a discharge is passed from a Leyden jar battery. The polarity acquired by the needle is indefinite, owing to the oscillatory nature of the discharge; the polarity, in any case, is determined by the direction of the *maximum current*, which may be expressed as the ratio of the quantity of electricity to the time of transmission, and the time of transmission of consecutive oscillations of the discharge may vary considerably.

SUMMARY

The action of an electrical machine may be due either to *friction* or to *induction*. The machines described may therefore be classified thus:

Frictional Machines.—The glass-cylinder machine.

Induction Machines.—The Electrophorus.
Kelvin's Water-dropping Apparatus.
,, Replenisher.
Toepler's Machine.
Wimshurst's ,,

The quantity of electricity generated in a given time is proportional to the speed of revolution, but the P.D. is independent of the speed.

The Action of Points.—*The density of the charge at a point of the surface of an irregular conductor is inversely proportional to the radius of curvature of the surface of that point.*

Spark discharge.—The length of spark depends upon (i) *the P.D. between the terminals*, (ii) *the dielectric*, (iii) *the shape of the terminals*, and (iv) *the pressure* (in the case of gases). The relationship between the P.D. and the spark length, through air at 760 mms. and 15° C., may be expressed thus:

$$V = 1500 + 30,000\, d$$

where V is in *volts* and *d* in *centimetres*.

The **dielectric strength** of any medium is the maximum strain, expressed in *volts per cm.*, which it can bear.

Paschen's Law.—*If the product of the pressure of the gas and the length of the spark is kept constant the P.D. required to produce the spark is also constant.*

Effects due to an electrical discharge.—

(i) **Mechanical.**—*Examples.*—Glass or cardboard punctured.

(ii) **Chemical.**—*Examples.*—(*a*) Decomposition of potassium iodide; (*b*) formation of ozone; (*c*) electrolysis of water.

(iii) **Heat.**—*Examples.*—(*a*) Fusion of thin metal wires; (*b*) ignition of gunpowder, ether, and coal gas.

(iv) **Magnetic.**—A steel needle may be magnetised by passing a discharge through a spiral surrounding the needle.

QUESTIONS ON CHAPTER XII

1. Describe some apparatus by which an indefinitely large quantity of electricity can be obtained by means of electrostatic induction from a minute initial charge.

2. State and explain the action of Lord Kelvin's Replenisher.

3. Describe a simple form of electrical induction machine.

4. Briefly enumerate the different kinds of electric discharge and describe their luminous and mechanical differences.

CHAPTER XIII

ATMOSPHERIC ELECTRICITY

Lightning.—Benjamin Franklin observed that *lightning* has all the properties—disruptive, heating, and chemical—which are characteristic of an electric discharge obtained by artificial means; the effects only differ in degree. He carried out, in 1752, his historic experiment with a kite, for the purpose of testing whether a charge might be derived from a thundercloud; a metal point attached to the kite was connected to one end of the string, the other end of which was tied to a key; a length of silk ribbon served to insulate the key from the hand. Standing under a roof (in order to keep the silk dry), he noticed that sparks were freely obtained from the key as soon as the string became wet, and that a Leyden jar could be charged by this means.

Lightning is an electric discharge taking place either between two clouds charged to different potentials, or between a charged cloud and the earth's surface. We may regard a charged cloud and the neighbouring surface of the earth as a condenser, in which the intervening air is the dielectric. When the potential difference acquires a certain magnitude the dielectric breaks down, and a discharge passes. Lightning assumes three forms: (i) *Fork*; (ii) *Globular*; and (iii) *Sheet.*

Sheet Lightning is probably the reflection from the surface of clouds of distant fork-lightning which is hidden from view by intervening clouds or by the horizon. **Globular Lightning** is a comparatively rare phenomenon, and is but little understood; it is usually described as a luminous ball which travels slowly, and finally bursts with considerable violence. It has occasionally been imitated on the small scale in electrical experiments. **Fork Lightning**, as observed in photographs, usually traces out a very irregular path with many ramifications. The path of the discharge is always that of least resistance, which may be determined either by the presence of dust, or by the path of a previous discharge. The flash is usually oscillatory; this effect may be observed in a photograph of the flash if, while exposing the plate, the camera is rotated rapidly in a direction perpendicular to that of the flash; in this manner two or more independent images of the flash are obtained which are identical in outline.

The duration of a flash is certainly less than 1/10,000 second. This has been proved by illuminating, by means of a flash, a rotating disc divided into sectors alternately black and white. When viewed in daylight the disc appears grey, while if in a dark room and only illuminated by an *instantaneous* flash, the disc appears to be at rest. If the duration is as long as the interval required for a black sector to move into the position occupied by the preceding white sector, the disc appears uniformly grey; if the duration is less, black and white sectors appear to be separated by grey ones. This is the appearance obtained with a disc of 60 sectors making 180 revolutions per second. Therefore, the duration of the flash must be less than $(1/60 \times 1/180)$ sec. $= 1/10,800$ sec.

Lightning Conductors.[1]—The modern type of lightning conductor consists of a broad *band* of copper, or preferably of iron, fixed to the outer side of a building, and terminating in a metal point which is slightly above the highest point of the structure; the lower end is connected to a metal plate buried in a stratum of earth *which is always wet*, so as to ensure good earth contact. All metal spouts and pipes outside the building should be independently connected by wires to a wet 'earth.'

The action of the lightning conductor is twofold: (i) A neighbouring cloud causes a stream of oppositely charged air to proceed from the point, thus slowly neutralising the charge on the cloud; and (ii) if a discharge should take place, it *probably* chooses the path of least resistance, and proceeds to earth without causing damage to the building.

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[1] See "Lightning, and its Protectors," Sir O. Lodge, *Journ. Inst. Elect. Engineers*, vol. xviii., No. 80.

The discharge is excessively rapid, and probably also of an oscillatory nature. The ordinary laws of current flow are therefore not obeyed, and the discharge is restricted almost entirely to the outer layer of the conductor ; also, since it is advantageous to retard the rapidity of the discharge, the capacity of the conductor should be as great as possible. For these reasons a flat metal band is preferable to a circular wire (which has less surface for a given mass of metal).

If any part of the conductor is in the form of a coil or convolution, the self-induction accompanying a sudden discharge may be so great as to make the coil practically a non-conductor ; the discharge may then spark across any short air gap, either to the building or to a neighbouring metallic conductor. The lightning conductor should therefore be as *direct* as possible. This explains the principle by which telegraphic or telephonic instruments are protected from lightning ; two metal plates or discs are placed face to face and separated by a thin sheet of perforated mica ; if one plate is connected to earth and the other to the end of the line wire, any sudden discharge proceeding from the line wire will pass more readily across the short air gap between the plates than through the coils of the instrument.

Origin of the Electrification of Clouds.—It has been suggested that the electric charge in clouds originates from evaporation proceeding from the earth's surface ; but the observations of several investigators do not agree in proving that the vapour rising from a charged liquid is either charged or uncharged. It may arise from the friction between solid and liquid particles, or between these and the earth's surface, or again from friction of masses of air at different temperatures.

Lord Rayleigh has shown that drops of water tend to coalesce when slightly electrified. The coalescence will cause a reduction in the total surface of the water drops, and therefore the density of the charge will increase ; but this will not necessarily increase the potential, since the potential of a charged drop depends so largely upon the neighbourhood of other charged drops. Holmgren has shown that when the area of a water surface is changing rapidly (*e.g.* when ripples are passing over it) that the water becomes positively charged, and the air negatively charged. Again, if small drops of water coalesce the total surface is diminished, and the work done by surface tension in diminishing the surface may reappear as electrification. Since a cloud acts as a moderately good conductor the charge on the individual drops of condensed moisture will distribute itself eventually over the surface of the cloud. Large rain-drops are characteristic of thunderstorms, and Sir G. Stokes has suggested that the drops may be the cause, rather than the consequence, of the storm.

Ordinary Electrification of the Air.—Lemonnier observed, in 1752, that a long insulated conductor fixed vertically becomes electrified, even when the sky is clear. This proves the presence of an electric field of force in the atmosphere. If the upper end of the conductor terminates in a point, discharge takes place (p. 218) until the potential of the point, and therefore that of the conductor, is the same as that of the layer of air in which the point is situated. Volta, in 1787, substituted a flame for the conducting point; but the method is not so trustworthy, owing to a small potential difference, perhaps amounting to 0.5 volt, produced by the combustion. A sponge dipped in spirit and ignited has been used; also, Kelvin has used spills made of paper impregnated with lead nitrate. The potential of the conductor is observed by connecting its lower end to opposite quadrants of a quadrant-electrometer.

The most suitable form of apparatus for equalising the potentials of a conductor and the surrounding air is the **water-dropper** (Fig. 155) devised by Kelvin. This consists of a copper vessel supported on a stout glass-rod, which is kept dry by surrounding it with pumice and sulphuric acid. The water in the vessel is allowed to drop slowly from the end of a long horizontal tube, which projects through a hole in a window. Each drop becomes charged oppositely to that of the surrounding air, and on falling off conveys away this opposite charge; this action continues until the drops of water, and the cistern, have the same potential as the air near to the nozzle. The reservoir is connected to the needle of an electrometer, the opposite pairs of quadrants being connected to the poles of a dry pile, or to the terminals of a battery of which the centre is earth-connected. The deflection of the needle indicates the potential of the air at every instant, except when the variations are very rapid. In cold weather, glycerine or alcohol are added to the water to prevent freezing.

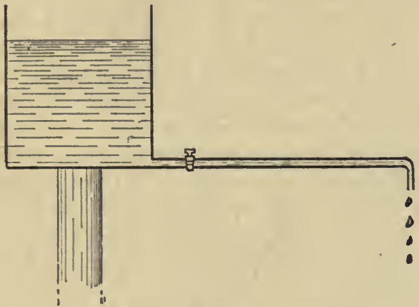

FIG. 155.—Kelvin's Water-dropper.

It has been suggested, more recently, that radio-active compounds, *e.g.* salts of uranium or radium, may be used as equalisers of potential instead of a flame or water-drops. A few decigrams of such a compound might be supported at the upper end of an insulated rod, and protected from rain by covering with a thin piece of aluminium foil (which would be practically transparent to the radiation).

In fine weather the air always has a *positive* potential which increases with the height above the earth's surface. The *potential-gradient* varies between wide limits; near to the earth's surface it usually varies from 100 volts to 200 volts per metre, but at higher levels the gradient is less, thus at altitudes of 1140 metres and 3360 metres, gradients of 42.6 and 18.7 volts respectively per metre have been observed. In cloudy weather the potential of the air is invariably lower than when the air is clear; in wet weather a temporary *negative* potential is frequently observed. It may be added that the potential of the air near a waterfall is always negative, and that the effect is greater when the water is pure (p. 126). The variations of potential are so frequent and marked as to suggest that the masses of air conveyed past the apparatus by the wind are themselves electrified.

When the air has a positive potential which increases with the altitude, the electric conditions resemble those found in the field of force of an air condenser; the air in the upper regions is very rarefied, and is an electric conductor, so also is the earth, while the air between is a non-conductor if dry. Hence, we may regard the upper stratum of the air as a positively charged conductor, while the earth is negatively charged.

Since the earth's surface within limited areas has uniform electric potential, the equipotential surfaces in the field of force are parallel if above an open plain. But over elevated parts of the earth's surface they approach each other to an extent depending upon the height and abruptness of the elevations. For, if σ = the surface density of the charge on the earth's surface, the intensity (F) of the electric force near the earth is equal to $4\pi\sigma$ (p. 142). On summits of hills or on house-tops σ has a higher value, and F is greater; but $(V_1 - V_2) = F \times d$ (p. 147), hence the distance apart of the equipotential surfaces will be less. At higher altitudes the effect of these irregularities in the earth's contour disappears and the equipotential surfaces are again equidistant.

Elster and Geitel have suggested that the electrification of the air is due to the ultra-violet rays of sunlight which have the peculiar property of dissipating a negative charge from the surface of certain substances; the effect is very marked in the case of freshly-polished zinc, and *dry* ice is also sensitive to the rays in the same way.

Other observers lay stress upon the fact that air is partially *ionised* (p. 509) when traversed by ultra-violet rays, and that the air then conducts electricity in a manner similar to an electrolyte (p. 464). It has recently been observed[1] that the oppositely-charged ions serve, like dust particles, as nuclei round which aqueous vapour may condense to form rain drops, and that the negative ions act in this manner far more readily than the positive ions; consequently, rain so formed would carry a charge which is chiefly negative.

The Energy of Discharge of a Cloud.—Suppose that the potential-gradient is 10 volts per cm., and that the equipotential surfaces are horizontal, then $(V_1 - V_2) = 10$ volts $= 10 \times 10^8$ E.M.[*] units $= 10^9/(3 \times 10^{10})$ E.S. units (p. 521). But $F \times d = (V_1 - V_2)$; hence $F = 1/30$ E.S. units. By Coulomb's Law, $F = 4\pi\sigma$, hence $\sigma = F/4\pi = 1/120\pi$ E.S. units $= 1/(120\pi \times 3 \times 10^{10})$ E.M. units $= 10^{-12}$ coulombs approximately. Let the area of the cloud-surface be 1 sq. kilometre, and its height above the earth's surface be 0.5 kilometre. The total charge on the cloud $= (10^{-12} \times 10^{10})$ coulombs, and its potential exceeds that of the earth by (5×10^5) volts. Hence the energy of the discharge, $\frac{1}{2}QV$, is (2.5×10^3) joules $= (2.5 \times 10^3 \times 10^7)$ ergs $= (2.5 \times 10^2)$ kilogram-metres. Or, the energy is sufficient to raise 250 kilograms to a height of 1 metre.

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[1] C. T. R. Wilson, *Phil. Trans.*, cxciii.

The Aurora.—The *aurora* is a rare phenomenon in our latitudes, but is frequently seen in arctic and antarctic regions. It assumes various forms, but often appears as a whitish bow over a dark segment in the northern sky, and becomes covered by extended masses of red or greenish-yellow feathered clouds radiating outwards from a point situated in the direction of the magnetic pole ; these rays lengthen and shorten, increase in brilliancy or disappear, and perhaps again appear. There is a close resemblance between the spectrum of the aurora and that of an electric discharge through rarefied air : this suggests that the phenomenon is electric and situated in the higher regions of the air. The aurora is usually accompanied by magnetic storms and is more frequent during periods of maximum sunspots. Franklin attributed the phenomenon to electric discharges caused by the contact of currents of warm air from the tropics with the cold air of the polar region. A more recent theory suggests that the phenomenon is due to streams of *electrons* discharged from the sun, the paths of which streams are influenced by the earth's magnetic field.

Earth Currents.—It is often observed that telegraphic lines are traversed by spontaneous currents which may be sufficiently strong to disturb the transmission of signals. They are more frequently observed in lines running north and south, and are observed both in aerial and underground lines. A spontaneous E.M.F. of 120 volts in a line 220 miles long was recorded by Sir W. H. Preece in 1881. Prof. Schuster[†] has shown that these currents are due principally, if not entirely, to electromagnetic induction.

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[*] The abbreviations E.M. and E.S. refer to the *electromagnetic* and *electrostatic* systems of units.

[†] *Brit. Assoc. Report*, 1890.

SUMMARY

Lightning is an electric discharge taking place either between two clouds charged to different potentials or between a charged cloud and the earth's surface. The discharge is usually *oscillatory*, and its duration is certainly less than 1/10,000 second.

Lightning Conductors.—The silent discharge from the point of an earth-connected conductor tends to diminish the potential-difference between the conductor and a neighbouring charged cloud. If a spark discharge takes place the conductor may serve to conduct the discharge to earth. A flat metal band is more efficient than the same weight of metal distributed as a circular wire.

The origin of the electrification of clouds is not known with certainty.

Electric condition of the Atmosphere.—In fine weather the air always has a positive potential which increases with the height above the earth's surface; in cloudy weather the potential gradient is diminished, and the potential is often negative in wet weather.

The potential of the air is determined by connecting the needle of an insulated electrometer to a point or to a flame placed in the region of which the potential is required. Instead of a flame, Lord Kelvin has used the *water dropper*, which consists of an insulated metal vessel with a long horizontal outlet from which water is allowed to fall in drops. In any case the electrometer gradually acquires the potential of the air space under examination.

PART III

VOLTAIC ELECTRICITY

CHAPTER XIV

VOLTAIC CELLS

The Electric Current.—If two conductors A and B are at different potentials, A having a positive and B an equal negative charge, and are then connected by a conducting wire, positive electricity will proceed along the wire from A to B and, after a very short interval of time, the potentials will be equalised. The upper part of Fig. 156 represents the initial distribution of the tubes of force between two oppositely charged spheres. The lower part represents the distribution immediately after joining the conductors by a wire; the ends of the tube of force *ab* have now been transferred to the wire, as shown at *a′b′*, and are proceeding along the wire in opposite directions; and when the opposite ends meet, the tube will have shrunk into the wire. This process is repeated with all the tubes of force, and the field finally disappears.

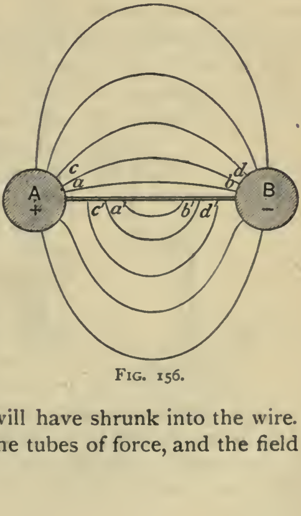

FIG. 156.

During the discharge along the conducting wire two phenomena occur: (i) *the wire is heated*, and (ii) *a magnetic field is set up* round the wire; but the discharge is so rapid that it would be difficult to observe these effects. When such phenomena accompany the transference of electricity we say that the wire is traversed by an **electric current.** What really takes place may be described as a transference of energy from the medium into the wire. The work done in creating the electrostatic field disappears as heat in the wire; the magnetic field, which also represents a certain amount of energy, is only created during the growth of the current and, as the current diminishes, the lines of force gradually shrink into the wire and give up their energy to it.

It is usual to say that, in this case, *a current of electricity traverses the wire from* A *to* B; but it is important to bear in mind that this is only an accepted mode of speech, for we might refer equally well to the transference of *negative electricity from* B *to* A. Recent experimental investigation has shown the probability, and perhaps the certainty, that an electric current consists of the transference of *negative electrons* (p. 505) along the wire from B to A; it is nevertheless advisable to adhere to the recognised nomenclature, and to describe the current as proceeding from points of higher to points of lower potential.

In order to study the phenomena of current electricity any such arrangement as shown in Fig. 156 would be inconvenient, and it is necessary to make use of sources of electrical energy, such as voltaic cells, which are capable of maintaining a constant difference of potential.

The Simple Voltaic Cell.—If plates of zinc and copper are dipped into dilute sulphuric acid, a difference of potential is set up between the two metals, the copper being at a higher potential than the zinc; in fact the acid is at a higher potential than the zinc, and the copper is at a higher potential than the acid (p. 476). If the zinc plate is connected to earth by a wire, the relative potentials of the metals and the acid may be indicated by means of

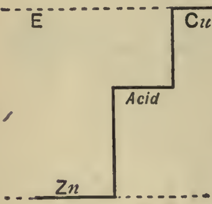

FIG. 157.

a diagram (Fig. 157); the P.D. between the copper and zinc plates is represented by the height of the dotted line E above the zero line, and it may be observed readily by means of a condensing electroscope, if several such cells are connected together *in series*, *i.e.* with the copper plate of one cell joined to the zinc plate of the next, and so on; the P.D. between the extreme ends of the series will then be equal to that between the plates of a single cell multiplied by the number of cells in series.

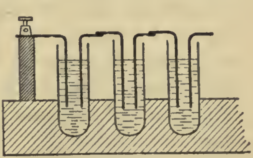

FIG. 158.

EXPT. 64.—Construct a series of simple cells (Fig. 158) by fixing a row of small glass tubes (2 inch × ½ inch) in a block of wood. Solder together a number of copper and zinc strips, and bend each compound strip so that the copper dips into one tube and the zinc into the next. Nearly fill each tube with very dilute sulphuric acid. Connect the zinc plate of the first cell to earth, attach a copper wire to the opposite terminal, and wrap the free end of this wire round a vulcanite rod, which will serve as a convenient handle. Fit up the condensing electroscope[1] (Fig. 159). Connect plate A to earth, and *momentarily*

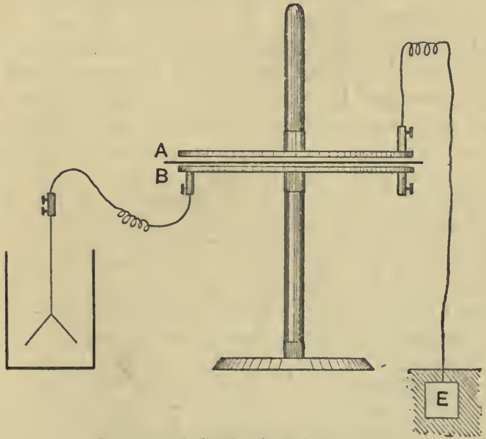

FIG. 159.—A Condensing Electroscope.

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[1] A condensing electroscope consists of two brass discs, about 20 cms. diameter. The lower disc is supported on a vertical rod of unpolished vulcanite, and is con-

touch plate B with the free end of the copper wire; A and B now have the same P.D. as that between the terminals of the cells. On raising plate A the capacity of the lower plate is diminished, and the potential of B is increased correspondingly; the potential of B is now sufficient to cause a divergence of the leaves. Verify that the charge on the leaves is *positive*. Discharge the condenser, reverse the battery, and repeat the observations; verify that the plate B now acquires a *negative* charge.

A fairly steady current will traverse a wire connecting the terminals[1]: this may be proved by means of its magnetic effect, *e.g.* by inserting a galvanoscope (p. 272) into the circuit. The P.D. is maintained by the energy which arises from the chemical changes now proceeding in the cell. It is usual to apply the term *electro-motive force*, usually written E.M.F., to the potential difference between the plates, although the term E.M.F. should be restricted to the forces within the cell which give rise to the P.D. between the terminals. The chemical action in the cell involves the solution of the zinc in the dilute acid—strictly speaking, the zinc initially becomes oxidised to zinc oxide, which is dissolved in the acid with formation of zinc sulphate. The P.D. is maintained until either all the zinc or all the acid has been used up.

Local Action.—In a simple voltaic cell, of which the plates are copper and *commercial* zinc, it will be noticed when a current is being generated by the cell that bubbles of gas are being liberated from *both* plates, and even when the circuit is broken that a gas is still given off from the zinc plate; but if *pure* zinc is used, then no gas is set free from the zinc plate. This effect with commercial zinc is termed **local action**, and is due to the presence of impurities, usually carbon and iron.

Local action is prevented by amalgamating the zinc by immersing the plate for a short time in dilute sulphuric or hydrochloric acid, and afterwards rubbing mercury over the surface with a rag soaked in the acid. A more satisfactory method is to add 4% mercury to the molten zinc when casting the plates.

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nected to a gold-leaf electroscope; the upper disc is supplied with a vulcanite handle, and is earth-connected. The plates are separated from each other by a thin sheet of paraffined paper.

[1] The terminals or plates of a voltaic cell are sometimes termed the *electrodes*.

Expt. 65 (i).—Add dilute acid to a fragment of pure zinc contained in a test-tube; no action takes place. Add a few iron filings; bubbles of gas are at once liberated from the filings, *but not from the zinc.* Charcoal or copper filings will act in the same manner. We have here, in fact, a simple voltaic cell, but the connecting wire is dispensed with since the metals are already in contact beneath the surface of the acid.

(ii) Add dilute acid to a fragment of *commercial* zinc, and observe the chemical action. The action ceases on adding a drop of mercury. Since the mercury dissolves zinc, but not iron, copper, or carbon, the film of mercury conveys *pure* zinc to the acid, and serves as a protecting layer to the particles of iron and carbon which may be present in the zinc, each of which would, if uncovered, form a minute voltaic cell with the zinc in which it is embedded.

Polarisation.—If the strength of the current derived from a simple cell is observed carefully, it will be evident that it slowly diminishes. It will also be observed that bubbles of hydrogen accumulate on the copper plate. This hydrogen has a two-fold effect: (i) it reduces the effective area of the copper plate, and (ii) it is, like zinc, a readily oxidisable element, and behaves in a voltaic cell in a manner similar to zinc, and tending to set up an E.M.F. which would generate a current in the *opposite* direction to that obtained in a simple voltaic cell. Hence the primary effect of the metal plates is reduced by the opposing effect of the hydrogen. This phenomenon is termed the **polarisation** of the cell. It is somewhat difficult to observe the effect with simple appliances, but the following experiment will suffice to prove its presence:

Expt. 66.—Construct a simple voltaic cell, using a stout copper wire instead of a copper plate. Immerse the wire, previously cleaned and *dried*, and the zinc plate in clean white sand saturated with dilute acid, and contained in a small beaker. Connect up the cell to a tangent galvanometer of low resistance (p. 274), bring the needle to rest as quickly as possible, and read the deflection at intervals of one minute; the diminution of deflection will prove the presence of polarisation.

Further details concerning polarisation are given on p. 479.

Various methods may be adopted to prevent polarisation; the most satisfactory are those in which the hydrogen is either oxidised or replaced by another element, and these constitute the numerous types of primary cells. These cells may be divided into two principal classes: (1) those which have only one fluid, and (2) those which have two fluids.

One-Fluid Cells.

The Bichromate Cell.—In this cell (Fig. 160) *potassium bichromate* $K_2Cr_2O_7$ is used as a depolarising agent. The chromic oxide CrO_3 is reduced by the hydrogen to the lower oxide Cr_2O_3, which is dissolved in the sulphuric acid, with which it is mixed, and forms chromium sulphate $Cr_2(SO_4)_3$: the change is accompanied by an alteration in colour from orange-red to dark green-blue. The elements used are carbon and zinc. The chemical change may be expressed thus:

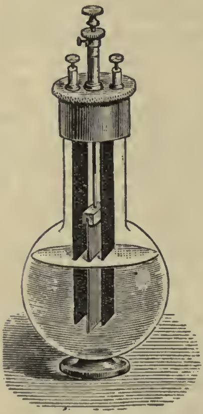
FIG. 160.—The Bichromate Cell.

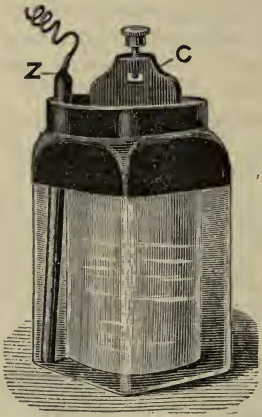

FIG. 161.—The Leclanché Cell.

$$3Zn + K_2Cr_2O_7 + 7H_2SO_4 = 3ZnSO_4 + K_2Cr_2(SO_4)_4 + 7H_2O.$$

The double sulphate of potassium and chromium, $K_2Cr_2(SO_4)_4$, is known as *chrome-alum*.

The solution may be made according to the following formula: $K_2Cr_2O_7$ 100 grams, concentrated sulphuric acid 200 grams, water 1200 grams. The zinc may be kept amalgamated by adding 2.5 grams of mercurous sulphate to each litre of solution. Chromic acid is perhaps preferable to potassium bichromate.

Leclanché Cell.—The Leclanché cell (Fig. 161) consists of a zinc rod and a carbon plate immersed in a strong solution (3 in 20) of ammonium chloride. The carbon plate is placed in a porous pot which is closely packed with a mixture of broken 'gas-carbon' and manganese dioxide, the latter acting as the depolariser. The comparative slowness with which the dioxide oxidises the liberated hydrogen causes polarisation to take place when strong currents are taken from the cell, which is therefore only suitable for weak or intermittent currents. The small surface of the zinc rod and the solid contents of the porous pot give to the cell a considerable internal resistance. The chemical action within the cell may be expressed thus:

$$Zn + 2NH_4Cl + 2MnO_2 = ZnCl_2 + 2NH_3 + H_2O + Mn_2O_3.$$

TWO-FLUID CELLS.

Daniell Cell.—The Daniell cell (Fig. 162) consists of a zinc rod surrounded by dilute sulphuric acid (1 to 10 by volume) contained in a porous pot, which is surrounded itself by a saturated solution of copper sulphate contained in a copper vessel which serves as the copper plate.

When in use, the hydrogen generated by the action of zinc on sulphuric acid passes through the porous pot and displaces *copper* from the solution of copper sulphate; hence pure copper, and not hydrogen, is deposited on the walls of the copper vessel. The solution of copper sulphate is kept saturated by placing crystals of the sulphate on a perforated copper shelf near the top of the outer vessel. A solution of zinc sulphate may be used instead of sulphuric acid, and it has the advantage that amalgamation of the zinc is not necessary in order to prevent local action.

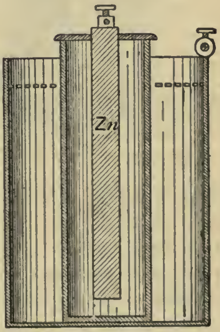

FIG. 162.—The Daniell Cell.

The E.M.F. of the Daniell cell varies from 1.07 to 1.14 volts (p. 243), depending upon the concentration of the solutions: it is diminished by increasing the strength of the zinc sulphate solution. Thus, with a saturated solution of copper sulphate, the E.M.F. is 1.146 volts if the zinc sulphate solution has $\frac{1}{12}$ gram of the crystals to 100 c.c. of water, and 1.081 volts if the solution is saturated.

If left standing for a long time some of the copper sulphate will pass through the porous pot and will be decomposed by the zinc, with deposition of copper on the zinc rod; this will cause *local action*.

The Grove Cell.—In the Grove cell (Fig. 163) the metals are zinc and platinum, and concentrated nitric acid is used as a depolariser. A thin sheet of platinum is surrounded by nitric acid contained in a flat porous pot. A zinc plate, bent into the form of the letter U, is immersed in dilute sulphuric acid (1 in 8) contained in an outer glass vessel. The hydrogen is oxidised by the nitric acid, which is reduced to nitric oxide: this gas is somewhat soluble in the acid, but if produced in excess it escapes into the air, and forms brown fumes of nitric peroxide. The E.M.F. of the cell varies from 1.9 to 2 volts.

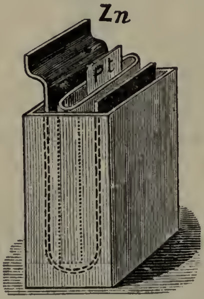

FIG. 163.—The Grove Cell.

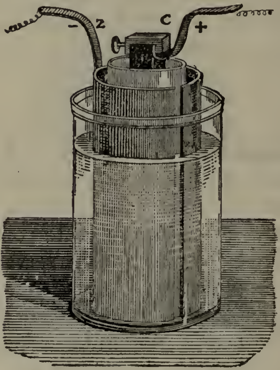

FIG. 164.—The Bunsen Cell.

The Bunsen Cell.—The Bunsen cell (Fig. 164) is similar to the Grove cell, except that the platinum is replaced by a rectangular block of hard gas-carbon.

Dry Cells.—Owing to the want of portability of ordinary cells containing fluids, it is often preferable to use *dry* cells. In reality these are by no means *dry*, and the success of such cells largely depends upon the contents being kept moist. All types are modifications of the Leclanché cell. A central carbon rod is surrounded by a stiff paste of manganese dioxide, plumbago, and gum ; this is surrounded by a zinc cylinder, the intervening space being filled with a thin paste of plaster of Paris and solution of sal-ammoniac.

STANDARD CELLS.

In order to determine the E.M.F. of any voltaic cell it is desirable that *standard* cells shall be available. A standard cell should always have the same E.M.F. even when constructed with chemicals obtained from different sources ; its E.M.F. should also be constant at a given temperature, and its *temperature-coefficient*, or rate of change of E.M.F. with change of temperature, should be known with great accuracy. Two standard cells are in frequent use, (i) the *Board of Trade Clark cell*, and (ii) the *Weston Cadmium cell.*

The **volt**, or practical unit of electromotive force, was defined by Order in Council (1894) as follows : "The volt, which has the value 10^8 in terms of the centimetre, the gramme, and the second of time, being the electrical pressure that if steadily applied to a conductor whose resistance is one ohm will produce a current of one ampere, and which is represented by 0.6974 (or $\frac{1000}{1434}$) of the electrical pressure at a temperature of 15° C. between the poles of the voltaic cell known as Clark's cell, set up in accordance with the specification appended hereto."

The Clark Cell.—This cell, devised by Latimer Clark, consists of electrodes of zinc and mercury immersed in a paste of mixed mercurous and zinc sulphates. The following details are abstracted from the above specification :

Preparation of the Materials.

The *Mercury* should be re-distilled.

The *Zinc* rod should be soldered at one end to a copper wire, cleaned with glass paper, dipped into dilute sulphuric acid, washed with distilled water, and dried.

Mercurous Sulphate.—To purify, shake up the pure salt with water and a little pure mercury in a bottle ; drain off the water and repeat twice.

Zinc Sulphate Solution.—Add to distilled water, in a flask, nearly twice its weight of pure crystals, and zinc oxide (2% of weight of crystals) to neutralise any acid. Warm to a temperature not above 30° C. Add mercurous sulphate, purified as above, to extent of 12% of the crystals taken, in order to neutralise any free zinc oxide remaining. Filter the solution, *while still warm*, into a stock bottle.

The Paste.—Mix the mercurous sulphate and the zinc sulphate solution, adding a few crystals of zinc sulphate *from the stock bottle*, to ensure saturation, and a small quantity of mercury. Shake these up to form a creamy paste. Warm the paste, but not above 30° C. Shake occasionally for an hour while cooling. This ensures a saturated solution of zinc and mercurous sulphates.

To set up the cell.

The cell (Fig. 165) may be set up in a small test-tube (5 cms. × 2 cms.). Pour in mercury to a depth of 0.5 cm., sufficient to cover the platinum wire which is sealed into the lower end of narrow-bore glass tubing supported by a cork C, which has been previously bored to carry the zinc rod and cut at the side so as to allow air to escape when inserted into the tube. Wash the cork well with hot water before use. Pass the zinc rod about 1 cm. through the cork, heat the exposed end of the platinum to red heat and insert it into the mercury. Shake up the paste, and pour into the tube to a depth of 1 cm. Insert the cork and zinc rod, and push the cork down until it nearly touches the liquid. Allow to stand for 24 hours, and finally cover the cork with melted marine glue.

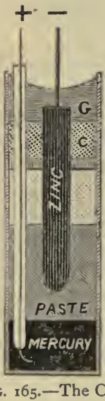

FIG. 165.—The Clark Standard Cell.

The electromotive-force of a Clark cell is 1.433 volts at 15° C., and the value at any other temperature t C. is given by the formula

$$E = 1.433\{1 - 0.00077(t - 15)\} \text{ volts.}$$

The Weston Cadmium Cell.—The high temperature-coefficient of the Clark cell is due to the considerable variation of the solubility of zinc sulphate with the temperature. Cadmium compounds have a far more constant solubility, and a cell in which cadmium sulphate is used instead of zinc sulphate will have a much smaller temperature-coefficient. The Weston cell resembles the Clark cell, except that the zinc rod is replaced by an amalgam of cadmium and mercury (12.7%

cadmium) and cadmium sulphate is used instead of zinc sulphate.

The cadmium sulphate is purified by digesting with cadmium hydrate $Cd(HO)_2$, prepared from $CdSO_4$ and $NaHO$. The solution is then filtered, shaken up with mercurous sulphate, again filtered, and evaporated to crystallisation. The other chemicals are purified in the same manner as described for the Clark cell; the paste of mercurous and cadmium sulphates is also made in the same manner.

Fig. 166 represents the most suitable form of Weston cell. Introduce the amalgam, which readily melts in a water bath, into the left limb, and fix the platinum terminal in position by the cork C, which is provided with a hole *a* for the subsequent introduction of crystals and solution. Pour mercury into the right limb, insert and fix the platinum terminal; then add the paste and crystals of $CdSO_4$. Introduce crystals of $CdSO_4$ through *a*, and finally fill up with saturated solution of cadmium sulphate, *which must be poured through the opening* *a*, nearly up to the corks. Insert glass wool in each limb, and close with another cork and marine glue.

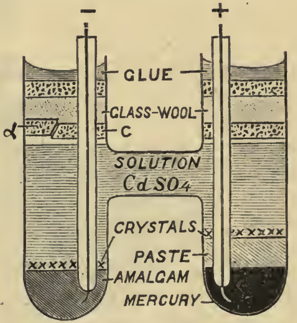

FIG. 166.—The Weston Standard Cell.

The electromotive-force of a Weston cell is 1.019 volts at 20° C., and the value at any temperature $t°$ C. is given by the formula

$$E = 1.019 - [3.8 \times 10^{-5}(t - 20)] \text{ volts.}$$

The Commutator, or Current Reverser.—It is desirable to have a simple appliance for reversing the direction of the current without interchanging the battery connections. Two forms of *commutator* are in frequent use:

(i) THE RUHMKORFF COMMUTATOR.—Fig. 167 represents the construction of a Ruhmkorff commutator. It consists of an ivory cylinder *v* mounted on brass spindles *a* and *b*, which are supported on metal pillars connected to terminals A and D. Two cylindrical brass quadrants, V and V′, are screwed to the ivory cylinder; one of them is connected to the spindle *b* by means of a screw, and the other is similarly connected to the spindle *a*. Two vertical springs, joined to terminals B and C, press against the ivory cylinder. The battery terminals are connected to A and D, and the external circuit to terminals B and C. In the position shown the circuit is open, and it is closed by rotating L to the right or left until the brass quadrants touch the vertical springs, the direction of the current depending upon the direction in which L is rotated.

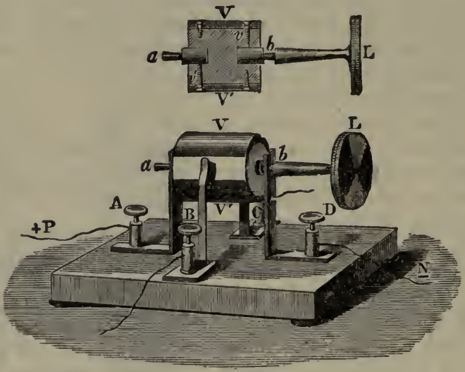

Fig. 167.—The Ruhmkorff-Commutator.

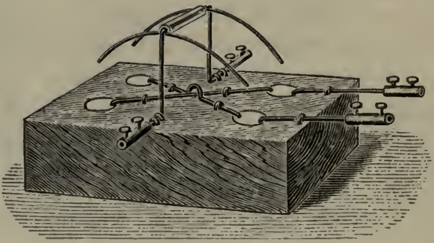

Fig. 168.—The Pohl Commutator.

(ii) The Pohl Commutator (Fig. 168).—This consists of a block of wood, or preferably of ebonite, with four mercury cups connected diagonally by means of thick copper wires. The battery terminals are connected to two wires which dip into the cups on one side of the block. The swinging arm consists of two pieces of bent wire which are insulated from each other by means of glass tubing: the arm carries two pieces of wire bent into an arc and which can be made to dip into either pair of cups by rotating the arm. The external circuit is joined by binding-screws to the ends of the arm.

SUMMARY.

The Electric Current.—When two conductors at different potentials are connected by a wire, an electric current traverses the wire so long as the P.D. is maintained. The direction from points of higher to points of lower potential is arbitrarily selected as the *direction* of the current. The passage of a current through a conductor is rendered evident by two phenomena: (i) *the conductor is heated*, and (ii) *a magnetic field is set up round the conductor.*

The Simple Voltaic Cell consists of plates of zinc and copper immersed in dilute sulphuric acid. The P.D. between the plates can be observed by means of a condensing electroscope. This cell is an unsatisfactory source of current owing to *local action* and *polarisation.*

Local action is observed when 'commercial' zinc is immersed in acid. Such zinc nearly always contains particles of impurity—usually carbon or iron—which, if on the surface of the zinc and touching the acid, constitute small voltaic cells. The chemical action associated with such cells involves the gradual solution of the zinc. Local action is prevented by amalgamating the surface of the zinc with mercury.

Polarisation.—The hydrogen which accumulates on the copper plate of a simple voltaic cell tends to diminish the strength of the available current, since it both reduces the effective area of the copper plate, and also *tends* to send a current through the cell in the reverse direction. Various chemical methods of preventing polarisation are available.

One-Fluid Cells.—The *Bichromate cell* consists of carbon and zinc plates immersed in dilute sulphuric acid to which potassium bichromate (or chromic acid) has been added. The bichromate acts as a 'depolariser' by oxidising the hydrogen.

The *Leclanché cell* consists of a carbon plate and a zinc rod immersed in a solution of sal-ammoniac. The carbon plate is surrounded with manganese dioxide, which serves as a 'depolariser.'

Two-Fluid Cells.—The *Daniell cell* may be represented by the symbol $Cu \mid CuSO_4, Aq \parallel H_2SO_4, Aq \mid Zn$. The two solutions are separated by a porous earthenware partition. The hydrogen displaces copper from the solution of copper sulphate, and pure copper is deposited on the copper plate.

The *Grove cell* is represented by the symbol

$$\mathrm{Pt} \mid \mathrm{HNO_3} \parallel \mathrm{H_2SO_4Aq} \mid \mathrm{Zn}.$$

The nitric acid oxidises the hydrogen, and so prevents polarisation. The two acids are separated by a porous partition.

The *Bunsen cell* is the same as the Grove cell, except that the platinum is replaced by a carbon plate.

Standard Cells.—The *Clark cell* consists of electrodes of zinc and mercury immersed in a paste of mixed mercurous and zinc sulphates. Its E.M.F. is 1.433 volts at 15° C.

The *Weston Cadmium cell* resembles the Clark cell except that the zinc rod is replaced by an amalgam of mercury and cadmium, and the zinc sulphate is replaced by cadmium sulphate. Its E.M.F. is 1.019 volts at 20° C.

QUESTIONS ON CHAPTER XIV

1. Explain the meaning of the term, and the cause of, polarisation of a voltaic cell. Give two instances of the use in cells of a depolariser; one being an instance of a cell with a single electrolyte, the other of a cell with two electrolytes.

2. Describe a condensing electroscope; explain clearly its special use and the method of using it.

3. A plate of zinc and a plate of copper are placed in dilute sulphuric acid. What action takes place, if any? How is the action modified if (1) the zinc plate is amalgamated, or (2) it is connected to the copper plate?

4. Describe the construction of the Daniell and Leclanché cells, and explain how the tendency to polarise is checked in these cases.

CHAPTER XV

MAGNETIC FIELDS DUE TO ELECTRIC CURRENTS

Oersted's Experiment.—Oersted, of Copenhagen, in the year 1820, carried out the first experiment which proved that there is a close relationship between magnetic and electrical phenomena. He observed that when a wire conveying a current is stretched horizontally in the magnetic meridian and immediately over a compass needle, the needle tends to set itself at right angles to the wire.

Expt. 67.—Connect the poles of a battery through a commutator to the ends of a long thin wire. Stretch the wire horizontally in the meridian over a compass needle. Observe how the needle is deflected. Reverse the direction of the current, and again observe the deflection. Repeat the observations with the wire *beneath* the compass needle.

Ampère's Rule.—From these observations the following rule may be deduced: **Suppose a man to be swimming in the wire in the same direction as the current, and with his face towards the needle; the north-seeking pole is deflected towards his left hand.**

The presence of a magnetic field near to a wire may also be detected by dipping a short piece of thin copper wire, traversed by a strong current, into iron filings; a bunch of filings will cling to the wire. Each fragment of iron becomes magnetised, and is held with its magnetic axis perpendicular to the wire.

The Field of Force due to a Linear Current.—Expt. 68.—Clamp a sheet of cardboard in a horizontal position, and bore a small circular hole through the centre. Fix a straight piece of thick copper wire vertically through the hole, and sprinkle iron filings on to the cardboard. Pass a strong current through the wire, and tap the cardboard. Notice that the filings have arranged themselves in circles concentric with the wire (Fig. 169). Note the direction of the current, and determine the *direction of the lines of force* by placing a compass needle to the north, south, east, and west of the wire. Observe the effect on the compass needle when the current is reversed.

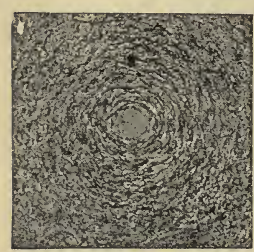

FIG. 169.

From this experiment we deduce the following rule: **The positive direction of the lines of force appears to be clockwise to an observer looking along a wire which is conveying a current away from him.**

Maxwell's Corkscrew Rule. — The same result is expressed in Maxwell's Corkscrew Rule as follows: **Imagine a corkscrew being screwed along the wire in the direction in which the current is passing. The direction in which the thumb rotates indicates the positive direction of the lines of force.**

The lines of force are circles described round the wire as centre, and with their planes perpendicular to the axis of the wire. Since the field of force must be symmetrical all round the wire, the magnitude of the magnetic force at all points of the same circumference must be the same.

Equipotential (magnetic) *surfaces are always perpendicular to the lines of force, and must therefore, in the case of a linear current, be planes, passing through the axis of the wire, radiating outwards and distributed at equiangular distances apart.*

When an iron wire is traversed by a current the surface is *transversely* magnetised, and if sawn longitudinally along its axis each section would show opposite polarity at its edges. If an iron wire is wrapped spirally round a copper wire conveying a current the iron is magnetised *longitudinally*, and would, when unwrapped, exhibit free poles at its ends.

The Magnetic Intensity varies inversely as the Distance.—For let V_1 and V_2 (Fig. 170) represent two equipotential surfaces in the field due to a current in the wire XY, also let a_1a_2 and b_1b_2 represent portions of two lines of force intercepted between V_1 and V_2. Since the work done in conveying unit pole along any line of force between V_1 and V_2 is equal to the product of the average force and the distance traversed,

$$(V_2 - V_1) = (f_a \times a_1a_2) = (f_b \times b_1b_2)$$

$$\text{or} \qquad f_a/f_b = b_1b_2/a_1a_2 = Ob_1/Oa_1.$$

Hence, **the intensity varies inversely as the distance.**

This result was proved experimentally by Biot and Savart by observing the rate of swing of a small magnet when situated at points to the magnetic east of a vertical current, and also when subject to the earth's field only. If the number of swings in a given time are n_1 and n_2 at distances d_1 and d_2, and n when due to the earth only, then experiment proves that

$$(n_1^2 - n^2)/(n_2^2 - n^2) = d_2/d_1.$$

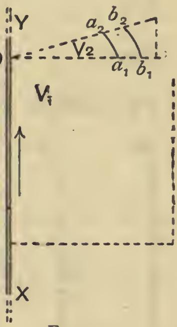

FIG. 170.

The same result may be deduced by assuming the truth of **Laplace's law**, viz: *The force exerted on a magnet pole by an infinitely short element of a conductor conveying a current is inversely proportional to the square of the distance* (d), *and directly proportional to the length* (ds) *of the element, to the current strength* (C), *to the pole strength* (m), *and to the sine of the angle of inclination of the element to the line joining the pole to the centre of the element.* Let AB (Fig. 171) represent an element of length ds: this may be resolved perpendicularly to, and along, the line of force F. Only the former component $ds \cdot \sin\alpha$ will exert magnetic force at m. Laplace's Law may be expressed thus:

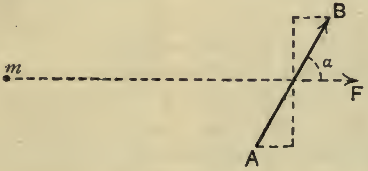

FIG. 171.

$$f = \frac{ds \cdot \sin\alpha \cdot C \cdot m}{d^2}.$$

Apply this to the case of two infinitely long wires AB and A′B′ (Fig. 172), carrying currents of equal strength, at distances d and d from m. The wires may be divided into pairs of elements similar to ds and ds', and

$$\frac{f}{f'}=\frac{ds \sin \alpha \, Cm}{d^2}\times\frac{d'^2}{ds' \sin \alpha \, . \, Cm}=\frac{ds}{ds'}\times\frac{d'^2}{d^2}.$$

But, $$\frac{ds}{ds'}=\frac{ma}{ma'}=\frac{d}{d'}.$$

Hence, $$\frac{f}{f'}=\frac{d}{d'}\times\frac{d'^2}{d^2}=\frac{d'}{d}.$$

The Magnetic Force varies directly as the Current Strength. For, if a second wire is introduced into the centre of the field so as to coincide with the first wire, and conveying a current of the same strength, the currents will produce identical fields of force, and the force at any point will be twice as great as that due to either of the currents. The two wires may be replaced by a single wire conveying a current of double strength without altering the other conditions. Hence *the force at any point varies directly as the current strength.*

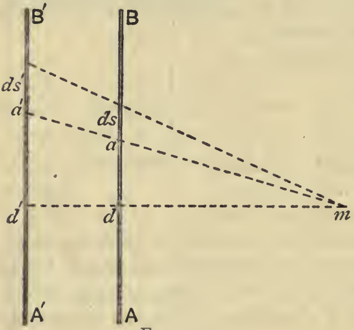

FIG. 172.

Magnetic Field due to a Current in a Circular Conductor.—By applying Maxwell's Rule it is evident that the lines of force within the space enclosed by a circular wire may be represented by Fig. 173, in which the current is supposed to be passing downwards at A and upwards at B. The magnetic field due to the current closely resembles that of a magnetised disc of steel, of which the contour is the same as that of the wire circle, and of which the thickness is equal to the diameter of the wire. If the degree of magnetisation is such that the magnetic fields are identical in every

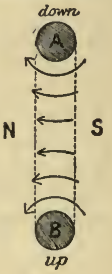

FIG. 173.

sense the magnetised disc may be termed *the equivalent magnetic shell of the circuit.*

EXPT. 69 (i) The magnetic properties of a circular conductor conveying a current may be observed by means of **De la Rive's Floating Battery,** which consists of a flat coil of thin insulated copper wire, the ends of which pass through a large cork, and are connected below to plates of copper and zinc. The coil should be vertical when the cork is floating on a liquid. If dilute sulphuric acid is used, a current traverses the coil, which will tend to set with its plane perpendicular to the meridian. If the direction of the current is traced out the following rule may be verified :

If the coil is held so that its face is perpendicular to the line of sight, and if the current appears to pass round the coil in a clockwise direction, then that face will have south-seeking polarity. If the direction is anti-clockwise, then the face will have north-seeking polarity.

(ii) Hold the pole of a bar-magnet near to the coil, and observe how the latter is either attracted or repelled, according to which face is directed towards the magnet.

The mutual action of the magnet and coil, may be more readily understood by maps of the combined fields of force.

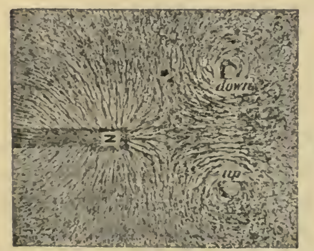

FIG. 174.

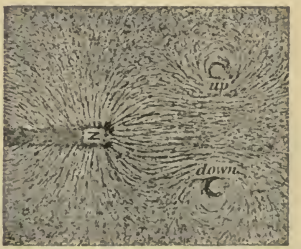

FIG. 175.

Fig. 174 represents the field of force when repulsion takes place, and Fig. 175 indicates mutual *attraction.* In the latter case it is evident that the tension of the lines of force will tend to move the coil towards the centre of the magnet.

Strength of Magnetic Field due to a circular current.—Let ds (Fig. 176) represent a small element of a conductor bent into the form of a circle of radius *a*. If A is a point on the axis of the circle, and distant *x* from the centre of the circle, the magnetic force at A due to the element *ds* will be perpendicular to the line joining A to *ds*, and may be represented by A*f*. This force may be resolved into two components, A*f*. $\cos\theta$ and A*f*. $\sin\theta$ (represented by A*v* and A*h* respectively), where θ is the angle subtended at A by the radius of the circle. An element diametrically opposite to *ds* will produce a force at A which may be resolved into two components, one equal and *opposite* to A*v*, the other equal to and in the *same direction* as A*h*. The entire circle may be divided into a number of pairs of such elements, the components similar to A*v* neutralising each other. The resultant force is equal to the sum of the components A*h*, and the direction of the force coincides with the axis of the circle. If a magnet pole of strength *m* is situated at A, the force in the direction of A*h* acting on the pole, and due to the current in *ds* is

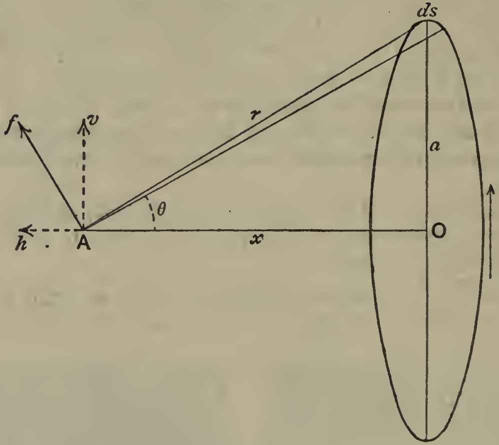

FIG. 176.

$$\frac{mC\,.\,ds}{r^2} \times \sin\theta.$$

The resultant force is obtained by substituting the entire length of the circle for the element *ds* ; hence

$$\text{Resultant Force} = \frac{mC \cdot 2\pi a}{r^2} \times \frac{a}{r} = \frac{mC \times 2\pi a^2}{(a^2+x^2)^{\frac{3}{2}}}. \quad \ldots\ldots\ldots(1)$$

If $m=1$, the resultant force measures the *Intensity of the Field*, hence Intensity of the Field $= \dfrac{C \times 2\pi a^2}{(a^2+x^2)^{\frac{3}{2}}}$.

At the centre of the circle, $x=0$, hence Intensity of the Field at the centre $= \dfrac{C \times 2\pi}{a}$.

This result may also be obtained in the following manner. If V = magnetic potential at A, C = the current strength (in C.G.S. units), ω = the solid angle subtended by the coil at A, then, from p. 77,

$$V = C\omega.$$

But $$\omega = 2\pi(1-\cos\theta), \qquad \text{(p. 563)}$$

Hence $$V = 2\pi C(1-\cos\theta) = 2\pi C\left(1 - \frac{x}{(x^2+a^2)^{\frac{1}{2}}}\right).$$

The intensity (F) of the field is equal to the rate of change of potential with respect to distance. Therefore

$$\begin{aligned} F = -\frac{dV}{dx} &= 2\pi C \cdot \frac{d}{dx}\left[x(x^2+a^2)^{-\frac{1}{2}}\right] \\ &= 2\pi C\left[(x^2+a^2)^{-\frac{1}{2}} - \frac{2x^2}{2}(x^2+a^2)^{-\frac{3}{2}}\right] \\ &= 2\pi C \cdot \frac{a^2}{(x^2+a^2)^{\frac{3}{2}}}. \end{aligned}$$

The C.G.S. Electro-magnetic Unit of Current.—In the above formula, if $a=1$ cm., and if the wire only occupies $\frac{1}{2\pi}$ of the circumference of the circle (*i.e.* a length equal to the radius), then

$$\text{Intensity of the Field at } O = C.$$

The **Unit of Current** may therefore be defined thus : **A current has unit strength when 1 cm. length of the circuit, bent into the form of an arc, of radius = 1 cm., exerts a force of 1 dyne on a unit magnet pole placed at the centre of the arc.**

The 'practical' unit of current, termed the ampere (pp. 293 and 523), is equal to $\frac{1}{10}$ part of the C.G.S. unit.

Equivalent Magnetic Shell. Ampère's Theorem.—Consider any closed circuit S (Fig. 177) traversed by a current of strength C. We can imagine the circuit to be replaced by an infinitely large number of small elements similar to *a*, each of which is traversed by a current of the same strength C; for each line separating two contiguous elements is traversed by equal currents in opposite directions, which neutralise each other, and it is only where the elements touch the contour of S that the currents remain un-neutralised. Further, we can imagine that each of these elements is replaced by a small *magnetic shell*, the contour of which coincides with that of the elementary circuit, and the magnetic moment of which is such that its effect at a distant point is the same as that due to the current element which it replaces.

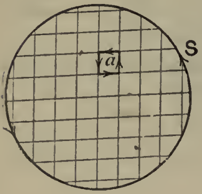

FIG. 177.

Let S (Fig. 178) represent an infinitely small circular circuit conveying a current of strength C; O is a point on the axis of the circuit, and distant *d* cms. from its centre. Since the circuit is very small, the distance of all parts of the circuit from O is equal approximately to *d*; hence, by equation (1), p. 255,

FIG. 178.

Intensity of field at $O = 2\pi a^2 C/d^3$.

Replace the circuit by a magnetised lamina, of which the magnetic moment is M, then, from p. 78,

Intensity at O, due to the magnet, $= 2M/d^3$.

Hence the lamina is *equivalent* to the circuit when $(\pi a^2 C) = M$.

If l = thickness of the shell, m = pole-strength, and σ = pole-strength of unit area, then

$$\pi a^2 C = m \times l,$$

$$\text{or} \qquad C = ml/\pi a^2 = \sigma l.$$

The same holds good for all other elements into which the circuit is resolved. But (p. 76) the product of the pole-strength per unit area and the thickness of the shell is termed the *Strength* (φ) *of the Shell.* Hence **the magnetic force due to a current in a closed circuit is identical with that of a magnetic shell**

of the same contour when the current (*in C.G.S. units*) **is equal numerically to the strength of the shell.** This is known as **Ampère's Theorem.**

Magnetic Moment of a Circular Current.—The magnetic moment M of a circular current may be regarded as identical with that of its equivalent magnetic shell. If a = radius of the coil, and C = current strength (in C.G.S. units), σ = magnetic density on the surface of the shell, and t = thickness of the shell, then the magnetic moment $=(\pi a^2\sigma \times t)=\pi a^2\phi=\pi a^2 C$. Hence

Magnetic Moment = Area × Current (*in C.G.S. units*).

If the plane of the coil is parallel to the lines of force of a uniform magnetic field of intensity H, the magnetic axis of the shell is perpendicular to the lines of force, and, from p. 75, the turning couple $=\pi a^2 C \cdot H$. If the plane of the coil makes an angle θ with the lines of force, the magnetic axis of the coil makes an angle $(90-\theta)$ with the lines of force, and the turning couple

$$=\pi a^2 CH \sin(90-\theta)$$
$$=\pi a^2 CH \cos\theta.$$

Intensity of Field due to a Linear Current.—Let A and B (Fig. 179) represent cross-sections through two infinitely long parallel wires forming two sides of a closed rectangular circuit, which is traversed by a current of C C.G.S. units. The solid angle at O is a *lune ab* on the unit sphere described round O, and its area = area of sphere × $\alpha/2\pi$ $=4\pi\times\alpha/2\pi=2\alpha$. If B is removed to infinity, ∠AOB becomes ∠AOC, and the solid angle becomes 2β.

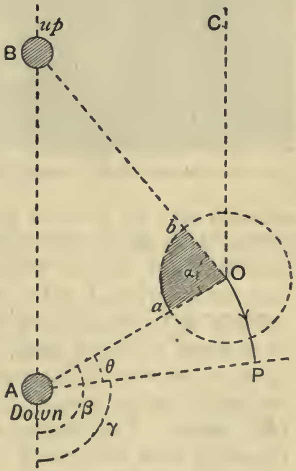

FIG. 179.

Since the potential at any point is equal to the product of the solid angle subtended at the point by the circuit and the current strength (pp. 77 and 256), the potential at O $=2\beta$C.

Since the lines of force are circles described round A, then OP will represent the line of force through O; also AO and AP represent two equi-potential surfaces. If V_1 and V_2 represent the potentials at O and P, and F = the intensity of the field at O, then

$$F \times OP = V_1 - V_2$$

or

$$F = \frac{V_1 - V_2}{OP} = \frac{2\beta C - 2\gamma C}{OP} = \frac{2\theta C}{OP}.$$

But

$$\theta = OP/AO,$$

hence

$$F = \frac{2C}{OP} \times \frac{OP}{AO} = \frac{2C}{AO}.$$

Magnetic Field due to a Solenoid.—If a current traverses a wire which is coiled closely round a cylinder, the resultant magnetic field may be compared to that due to a row of magnetic shells with faces of opposite polarity in contact, each turn of wire being replaced by its equivalent magnetic shell; in other words, *a solenoid of wire conveying a current should have the magnetic properties of a bar magnet.* Strictly speaking, each turn of the solenoid may be resolved into one turn exactly perpendicular to the axis of the solenoid, and a short length coinciding with the axis; but the magnetic effect of the axial component may be neutralised either by carrying the wire back along the axis or by winding the wire back to the starting point, so as to form a second layer, and so that the current traverses all turns of wire in the same direction; in the latter case the axial components of the two layers will now neutralise each other. Fig. 180 represents an iron-filing map of the magnetic field inside and outside a spiral of wire conveying a current.

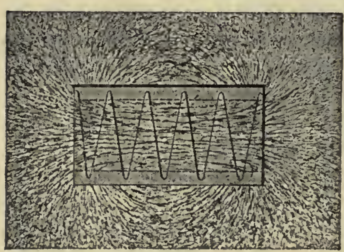

FIG. 180.—Magnetic Field due to a Solenoid.

Intensity of Field inside a Solenoid.—If each turn of the solenoid is replaced by its equivalent magnetic shell, then the *strength* of the shell is equal to the current-strength (p. 256), or

$$\phi = C.$$

If there are n turns of wire per cm. length of the solenoid, the thickness of each shell is $1/n$, and $\phi=\sigma/n$, where σ is the pole-strength per (cm.)^2. Hence

$$\phi=\sigma\times\frac{1}{n}=C,$$

or

$$\sigma=nC.$$

Therefore the pole-strength (m) at either end of the solenoid $=\pm\pi a^2\sigma=\pm\pi a^2 nC$. Since the number of lines of force originating from a magnet pole of strength m is equal to $4\pi m$ (p. 51), the number of lines emerging from the ends of the solenoid $=4\pi\times\pi a^2 nC=4\pi^2 a^2 nC$.

The intensity of the field within the spiral is equal numerically to the number of lines of force per cm.^{2}, hence

Intensity of field $=4\pi^2 a^2 nC/\pi a^2=4\pi nC$ C.G.S. units.

The intensity of the field near to the end of a long solenoid. It has been shown on p. 255 that the force acting on a magnet pole of strength m situated on the axis of a circular wire traversed by a current of C C.G.S. units is equal to

$$\frac{mC\,.\,2\pi a}{r^2}\times\sin\theta,$$

where θ is the angle subtended by the radius a of the circular circuit and r is the distance of m from any point of the circuit. Hence, the intensity of the field at the same point is

$$\frac{C\,.\,2\pi a\,.\,\sin\theta}{r^2}.\quad\dots\dots\dots\dots(1)$$

Consider the intensity of the field at a point O (Fig. 181) on the axis of a short solenoid AB traversed by a current of C C.G.S. units. The

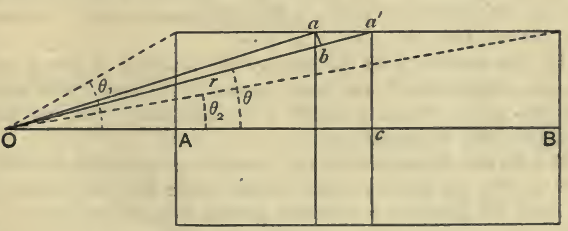

FIG. 181.

problem is solved by determining the intensity due to a *thin* section, such as aa', and integrating the expression so as to include the whole solenoid. If C' is the total current traversing unit length of the solenoid, and if there are n turns of wire per unit length, then $C' = nC$; and the total current traversing the section aa' is $C' \times aa' = nC \times aa'$. By formula (1) the intensity at O due to the current in the section aa' is

$$(nC \times aa') \,.\, 2\pi a \,.\, \sin\theta / r^2. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If the small angle aOa' is represented by $d\theta$, and if ab is drawn perpendicular to Oa', then $ab = Oa \,.\, d\theta$; and since the triangles $aa'b$ and $a'Oc$ are similar, $ab/aa' = a'c/a'O$, or

$$aa' = (ab \times a'O)/a'c = Oa \,.\, d\theta \,.\, r/a = r^2 \,.\, d\theta/a.$$

Substituting this value of aa' in formula (2), the intensity at O is

$$2\pi nC \,.\, \sin\theta \,.\, d\theta.$$

The intensity due to the whole solenoid is obtained by integrating this between the limits $\theta = \theta_2$ and $\theta = \theta_1$; or, the total intensity is

$$2\pi nC \int_{\theta_2}^{\theta_1} \sin\theta \,.\, d\theta$$

$$= 2\pi nC \Big[-\cos\theta \Big]_{\theta_2}^{\theta_1}$$

$$= 2\pi nC (\cos\theta_2 - \cos\theta_1).$$

If the point O is *within* the solenoid, $\cos\theta_1$ is *negative*, and the intensity is equal to $2\pi nC(\cos\theta_2 + \cos\theta_1)$; and if the solenoid is long and if O is at a considerable distance from either end then $\cos\theta_2 = \cos\theta_1 = 1$, and the intensity is equal to $4\pi nC$. At the extreme end of a long solenoid, $\cos\theta_1 = 0$, and the intensity is equal to $2\pi nC$. Hence the intensity at the end of a long solenoid is equal only to one-half the intensity at points inside the solenoid and at a considerable distance from the ends.

Fig. 182 represents the variation of the intensity at points on the axis of a long solenoid and near to one of the ends; it is assumed that the radius of the solenoid is 10 cms., and that the current strength is such that $2\pi nC = 1$.

Ring-shaped Solenoid.—In the case of a ring-shaped solenoid the turns of wire are closer together on the inside than on the outside of the ring; the intensity is therefore greater towards the inside. If r_1 and r_2 are the inner and outer radii of the ring, and if N is the total number of turns, then the number of turns per unit length are $N/2\pi r_1$ and $N/2\pi r_2$ respectively, and the intensity varies from $4\pi CN/2\pi r_1$ to $4\pi CN/2\pi r_2$.

Definitions.—If n is the number of turns per unit length, and if C is the current strength in *amperes*, the product nC is termed the **ampere-turns** per unit length.

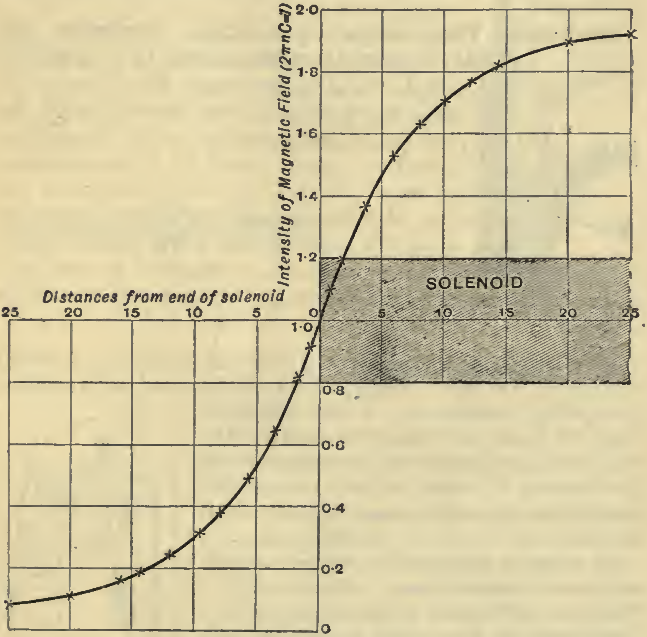

FIG. 182.

The **Magneto-motive force** (p. 400) of a current traversing a solenoid may be defined as *the line integral of the magnetising forces*, and it is numerically equal to the product of the intensity, and the length of the field, or

$$\text{Magneto-motive force} = (4\pi n\text{C} \times l) = 4\pi\text{NC}.$$

ELECTRO-DYNAMICS.

The following paragraphs are devoted to a consideration of the mechanical forces which act upon linear conductors conveying currents, when placed either in a uniform magnetic field or in a field due to a neighbouring current. The term **Electrodynamics** has been applied to this branch of the subject by Ampère.

Mechanical Force acting on a Linear Conductor conveying a current when placed in a magnetic field.—If A (Fig. 183) represents the cross-section of a wire conveying a current *down* through the paper, a single north-seeking pole, situated at *n*, will tend to rotate round A in a clock-wise direction to *n′*. But if the pole is fixed and the conductor is free to move, then the latter will move in such a manner that it will occupy subsequently the same position relatively to the pole as in the former case, and will rotate round *n* in a clock-wise direction.

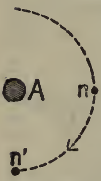

FIG. 183.

EXPT. 70 (i), Fig. 184 represents a glass tube (G) closed at both ends with corks. The north-seeking pole of a cylindrical bar magnet projects through the lower cork. A wire suspended from the upper cork dips into mercury (H). Pass a strong current down the wire, and observe the direction in which the wire rotates; also observe the direction when the current is reversed.

(ii) Clamp a cylindrical bar magnet vertically and some distance above the table. Hold a long piece of tinsel by its upper end so that it hangs vertically, close to the magnet, and with its lower end as free as possible. Notice how the tinsel, when traversed by a strong current, tends to coil itself round the magnet.

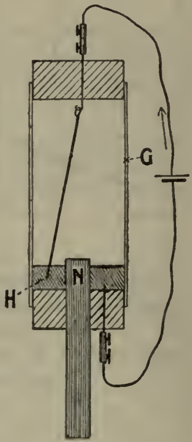

FIG. 184.

Rule.—The following rule, due to Prof. Fleming, will be found useful in determining the direction of motion of a linear current when placed in a magnetic field: **Hold the thumb and first finger of the left hand as fully extended as possible, and bend the second finger at right angles to the palm. If the first finger represents the direction of the lines of force, and the second**

finger that of the current, then the thumb will indicate the direction of motion (Fig. 185).[1]

By Laplace's Law (p. 251) the force acting on a short element ds of a wire conveying a current C. C.G.S. units is $ds\,.\,C\,.\,m/r^2$ when the element is perpendicular to the lines of force due to a magnet-pole of strength m distant r cms. from the element. But m/r^2 is the *intensity* of the field due to m. *If the wire is l cms. long, and placed within a uniform field of intensity* H, *the force tending to move the wire in a direction perpendicular to the lines of force is* lCH. The direction of the force may be deduced by applying Fleming's Rule.

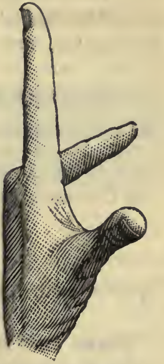

FIG. 185.

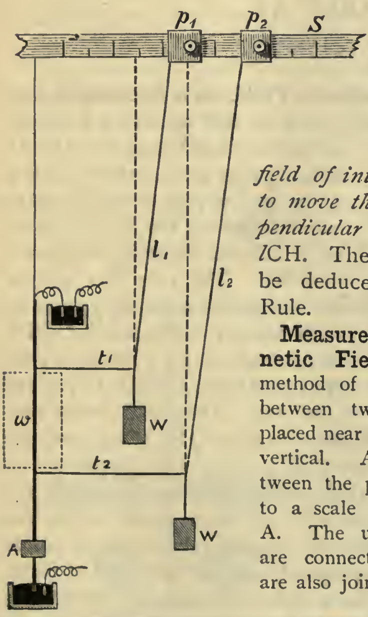

FIG. 186.

Measurement of an Intense Magnetic Field.—Fig. 186 represents a method of measuring the magnetic field between two long straight pole faces placed near together and with their lengths vertical. A wire w is suspended between the pole-faces by a cord attached to a scale S and stretched by a weight A. The upper and lower ends of w are connected to mercury cups, which are also joined to a battery and ammeter.

---

[1] Compare this rule with the ***right-hand rule*** (p. 413) for determining the direction of ***induced*** currents. In each case, the thumb represents 'direction of motion,' the first finger 'direction of lines of force,' and the second finger 'direction of current.' It is useful to associate the *m* in thu*m*b with *m* in *m*otion, and the *f* in *f*irst finger with the *f* in magnetic *f*ield.

Horizontal threads, t_1 and t_2, are connected to vertical threads, each stretched by a weight W grams, and attached to sliders p_1 and p_2. The direction of the current is arranged so that w is deflected to the left, and the sliders are moved to the right until w is brought back to its original position. If d_1 and d_2 are the distances through which the sliders have to be moved, the total force exerted by the threads on w will be $Wg\left(\frac{d_1}{l_1}+\frac{d_2}{l_2}\right)$ approximately. The magnetic force acting on w is HLC, where L = vertical length of the pole-face, and C = current strength (in C.G.S. units). Hence

$$H=\frac{Wg}{LC}\left(\frac{d_1}{l_1}+\frac{d_2}{l_2}\right).$$

Another method is described on p. 426.

Force exerted by a Magnetic Field on a Rectangle conveying a Current.—Let ABCD (Fig. 187) represent a rectangular circuit of length b and width a, conveying a current of C. C.G.S. units, and placed with its plane coinciding with the lines of force of a uniform horizontal magnetic field H. The side CD will be acted upon by a force f_2, equal to bCH dynes, the direction of which is determined by Fleming's rule. An equal force f_1 will act upon the side AB. *These two forces constitute a couple of which the moment is ba*CH. (Compare this with the Moment of a Circular Coil, p. 257). No forces act on the sides AD and BC in their initial position, but when the rectangle is deflected forces will act upon them which are directed upwards and downwards respectively. The rectangle will rotate until its plane is perpendicular to the lines of force. In this position the lines of force due to the current traverse the rectangle in the same direction as those due to the field ; or, *the number of lines of force traversing the rectangle is then a maximum.* Moreover, *all the forces acting on the conductor tend to enlarge its area.* The following rule, due to Clerk Maxwell, may be applied to all such cases : **Every circuit is acted upon by**

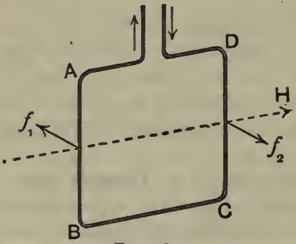

FIG. 187.

forces which tend to alter its position or its configuration so as to make it embrace a maximum number of lines of force.

An experiment, due to Ampère, clearly exhibits the truth of Maxwell's rule. A shallow wooden trough (Fig. 188) is divided lengthwise by a wooden partition, and filled with clean mercury. The two compartments are connected by a bent iron wire *ab* floating on the mercury. When the terminals are connected to the poles of a battery, the floating bridge moves away from the terminals, thus increasing the area of the circuit, and consequently the total flux through the area. In this case the flux originates from the current in the circuit itself.

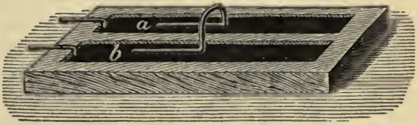

FIG. 188.

In further reference to Fig. 187, the work done by the forces in rotating the rectangle from its initial to its final position is equal to $2\left(b\text{CH}\times\frac{a}{2}\right)=ab\text{CH}$ ergs. But $ab\text{H}$ is the total number of lines of force cut by the rectangle. Hence

$$\textbf{work done}=\text{NC ergs.}$$

Or, **the work done when a current moves across a magnetic field is equal to the product of the current strength and the total number of lines of force cut.**

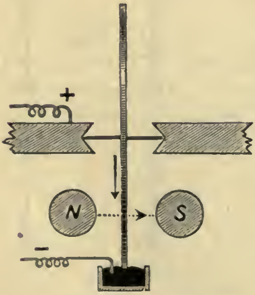

FIG. 189.—Barlow's Wheel.

Barlow's Wheel.—Barlow's wheel (Fig. 189) is an instructive application of the principle explained in the previous paragraph. It consists of a star-shaped metal wheel supported on horizontal metal bearings, and the spokes of the wheel, in their lowest position, dip into a mercury trough. If a current passes from the axle to the mercury trough, and if the poles of an electro-magnet are placed on either side of the lower half of the wheel, the wheel rotates in a direction which may be deduced by Fleming's rule. This apparatus is a simple application of the conversion of electrical into mechanical energy.

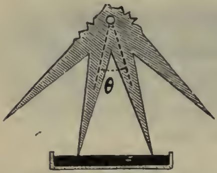

FIG. 190.

If the mercury trough is so arranged that one spoke is just leaving the mercury when the next spoke is about to make contact, the area traced out by a spoke during contact with the mercury is approximately $\frac{1}{2}a\theta . a$, where θ is the circular measure of the angle between consecutive spokes (Fig. 190), and a is the length of a spoke. If the field has a uniform intensity H, and if C = current strength, the number of lines of force cut is equal to $H . a^2\theta/2$, and the work done is $HC . a^2\theta/2$. The work done in one complete rotation is $HCa^2 . 2\pi/2$ or πa^2HC.

MUTUAL ACTION OF PARALLEL AND OBLIQUE CURRENTS.

(i) **Two Parallel Currents.**—Let AB and CD (Fig. 191) represent two parallel wires conveying currents of strengths C and C′ C.G.S. units. Consider AB to be fixed, and CD free to move. The direction of the magnetic field at P due to the current in AB may be represented by PH, which is perpendicular both to CD and to OP. By Fleming's rule, CD will tend to move along PO towards AB. If the current in CD is reversed, CD will be repelled from AB. Hence, **two parallel conductors attract each other if currents traverse them in the same direction, and repel each other if the currents are in opposite directions.**

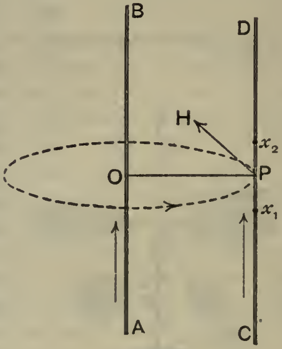

FIG. 191.

If the perpendicular distance between the wires is r cms., the intensity of the field at P is $2C/r$ (p. 257). The force acting on an element x_1x_2 of length l is $(lC' \times 2C/r)$ dynes.

EXPT. 71.—Bend a copper wire into a rectangular form ABCD (Fig. 192), and solder the ends to two lengths of tinsel, the upper ends of which are soldered to copper wires passing through the cork X. Clamp the cork at a convenient height, and include in the circuit a cell and a length of free wire EF. Hold EF near to and parallel to either of the sides of the rectangle. Observe the attraction or repulsion.

EXPT. 72.—Wind a close spiral (10 cms. long, 1 cm. diam.) of thin uncovered copper wire. Support it vertically by the upper end, and so that the lower end *just* dips into a vessel of mercury. Connect the upper end and the mercury to the terminals of a battery, and observe how the spiral vibrates up and down. Since parallel currents in the same direction are traversing consecutive turns of the spiral, attraction takes place, and the spiral is shortened sufficiently to break the circuit at the mercury cup, when the spiral again lengthens and completes the circuit. This apparatus is known as *Roget's Spiral*.

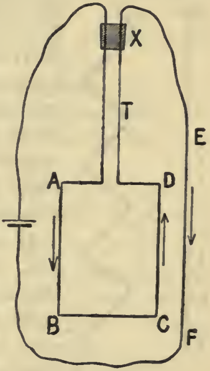

FIG. 192.

(ii) **Oblique Currents.**—If the circuit CD (Fig. 193), which is free to move, is perpendicular to the fixed circuit AB, and if the currents are in the directions shown, then, by Fleming's rule, CD will be urged in the direction *f* parallel to AB. If either of the currents is reversed CD will be urged in the opposite direction.

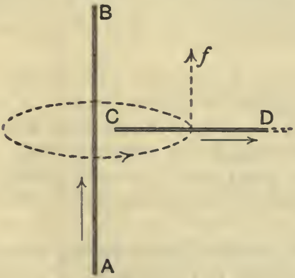

FIG. 193.

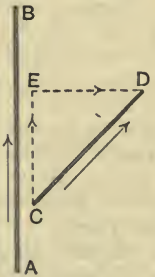

FIG. 194.

If CD is inclined at an angle to AB (Fig. 194), we may consider the circuit CD to be resolved into two components, CE and ED. The component CE will be attracted towards AB, and ED will be urged upwards and parallel to itself. These combined effects will tend to rotate CD into a position parallel to AB. If either of the currents is reversed the forces will tend to rotate CD away from AB. Hence, *two circuits crossing obliquely attract each other when currents traversing them proceed from or to the apparent point of intersection, but repel each other if one current proceeds from and the other towards that point.*

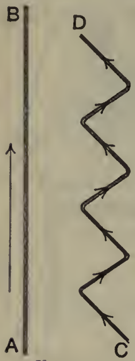

Fig. 195.

If one of the conductors is *sinuous*, as in Fig. 195, the mutual forces are the same as when the sinuous conductor is replaced by a straight conductor of the same length; for each element may be resolved into two components, of which that parallel to AB is effective, while the consecutive components perpendicular to AB will have opposite effects, and will neutralise each other.

Summary.

The magnetic field round a wire conveying a current.—Ampère's Rule:—*Suppose a man to be swimming in the wire with the current, and with his face towards a magnetised needle; the north-seeking pole is deflected toward his left hand.*

The lines of force are circular and concentric with the wire. The *positive* direction of the lines appears to be clock-wise to an observer looking along the wire when it is conveying a current away from him.

Maxwell's Cork-screw Rule:—*Imagine a corkscrew being screwed along the wire in the direction in which the current is passing. The direction in which the thumb rotates indicates the positive direction of the lines of force.*

The magnetic equi-potential surfaces are planes passing through the axis of the wire, radiating outwards, and distributed at equi-angular distances apart.

The magnetic force varies directly as the current strength and inversely as the distance, and, in the case of an infinitely long wire, is numerically equal to $2C/r$, where C is the current strength in C.G.S. units and r is the distance.

Laplace's Law :—*The force exerted on a magnet pole by an infinitely short element of a conductor conveying a current is inversely proportional to the square of the distance, and directly proportional to the length of the element, to the current strength, to the pole strength, and to the sine of the angle of inclination of the element to the line joining the pole to the centre of the element.*

Magnetic Field due to a current in a circular conductor.—*If a coil is held so that its face is perpendicular to the line of sight, and if the current appears to pass round the coil in a clockwise direction, then that face will have* **south-seeking** *polarity.*

The **intensity** of the field due to a circular current of C C.G.S. units at a point on the axis of the circuit and distant x from its centre is $C \times 2\pi a^2/(a^2+x^2)^{\frac{3}{2}}$. The intensity at the centre of the circle is $2\pi C/a$.

The C.G.S. Electro-magnetic Unit of Current.—*A current has unit strength when 1 cm. length of the circuit, bent into the form of an arc of* 1 *cm. radius, exerts a force of 1 dyne on a unit magnet pole placed at the centre of the arc.*

Equivalent Magnetic Shell.—A thin steel lamina, magnetised transversely, creates a magnetic field identical with that due to a current traversing a single closed circuit when its magnetic moment M is equal to the product of the area of the circuit and the current C (C.G.S. units), or when $M = \pi a^2 C$.

The **strength** of a magnetic shell is defined as the product of the pole-strength per unit area and the thickness of the shell.

Magnetic intensity due a solenoid.—The intensity at a point inside the solenoid, and at some distance from either end, is equal to $4\pi nC$, where n is the number of turns of wire per unit length. The intensity at either end is $2\pi nC$.

If C is the current strength in amperes, the product nC is termed the *ampere-turns* per unit length.

The *magneto-motive force* is the product of the intensity and the length of the field.

Mechanical force acting on linear conductors.—Fleming's Rule :—*Hold the thumb and first finger of the* **left hand** *as fully extended as possible, and bend the second finger at right angles to the palm. If the first finger represents the direction of the lines of force, and the second finger that of the current, then the thumb will indicate the direction of motion.*

Every circuit is acted upon by forces, which tend to alter its position, or its configuration, so as to make it embrace a maximum number of lines of force.

The work done when a current moves across a magnetic field is equal to the product of the current strength and the total number of lines of force cut.

Two parallel conductors **attract** *each other if currents traverse them in the* **same** *direction, and* **repel** *each other if the currents are in* **opposite** *direction.*

Two circuits crossing obliquely attract each other when currents traversing them proceed from or to the apparent point of intersection, but repel each other if one current proceeds from, and the other towards, that point.

QUESTIONS ON CHAPTER XV.

1. An electric current is flowing along a wire. You are given a pivotted compass needle, and are required to find out by its aid which way the current is flowing. How would you proceed (*a*) if the wire in question lies horizontally; (*b*) if the wire runs vertically; (*c*) if the wire is coiled up in a circular coil or open hank?

2. A road in the northern hemisphere runs magnetic north and south. At one point an insulated conductor passes beneath it in which an electric current flows from east to west. How will the indications of a dip circle be affected at points near to the conductor?

3. A wire is stretched from east to west (magnetic). How, without breaking it, can you test whether, and in what direction, an electric current is passing through it?

4. Define unit magnetic pole and unit electrical current in the electro-magnetic system, and state the relation of the ampere to the latter.

5. What would be the magnetic effect produced on a straight steel tube by the passage of a strong current through a straight wire placed along the axis of the tube? and how would you prove your statement?

6. What are the general laws of attraction and repulsion of currents, and how would you experimentally investigate them?

7. Experiments are to be arranged to find out how the conductor carrying an electric current tends to move in a magnetic field. What experiments would you arrange?

8. State the rules by which the force acting on a conductor carrying a current in a magnetic field can be determined.

A vertical circular ring, radius a, carrying a current i, is in equilibrium in the earth's magnetic field when perpendicular to the meridian. Find the work required to twist the ring round a vertical axis until its plane coincides with the meridian.

9. How would you show by experiment that the magnetic field due to a plane current circuit, at any distance great compared with the

dimensions of the circuit, depends not on the form but only on the area and the current, and that it is equal to that of a certain magnet set with axis perpendicular to the plane of the circuit? Show, by considering the case of a plane circular current, that the moment of the equivalent magnet is (area × current).

10. Describe a form of apparatus by means of which the fundamental laws of the attractions and repulsions observed between conductors, in which currents are flowing, may be demonstrated.

11. Find an expression for the magnetic force at any point on the axis of a circular coil carrying a steady current.

12. A current is sent through two equal and parallel rings of wire placed somewhat close together; draw a diagram showing the distribution of magnetic force due to the current, (i.) when it goes round the two rings in the same direction, (ii.) when it goes in opposite directions.

13. Find an expression for the strength of the magnetic field near the middle of a long uniform solenoid. Indicate by a diagram the manner in which the field varies near the ends, and find approximately the value of the field just inside the ends.

14. What is the strength of the magnetic field 5 cms. from a long straight wire conveying a current of 15 amperes?

15. Three parallel wires are each 10 cms. distant from the other two, and each is traversed by a current of 10 amperes in the same direction. Find the magnitude of the force acting on each centimetre length of one of the wires, and show its direction by means of a diagram.

16. What is the intensity of the magnetic field at the centre of the coil of a tangent galvanometer of 20 turns of wire and 25 cms. mean radius when traversed by a current of 0.1 ampere?

17. Find the intensity of the field near the centre of a solenoid, 80 cms. long, wound with four layers of wire of 400 turns each, when traversed by a current of 4 amperes.

18. How may the intensity of the magnetic force inside a solenoid be approximately calculated? What is it in one of 300 turns, 15 cms. long, which carries a current of 0.2 ampere? What effect has the diameter of the solenoid?

CHAPTER XVI

GALVANOMETERS AND ELECTRO-DYNAMOMETERS

THE mutual electro-magnetic forces between a wire conveying a current and a neighbouring magnet may be used in order to *detect* the presence of an electric current; an instrument devised for this purpose is termed a **Galvanoscope.** If the instrument is so designed that by its means a current may be *measured*, it is then termed a **Galvanometer.**

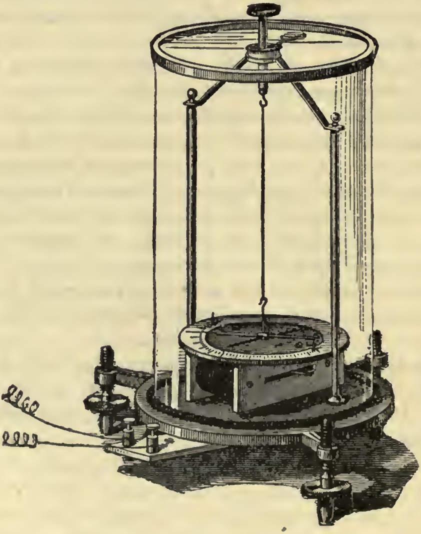

FIG. 196.—An Astatic Galvanometer.

Nobili's Astatic Galvanometer.—This instrument (Fig. 196) is really a special type of galvanoscope of which the *sensibility* (*i.e.* the amount of deflection obtained with a given current) is increased by using an astatic pair of needles suspended by means of a single silk fibre so that the lower needle moves within the space enclosed by a rectangular coil (Fig. 197). It is evident that the deflection of both needles due to a current will be in the same direction. In practice the needles are never of exactly the same size and magnetic moment; and, as explained on p. 86, the controlling force is $(m-m')$H, where m and m' are the pole-strengths and H is the horizontal intensity of the earth's field. Nevertheless this force is usually so small that the needles take up their position of rest in obedience to the torsion of the suspending fibre, which is therefore, in this instrument, the chief controlling force; the magnitude of this controlling force depends upon the force with which the twisted fibre tends to untwist.

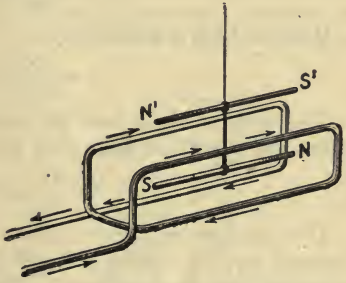

FIG. 197.—Principle of the Astatic Galvanometer.

The Tangent Galvanometer.—In order that a galvanometer may obey the tangent-law (p. 18) it is necessary that the field created by the current shall be uniform within the region in which the needle is capable of moving, and that the controlling force shall be due to a uniform magnetic field. In a properly constructed tangent galvanometer the torsion of the suspending fibre should be negligible, and this is obtained if the fibre is sufficiently long and narrow. The field due to the current is uniform if the coil is circular and of considerable diameter; but even in this case it is necessary to use a very short needle, so that when deflected it does not move into regions where the field is not uniform.

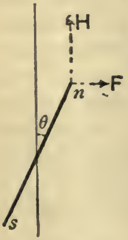

FIG. 198.

If ns (Fig. 198) represents the needle deflected through θ°, and if F and H are the intensities of the fields due to the current and the earth's horizontal component respectively, then, by taking moments round the centre of the needle, $F = H \tan \theta$. But, by p. 255, $F = 2\pi C/a$, where C is the current in C.G.S. units and a is the radius of the coil. Hence,

$$C = \frac{aH}{2\pi} \tan \theta.$$

If the coil has n turns, then

$$C = \frac{aH}{2\pi n} \cdot \tan \theta.$$

The quantity $2\pi n/a$ is called the **true constant**[1] of the galvanometer and is denoted usually by the symbol G. Hence,

$$C = \frac{H}{G} \tan \theta.$$

The quantity $aH/2\pi n$ is called the **reduction factor** and is denoted by the symbol k. (If the current is expressed in amperes then $k = 10aH/2\pi n$.) The equation for the tangent galvanometer may therefore be written

$$C = k \tan \theta.$$

The value of the constant G is determined by measuring the dimensions while the coil is being wound. If the coil consists of several turns of wire, and if considerable accuracy is required, allowance must be made for the fact that all the turns cannot be at the same distance from the centre.

The construction of a tangent galvanometer is identical with that of a *sine*-galvanometer (Fig. 200) except that the horizontal scale on the base of the instrument is omitted. A useful type of instrument is constructed by winding two or more separate coils on the circular wooden frame. The magnetised needle is suspended, by means of a single silk or quartz fibre, at the centre of the coils and above the centre of a circular scale. A long pointer is attached to the centre of the needle and at right angles to its axis. The needle is protected from air currents by placing a glass shade over the instrument.

It may be proved that a tangent galvanometer is most *sensitive* when the deflection is practically zero, and that a small error in reading the deflection will introduce a minimum error into the calculation when the deflection is 45°. Error in observation may arise from three causes :

---

[1] Since the intensity of the field at the centre of the coil is $2\pi nC/a$ (see p. 255), the *true constant* may be defined as the intensity of the field at the centre of the coil when traversed by 1 C.G.S. unit of current.

(i) *Parallax*: the eye should be vertically over the needle when reading the deflection. This is ensured by mounting the circular paper scale, of which the central portion is removed, on a plane mirror ; the eye is moved until the pointer hides its image reflected from the mirror.

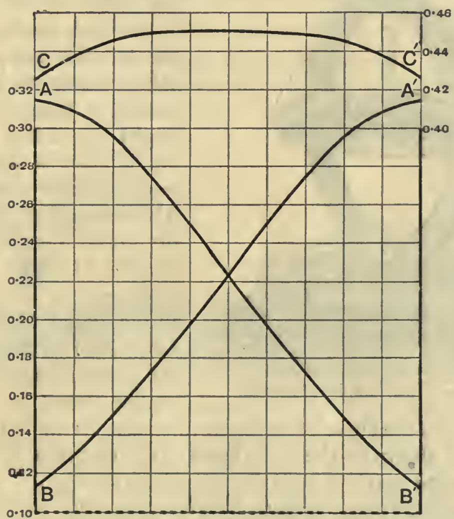

Fig. 199.—The Magnetic Field of a Helmholtz Tangent Galvanometer.

(ii) *The suspending fibre may not coincide with the centre of the circular scale*: errors due to this are eliminated by taking the mean of the readings of both ends of the pointer (cf. p. 100, § *b*).

(iii) *The axis of the magnet and the plane of the coil may not coincide with the meridian* (the former due to torsion in the fibre): in this case the errors are eliminated by observing deflections when the current is reversed, and taking the mean of the four readings obtained. Deflections are observed more accurately if a mirror is attached to the magnet, and a beam of light reflected from the mirror on to a scale (p. 277).

A greater uniformity of the field due to the current is obtained in a type of galvanometer, due to Helmholtz, in which two equal circular coils are fixed so that their axes coincide and at a distance apart equal to their common radius. The needle is suspended at a point midway between the coils. The field extending to a distance equal to one-tenth the radius on either side of this point is practically uniform. This may be understood clearly by plotting on squared paper the value of the intensity of the field, $2\pi a^2C/(a+x^2)^{\frac{3}{2}}$, due to each coil at various points along the axis; the resultant intensity is obtained by adding together the intensities due to the separate coils. In Fig. 199 AB′ represents the intensity curve for a single turn of wire, of radius 20 cms., traversed by unit current and situated in the plane AB. Also A′B represents the curve obtained from a similar circuit in the plane A′B′. CC′ is the curve of resultant intensity, the numerical value of which is shown on the right hand of the diagram, and its contour indicates that the intensity is uniform over a considerable distance midway between the coils.

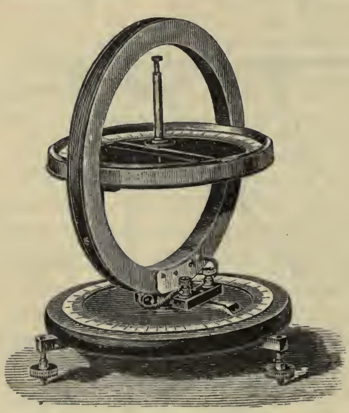
Fig. 200.—The Sine Galvanometer.

The Sine Galvanometer.—A *sine galvanometer* (Fig. 200) only differs from a tangent galvanometer in that the coil can be rotated round a vertical axis passing through its centre, and the amount of rotation can be observed by means of a horizontal scale fixed below the coil. The coil is initially adjusted so that its plane is in the meridian; when the needle is deflected by a current the coil is rotated until its plane coincides with the magnetic axis of the needle. In this position the deflecting force F (Fig. 201) due to the current is perpendicular

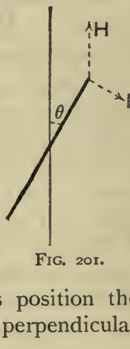

Fig. 201.

to the axis of the needle, and the equation of equilibrium is $F = H \sin\theta$. The general equation is therefore

$$C = \frac{H}{G} \sin\theta.$$

The advantage of this instrument is that the needle is always in the same position relatively to the coil, and therefore the deflection is not liable to error due to want of uniformity of the field; also the suspending fibre remains untwisted, and therefore no error due to torsion is introduced. But since the maximum value of $\sin\theta$ is 1, currents greater than $\frac{H}{G}$ units cannot be measured by means of the instrument.

Mirror Galvanometers.—In principle these instruments may resemble either a galvanoscope or a tangent galvanometer, except that the pointer is replaced by a circular mirror (about 1 cm. diameter) attached to the needle, from which a beam of light is reflected on to a horizontal paper scale placed at some distance from the mirror. The beam of light is equivalent to a pointer of length equal to twice the distance between the mirror and the scale. In Fig. 202 the beam of light is derived from an electric incandescent filament enclosed in a metal case fitted with a horizontal adjustable tube, in which a lens is fitted. The beam of light is directed upon the mirror of the galvanometer and reflected back to the divided scale; and deflections are observed by focussing the image of a fine vertical line scratched on the surface of the lens. A *concave* mirror is generally used, and the cross-line can then be focussed without using an auxiliary lens. In using this method of observa-

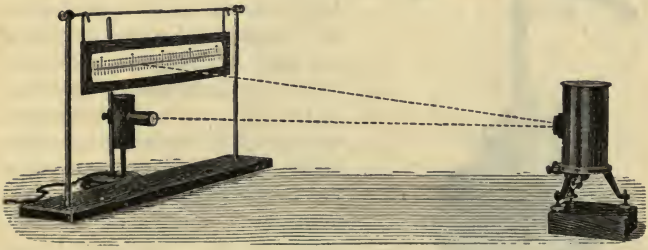

FIG. 202.—Galvanometer, with Lamp and Scale.

tion it is necessary to remember that *the reflected beam moves through an angle which is twice as great as that through which the mirror moves* (p. 96). If α and δ are the angles through which the mirror and the beam of light are deflected, then $\delta = 2\alpha$; and, if δ is small, $\tan\delta = 2\tan\alpha$.

$$\text{Hence}\quad \tan\alpha = \frac{\text{deflection in cms.}}{2 \times \text{distance of mirror from scale}}.$$

The angular deflection of the needle is usually very small, and it may therefore be assumed that the field due to the current is uniform within the range of movement of the needle; hence the instrument will obey the *tangent law*, and currents traversing the coil are proportional to the tangent of the angle of deflection, and therefore to the deflection of the image on the paper scale. Nevertheless, owing to the indefinite dimensions of the coil and to the nearness of the windings to the magnet, it is not possible to calculate the 'constant' or the 'reduction factor' from the dimensions of the coil as in the case of a tangent galvanometer. It is necessary to measure the current to which an observed deflection is due by an independent method, *e.g.* by means of a copper or silver voltameter (p. 457).

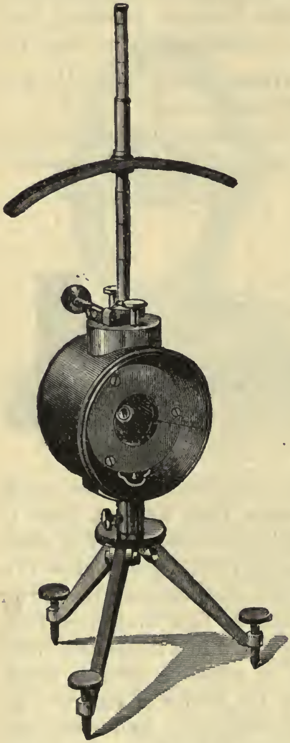

FIG. 203.—Kelvin's Mirror Galvanometer.

The current strength necessary to give a deflection of one scale-division is termed the **sensibility** of the galvanometer. But a statement of the sensibility is incomplete unless the length of a scale-division and the distance of the scale from the mirror are given.

Fig. 203 represents **Kelvin's Mirror Galvanometer**, which consists of a hollow circular coil of silk-covered copper wire, in the centre of which the mirror is suspended by means of a single silk fibre. The needle consists of two or three short lengths of magnetised watch spring attached to the back of the mirror. A *controlling magnet*, which may be raised or lowered along a vertical rod above the instrument serves to modify the magnitude and direction of the earth's magnetic field. Without the controlling magnet it would always be necessary to place the galvanometer with the plane of its coils in the magnetic meridian, and it would be impossible to modify the sensibility of the instrument.

A still more sensitive instrument is obtained by constructing a mirror galvanometer with an astatic system of needles. Each set of needles is surrounded by its own coil, and the coils are connected together in series as shown in Fig. 204. The highest sensibility is obtained by adjusting a controlling magnet so that the field is not quite uniform over the whole magnetic system.

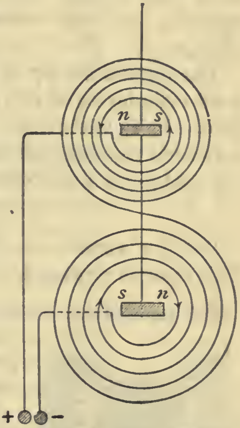

FIG. 204.—An Astatic Mirror Galvanometer.

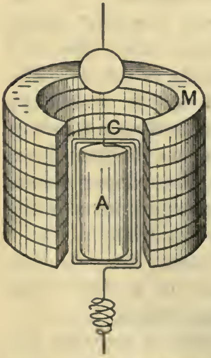

FIG. 205.—The d'Arsonval Galvanometer.

Suspended Coil Galvanometers.—It has been shown (p. 257) that the moment of the couple acting on a rectangular coil, traversed by a current C and suspended with its plane coinciding with the direction of a uniform magnetic field H, is equal to ACH, where A is the total area of all turns of the coil. This principle is applied in a type of galvanometer of which the **d'Arsonval** (Fig. 205) is the most familiar example. The magnetic field is derived from a cylindrical magnet M built up of highly magnetised rings of tungsten steel. The coil C is hung on a stretched strip of phosphor-bronze which conveys the current to and from the coil; a strip is preferable to a wire, since the cooling surface is greater, the temperature more uniform, and the torsional rigidity therefore less liable to change.

The torsion set up in the strip is the controlling force. The fixed cylinder A, of soft iron, serves to concentrate the magnetic field. If T_0 is the torsional couple set up when the suspension is twisted through unit angle, then, when a current traverses the coil and gives a deflection α,

$$CH \times A \cos \alpha = T_0 \alpha.$$

But, if α is small, $\alpha = \sin \alpha$; hence $C = T_0 \tan \alpha / AH$ C.G.S. units. This may be written $C = K \tan \alpha$, where $K = T_0 / AH$.

This assumes that the magnetic field is uniform in direction. If the pole-faces of the field magnet are curved, and concentric with the cylinder A (Fig. 206), the moment of the deflecting couple is constant, since in all positions the direction of the field coincides with the plane of the coil. The current is then proportional exactly to the deflection observed.

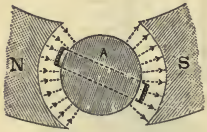

FIG. 206.

The special merits of this type are as follows: (i) The deflections are scarcely affected by external magnetic fields. (ii) The instrument may face in any direction, since the zero position of the coil is independent of the direction of the magnetic field in which it is suspended. (iii) The coil is practically *aperiodic* (or *deadbeat*), particularly if it is wound on a frame of sheet copper, when the movements of the coil are damped down by the eddy-currents (p. 416) induced in the copper frame. (iv) The instrument is adapted for *ballistic* experiments if the coil is wound on a frame of non-conducting material.

The disadvantages of this type of instrument are: (i) The sensitiveness cannot be modified by means of a controlling magnet, and (ii) The *damping* is usually appreciable.

Ballistic Galvanometers.—If it is desired to measure the *quantity* of electricity which passes when a current of extremely brief duration traverses the coil of the instrument, it is necessary that the needle (or coil, if a suspended-coil galvanometer, is used) shall be so constructed that the duration of the current is small compared with the time of vibration of the needle; also the retardation or *damping* of the swing of the needle must be very slight. With such an arrangement the momentary current will pass before the needle has appreciably moved from its position of rest; but the needle will have received an *impulse*, a measure of which is obtained by observing the extent of the first swing.

This type of instrument is termed a **ballistic galvanometer.** The following paragraphs show how, in the case of the needle instrument, the quantity Q of electricity may be deduced from an observation of the angle of throw of the needle.

In Fig. 207 let OA represent the semi-length of the needle, and OA′ its position at the limit of its first swing. If m is the pole-strength, H the field-strength, and $2l$ the length of the magnet, then *the work done against magnetic forces* due to the field is

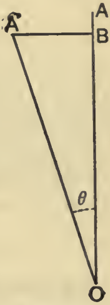

FIG. 207.

$$2m\mathrm{H}\times\mathrm{AB}=2m\mathrm{H}(\mathrm{OA}-\mathrm{OB})=\mathrm{MH}(1-\cos\theta)=\mathrm{MH}\,.\,2\sin^2\frac{\theta}{2}.$$

This must be equal to *the initial kinetic energy* of the needle, or

$$\tfrac{1}{2}\mathrm{I}\omega^2=\mathrm{MH}\,.\,2\sin^2\frac{\theta}{2},$$

or

$$\omega=2\sqrt{\mathrm{MH}/\mathrm{I}}\,.\,\sin\frac{\theta}{2}.\quad\text{.....................(i)}$$

If C is the mean current (in C.G.S. units) during the short time interval dt occupied by the discharge, and if G is the constant of the galvanometer, then GC is the intensity of the field at the centre of the coil, and $\mathrm{GC}m$ is the force acting on each pole. The *impulse* on each pole is $\mathrm{GC}m\times dt$; and the *moment of the impulse* is $\mathrm{GC}m\,.\,dt\times 2l=\mathrm{MGC}\,.\,dt=\mathrm{MGQ}$. But the moment of the impulse must be equal to the initial *moment of momentum* (Iω); hence

$$\mathrm{MGQ}=\mathrm{I}\omega,$$

or

$$\omega=\mathrm{MGQ}/\mathrm{I}.\quad\text{..........................(ii)}$$

Hence, from equations (i) and (ii),

$$\mathrm{MGQ}/\mathrm{I}=2\sqrt{\mathrm{MH}/\mathrm{I}}\,.\,\sin\frac{\theta}{2},$$

or

$$\mathrm{Q}=\frac{2}{\mathrm{G}}\sqrt{\frac{\mathrm{IH}}{\mathrm{M}}}\,.\,\sin\frac{\theta}{2}.$$

The quantities I and M may be eliminated by substituting their values expressed in terms of T, the time of one complete oscillation, where $T=2\pi\sqrt{I/MH}$. Hence

$$Q=\frac{TH}{G\pi}\cdot\sin\frac{\theta}{2}$$
$$=\frac{TH}{G\pi}\times\frac{s}{4L},$$

where s is the deflection in scale divisions and L is the distance, in the same units, between the mirror and the scale.

In practice it is usual to eliminate H/G by observing the steady deflection α produced by a known current A C.G.S. units, when $H/G=A/\tan\alpha$. Hence

$$Q=\frac{A}{\tan\alpha}\cdot\frac{T}{\pi}\cdot\frac{s}{4L}.$$

Logarithmic Decrement.—In the above formulae for the ballistic galvanometer it is assumed that the magnetic couple due to the earth's field is the only cause tending to bring the needle to rest, and no allowance has been made for the damping of the movement due to air-friction and to induced currents set up in the surrounding metal-work. It is known that the amplitude of successive swings of a galvanometer needle diminish in geometrical ratio, and if s_1, s_2, ... are the amplitudes of successive swings, then

$$\frac{s_1}{s_2}=\frac{s_2}{s_3}=\dots\quad\frac{s_{n-1}}{s_n}=\rho \text{ (a constant).}$$

The relation between the observed amplitude s of the first throw, and the amplitude s_0 which would have been observed *in the absence of any damping*, is indicated by the fact that the damping has been in operation during only one-half of one swing; hence, in this case, the damping factor is $\rho^{\frac{1}{2}}$; or $s_0=\rho^{\frac{1}{2}}s$.

If the constant ρ is denoted by e^{λ}, where λ is the Napierian logarithm of ρ, then $\rho^{\frac{1}{2}}=e^{\frac{\lambda}{2}}=1+\frac{\lambda}{2}+\dots$. Hence $s_0=s\left(1+\frac{\lambda}{2}\right)$. The quantity λ is called the *logarithmic decrement*.

In practice, the value of ρ is determined accurately by observing the amplitudes of two swings separated from each other by an observed number of swings; thus, if the 1st and the 9th swings are observed, $\rho^8=s_1/s_9$.

Ballistic Galvanometer (suspended coil pattern).—The work done against the torsion of the suspension must be equal to the initial kinetic energy of the coil. If T_0 is the moment of torsion of the suspension, and θ the final deflection of the coil, the resistance of the suspension at the end of the swing is $T_0\theta$, and the *mean* resistance is $T_0\theta/2$. Hence the work done against torsion is $T_0\theta^2/2$. The initial kinetic energy is $I\omega^2/2$. Hence

$$\tfrac{1}{2}I\omega^2 - \tfrac{1}{2}T_0\theta^2,$$

or

$$\omega = \theta\sqrt{T_0/I}. \quad \text{.............................(iii)}$$

If c is the mean current during the short period dt of the discharge, the mean moment of the force tending to twist the coil about a vertical axis is cHA, where A is the total area of the coil, and H is the intensity of the magnetic field; and the mean moment of the impulse is $cHA\,.\,dt$, or QHA. But the moment of the impulse is equal to the moment of momentum ($I\omega$) of the coil; hence

$$QHA = I\omega,$$

or

$$Q = I\omega/HA. \quad \text{..........................(iv)}$$

But, from (iii), $\omega = \theta\sqrt{T_0/I}$,

and from p. 280, $AH = T_0\alpha/C\,.\cos\alpha$,

where α is the deflection due to a constant current C.

Hence
$$Q = I\,.\,\theta\sqrt{\frac{T_0}{I}} \times \frac{C\,.\cos\alpha}{T_0\alpha}$$
$$= \frac{\theta\,.\,C\cos\alpha}{\alpha}\sqrt{\frac{I}{T_0}}.$$

The quantities I and T_0 may be eliminated by substituting their value in terms of T, the time of one complete oscillation, where $T = 2\pi\sqrt{I/T_0}$.

Hence
$$Q = \frac{\theta\,.\,T\,.\,C\cos\alpha}{2\pi\alpha} = \frac{\theta\,.\,T\,.\,K}{2\pi}.$$

As in the case of the needle instrument, a correction for damping must be made in the value of θ.

EXAMPLE.—In order to find the capacity of a condenser, it was charged to a potential-difference of 1.43 volt, and discharged through a ballistic galvanometer (suspended coil type). The mean deflection was 17.5 cms. on a scale placed 110 cms. from the mirror. The time of 15 complete oscillations was found to be 175 secs.; and the amplitudes of the 5th and 30th swings were observed to be 16.3 cms. and 3.5 cms. respectively. The constant (K) of the galvanometer was determined by connecting its terminals to a potential-difference of 2.14×10^{-4} volt; and the observed deflection was 22.6 cms. The resistance of the galvanometer was 650 ohms.

$\theta = 17.5/(2 \times 110) = 0.0795$ radians; $T = 175/15 = 11.67$ secs.;
$\rho = \sqrt[25]{16.3/3.5} = 1.0635$, and $\rho^{\frac{1}{2}} = 1.0312$.

In the determination of K,

the tangent of the angle of deviation of the spot of light $=\frac{22.6}{110}=0.2054$;

hence, the angle of deviation ,, ,, $=11^\circ.6$,

and ,, ,, of the needle $=5^\circ.8$.

$$\text{Therefore,}\quad K=\frac{c}{\tan 5^\circ.8}=\frac{2.14\times10^{-4}}{650\times0.1016}=3.24\times10^{-6}\text{ amp.}$$

Using the equation $Q=\theta TK/2\pi$,

$$Q=(0.0795\times11.67\times3.24\times10^{-6})/2\pi=4.787\times10^{-7}\text{ coulomb.}$$

Hence, **capacity** $=Q/V=(4.787\times10^{-7})/1.43=0.335\times10^{-6}$ farad $=0.335$ microfarad.

Weber's Electro-dynamometer.—This instrument may be regarded as a standard tangent galvanometer in which the needle is replaced by a small coil suspended by a bifilar suspension so that, when no current is traversing the coils, its plane is at right angles to that of the large coil. The usual type of instrument (Fig. 208) resembles Helmholtz's arrangement of two parallel coils as described on p. 276.

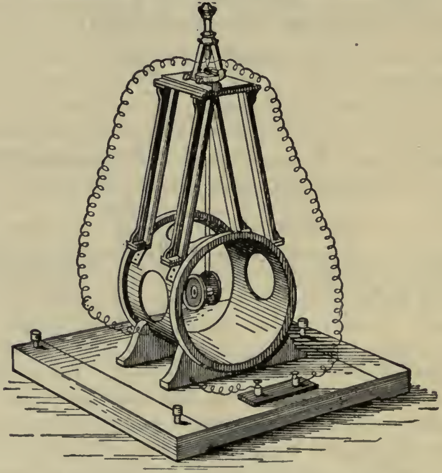

FIG. 208.—Weber's Electro-dynamometer.

The fixed and suspended coils are connected in series, and the passage of a current tends to turn the suspended coil so that its plane coincides with that of the fixed coils. The deflection is determined by the relative magnitude of the electromagnetic effects and of the restoring couples due to the earth's field and to the suspension.

By p. 255 the strength of the field at a point mid-way between the centres of the fixed coils is $4\pi na^2\mathrm{C}\Big/\left(\frac{5a^2}{4}\right)^{\frac{3}{2}}=\mathrm{GC}$, where n is the number of turns of wire in each coil and G is the true constant of the coils. Also (p. 257) the magnetic moment of the moving coil is ($\pi a_1{}^2\mathrm{C}\times$ the number of turns) $=\mathrm{SC}$, where S is the total area of all the turns of wire. If θ_1 is the angular deflection of the suspended coil, the deflecting couple is $\mathrm{GC^2S}\cos\theta_1$. The restoring couple due to the earth is $\mathrm{SCH}\sin\theta_1$; and that due to the bifilar is $\mu\sin\theta_1$, where μ is a constant depending upon the dimensions of the bifilar. Therefore,

$$\mathrm{GC^2S}\cos\theta_1=\mathrm{SCH}\sin\theta_1+\mu\sin\theta_1$$

or

$$\tan\theta_1=\frac{\mathrm{GC^2S}}{\mathrm{SCH}+\mu}$$

$$=\frac{\mathrm{GC^2S}}{\mu}\left(1+\frac{\mathrm{SCH}}{\mu}\right)^{-1}=\frac{\mathrm{GC^2S}}{\mu}-\frac{\mathrm{GSC^3H}}{\mu^2}$$

(the higher terms being neglected, since the product SCH is always small compared with μ). If the current is now reversed, the deflection is slightly altered, since the sign of the restoring couple due to the earth is reversed; if the deflection is now θ_2,

$$\tan\theta_2=\frac{\mathrm{GC^2S}}{\mu}+\frac{\mathrm{GSC^3H}}{\mu^2}.$$

By addition,

$$\mathrm{C}^2=(\tan\theta_1+\tan\theta_2)\mu/2\mathrm{GS}.$$

The value of μ is determined in the following manner: Since the restoring couple due to the bifilar is proportional to the mass of the coil and to a constant depending upon the dimensions of the suspension (p. 105), we may write $\mu=\lambda\mathrm{M}$, where M is the mass of the coil. If the coil is made to vibrate round a vertical axis, then[1]

$$t_1=2\pi\sqrt{\mathrm{I}/\lambda\mathrm{M}}$$

---

[1] This formula may be deduced from that of the simple pendulum (p. 26), in which case $t=2\pi\sqrt{\frac{\mathrm{I}}{mgl\sin\theta/\theta}}$, where θ is the maximum angular displacement (in circular measure); in this case, $mgl\sin\theta/\theta$ is *the moment of the restoring force per unit angular displacement.* In the case of a bifilar suspension (p. 105), the moment of the restoring couple per unit angular displacement may be written $\lambda\mathrm{M}\sin\alpha/\alpha=\lambda\mathrm{M}$. Hence, $t_1=2\pi\sqrt{\mathrm{I}/\lambda\mathrm{M}}$.

where t_1 is the time of one complete vibration, and I is the moment of inertia of the coil. A metal bar, of moment of inertia i and mass m, is now attached to the coil, and the time of vibration again observed; then

$$t_2 = 2\pi\sqrt{(\mathrm{I}+i)/\lambda(\mathrm{M}+m)}.$$

From these equations the value of λ, and therefore of μ, can be calculated.[1]

This instrument can be used either for direct or alternating currents, since in the latter case the current is simultaneously reversed in all the coils.

Fig. 209 represents a type of ampere-balance used by Lord Rayleigh in his determination of the electro-chemical equivalent of silver (p. 456). The two fixed coils, C_1 and C_2, are horizontal; the movable coil c is suspended from one arm of a balance, and adjusted so that the axis of all the coils coincide. The coils are connected in series so that c is repelled by C_2 and attracted by C_1, and the force acting on c when unit current traverses the circuit can be calculated from the dimensions of the coils. Hence, if the force is counterpoised by adjusting weights in the scale-pan suspended from the other arm of the balance, the current strength can readily be calculated.

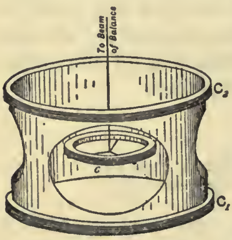

FIG. 209.

Kelvin's Ampere Balance.—Lord Kelvin has designed a series of instruments for measuring currents, the principle of which depends upon the mutual action between the fixed and movable portions of a circuit conveying a current. The movable portion of the circuit consists of one or more turns of wire, with their planes horizonal, and supported at opposite ends of a balance-beam free to move round a horizontal axis. The principle of the instrument designed to measure smaller currents, *e.g.* centi-amperes, is represented in Fig. 210 in which a, b, c, and d are fixed coils, e and f movable coils; all the coils are connected in series between the terminals T_1 and T_2. The ends of the movable coils are supported from two sets of thin copper wires, g and h, through which the current passes to and from these coils. All the coils are so connected that a current tends to raise one movable coil and to depress the other, and the couple tending to deflect the beam is proportional to the *square* of the current strength. Fig. 211 represents the **standard centi-ampere balance.** A horizontal scale is fixed to the beam, and is graduated from left to right. A carrier, which supports a standard weight, can be made to slide along the scale by means of a cord manipulated from outside the case of the instrument; also, a second weight may be placed in a V-shaped trough fixed at the right-hand end of the beam. The latter weight is adjusted so as to keep the beam horizontal when the sliding weight is at the zero of the scale, and when no current is passing. The

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[1] For further details, see G. F. C. Searle (*The Electrician*, Vol. 28, p. 273).

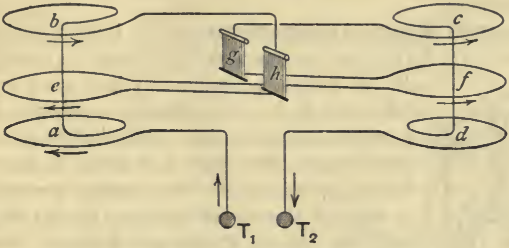

FIG. 210.—Principle of Kelvin's Ampere Balance.

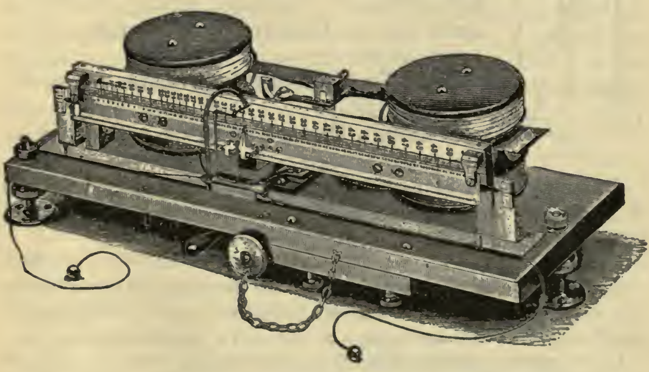

FIG. 211.—Kelvin's Centi-ampere Balance.

current strength is determined by observing the distance along the scale to which it is necessary to move the carrier in order to keep the beam horizontal. Other types of the same instrument have been designed to measure *deci-*, *deka-*, *hekto-*, and *kilo-amperes.*

Siemens's Electro-dynamometer.—This instrument consists of a fixed coil A (Fig. 212) connected in series with a movable coil B, of which the ends dip into mercury cups. The coil B is suspended by a silk thread, and is controlled by a fine spiral *s* of steel or phosphor bronze, the upper end of which is attached to a torsion-head *c*. The normal position of B is with its plane perpendicular to that of A, and with the pointer P mid-way between two stops inserted in the edge of the graduated disc. The passage of a current tends to rotate B with a force proportional to the *square* of the current, and P is brought back to zero by rotating *c*. Since the restoring force due to the spiral is proportional to the angle through which *c* is rotated, hence the latter is proportional to the square of the current.

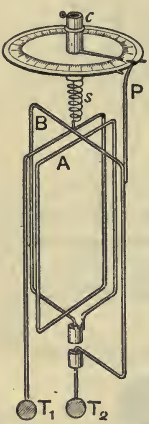

FIG. 212.—Siemens's Electro-dynamometer.

Duddell's Thermo-galvanometer.—(See Appendix, p. 566.)

Fleming's Thermo-ammeter.—(See Appendix, p. 567.)

Other methods of measuring current, depending upon the *heating* and *chemical* effects of the current, are described in Chaps. XIX. and XXIV.

SUMMARY

A **galvanometer** is an instrument for the detection and measurement of a current. Its principle depends upon the mutual electromagnetic forces between a wire conveying a current and a neighbouring magnet.

In the **astatic galvanometer** the sensibility is increased by using an astatic system of magnets, since the magnitude of the 'controlling' force is diminished.

The Tangent Galvanometer.—If a wire is bent into the form of a circle of radius a, and is traversed by a current of C C.G.S. units, the intensity of the field at the centre is $2\pi C/a$. If the plane of the coil coincides with the direction of a uniform magnetic field of intensity **H**, a short magnet suspended at the centre is deflected through an angle θ, such that $2\pi C/a = H \tan \theta$. Hence $C = a \tan \theta . H/2\pi$. If the coil consists of n turns of wire, then

$$C = a \tan \theta . H/2\pi n.$$

This is usually written $C = \tan \theta\, H/G$, where G is the *true constant* of the instrument.

A more accurate type of instrument, due to Helmholtz, consists of two circular coils of the same dimensions, and fixed in vertical parallel planes at a distance apart equal to their common radius. The magnetised needle is suspended mid-way between the centres of the coils.

The **sine galvanometer** is constructed in exactly the same manner as the tangent galvanometer, except that the coil can be rotated round a vertical axis, and the amount of rotation observed by means of a horizontal circular scale on the base of the instrument. If, when a current is traversing the instrument, the coil is rotated until its plane coincides with the magnetic axis of the needle, the current is proportional to the *sine* of the angle of deflection.

In a **mirror galvanometer** a reflected beam of light serves the purpose of a pointer. The angular deflection of the beam of light is twice that of the magnet to which the mirror is attached. The sensibility of the instrument may be increased by diminishing the controlling field by means of an external magnet or by using an astatic system of magnetised needles.

Suspended Coil Galvanometers.—In this type of instrument the magnet is fixed and the coil movable. The coil is practically *dead-beat* if it is wound on a frame of conducting material; on the other hand the instrument is suited for ballistic experiments if the frame is of non-conducting material.

The Ballistic Galvanometer.—A galvanometer may be used for ballistic experiments when the period of swing of the needle is considerable and when the 'damping' of the swing is slight.

Weber's Electrodynamometer, Kelvin's Ampere Balance, and **Siemens's Electrodynamometer** are types of instruments for the measurement of current strength. Each may be used for the measurement of either direct or alternating currents.

QUESTIONS ON CHAPTER XVI

1. Explain how the current in a tangent galvanometer properly arranged, is proportional to the tangent of the angle of deflection.

Describe some form of tangent galvanometer, and explain how the sensitiveness can be varied by suitably placing a magnet outside the galvanometer.

2. Describe the construction and use of a tangent galvanometer. Calculate the strength of the current in C.G.S. units and also in amperes from the following data: Radius of coil, 12 cms. Number of turns in coil, 10. Deflection of needle, 45°. Value of earth's horizontal force, 0.18.

3. Discuss the several forces or moments which act on the needle of a tangent galvanometer when deflected by the action of a current passing through the coil of the galvanometer, and deduce the law of action of the instrument.

4. A coil of six turns, each of which is 1 metre in diameter, deflects a compass needle at its centre through 45°. Find the strength of the current in amperes, having given that $H = 0.18$ C.G.S. units.

5. A current flows through two tangent galvanometers in series, each of which consists of a single ring of copper, the radius of one ring being three times that of the other. In which of the galvanometers will the deflection of the needle be greater? If the greater deflection be 60° what will the smaller be?

6. The coil of a tangent galvanometer is placed at right angles to the magnetic meridian and a steady current passes through it. The needle when set in vibration makes 5 oscillations in a given time, but only 3 in the same time when the direction of the current is reversed. Compare the magnetic force at the centre of the coil due to the current with that due to the earth.

7. Describe two methods of procuring an astatic system of magnetic needles, and two ways of controlling the sensitiveness of a galvanometer.

8. Of how many turns of wire must the coil of a tangent galvanometer consist, if the radius of the coil is 15 cms., and if a current of 0.01 ampere is to produce a deflection of 30°? ($H = 0.18$.)

9. The coil of a sine galvanometer consists of 100 turns of wire, and its mean diameter is 1 metre. What is the greatest current that it will measure at a locality where $H = 0.18$?

10. A sine galvanometer consists of a single coil of 49 turns of wire, the mean radius of which is 20 cms. A current of 0.08 ampere causes such a deflection that the coil has to be turned through 45° to bring the needle to its original position with regard to the coil. Determine the reduction-factor of the galvanometer, and the horizontal component of the earth's magnetic intensity.

CHAPTER XVII

RESISTANCE

Introductory.—When a steady current is traversing a metal wire the total quantity of electricity passing all sections of the wire in a given time is the same; it is also equally characteristic that there is a difference of potential between all points of the wire, and that the potential diminishes in the direction in which the current is flowing. The P.D. between any two points may be measured experimentally by means of a quadrant electrometer, which has been previously standardised by connecting opposite pairs of quadrants to a source of constant and known E.M.F., such as a Clark cell. A P.D. may be theoretically defined, in C.G.S. units, as the work in ergs which would be required in order to convey unit quantity of electricity from the point of lower to that of higher potential.

The terms E.M.F. and P.D. are often used synonymously; but the student is advised carefully to distinguish their meaning by regarding them as related to each other in the same sense as *cause* and *effect*. Thus, the term E.M.F. may be restricted to the *source* of the current in any circuit, and differences of potential are created in all parts of the circuit by the E.M.F. In this sense the E.M.F. is strictly local, while the P.D. is distributed. Further, we may say that an E.M.F. exists even when the circuit is not completed and when no current is flowing, and that it is therefore independent of the resistance in the circuit; but, as will be shown in the next paragraph, the P.D. between any two points depends entirely upon the resistance between those points.

The relationship between the current strength in a wire and the P.D. between its ends was first investigated by G. S. Ohm in 1826.

Ohm's Law.—From his experiments Ohm deduced the following law: "**In any wire at uniform temperature the current is directly proportional to the P.D. between its ends.**" If E and C represent the magnitude of the P.D. and current respectively, the ratio E/C is found to remain absolutely constant, providing that the temperature remains unchanged.

EXPT. 73.—If any two points A and C (Fig. 213), on a wire AB conveying a current, are touched by the ends of a long thin wire AR_2C, a weak current will traverse the latter wire from A to C, and it may be detected by a galvanometer MG. If another source of E.M.F. such as a standard cell E_2 is included in the circuit and placed so as to tend to send a current in the opposite direction, and if this opposing E.M.F. is equal to that due to the P.D. between A and C, then no current will traverse the galvanometer MG. By varying the position of the point of contact C, a point may be found such that no deflection is obtained, and the P.D. between A and C is then equal to the E.M.F. of E_2. The strength of the current along AB may be observed by means of a tangent galvanometer TG. If *two* standard cells in series are used instead of E_2, and if the point C remains fixed, it will be found necessary to *double* the strength of current along AB in order to obtain no deflection in MG. If *three* standard cells are used the current must be made *three* times as great. In all cases *the ratio between the number of standard cells used and the current strength is constant.*

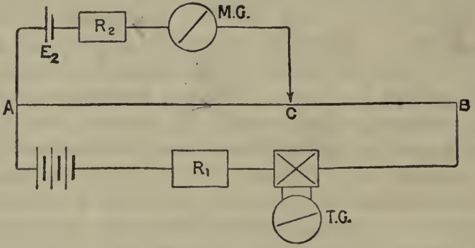

FIG. 213.—Demonstration of Ohm's Law.

Resistance.—The ratio E/C, which has a constant value for any conductor when the temperature is constant, is called the **resistance** of the conductor. The inverse ratio C/E may similarly be termed the **conductance** of the conductor. The numerical value of a resistance is expressed in C.G.S. units when both E and C are expressed in the same units.

The *practical* unit of resistance is called the **ohm**, and a conductor is said to have a resistance of one ohm when a P.D. of one volt between its ends creates in it a current of one ampere.

From Ohm's Law it is evident that if of the three quantities, resistance, current, and P.D., two can be defined and accurately determined, the third quantity can be obtained indirectly by means of the other two. The practical units were agreed to by the Paris Congress of 1881 : the *volt*, as the practical unit of P.D., was fixed as equal to 10^8 C.G.S. units (p. 414), perhaps because this was approximately the E.M.F. of a Daniell cell, which was then regarded as the most trustworthy standard of E.M.F. ; and the *ohm* was fixed as equal to 10^9 C.G.S. units of resistance (p. 431), because this was approximately the resistance of the standard unit previously in use. It therefore follows that 1 ampere $= 10^8/10^9 = 10^{-1}$ C.G.S. unit of current.

In experimental work a current and a resistance can be measured with far greater accuracy than a P.D. Hence the measurement of current and resistance are usually relied upon in order to determine a P.D. in the electromagnetic system.

Specific Resistance.—Ohm proved that **the resistance of a conductor depends upon the material, and that it is directly proportional to the length l, and inversely proportional to the cross-section s of the conductor.** Hence,

$$R = \rho \cdot \frac{l}{s},$$

where ρ is a constant depending upon the material, and called the **specific resistance** of the material. If both l and s are equal to unity, then $R = \rho$; consequently the specific resistance of any material may be defined as **the resistance of a conductor of that material 1 cm. long and 1 cm.^{2} cross-section**, or as **the resistance between opposite faces of a centimetre-cube of the material.**

The specific resistance depends upon the physical condition of the material, and, in the case of metals, it is modified by processes of hardening and annealing ; it is also considerably modified by the presence of traces of impurity. The specific resistance of metals always increases with an increase of temperature. If R_0 is the resistance of a metallic conductor at

0° C., its resistance at t° C. is approximately represented by the equation

$$R_t = R_0(1 + \alpha t),$$

where α is the **temperature coefficient** of the material.

From this formula, the increase of resistance appears to be proportional to the rise in temperature, and this is sufficiently true over limited ranges of temperature. Between wider limits of temperature, a formula involving *two* constants is generally required, which may be expressed thus:

$$R_t = R_0(1 + \alpha t + \beta t^2).$$

The values of the constants α and β for any given metal may be determined by measuring the resistance at three definite temperatures, *e.g.* the temperatures of melting ice, boiling water, and boiling sulphur (which boils at 444°.5 C. at normal pressure); in this manner three independent equations are obtained for the calculation of R_0, α and β. The most recent values obtained for the constants α and β for mercury are 0.0008878 and 0.000001037; also, for platinum,

$$\alpha = +0.003448 \text{ and } \beta = -0.000000533.$$

It is evident that if the series of values of R_t and of t are plotted on squared paper, the curve obtained will be a parabola, which is concave towards the axis of the temperatures; and the fact that the temperature coefficient of several pure metals varies from 0.0037 to 0.0040, which is practically the same as the coefficient of expansion (0.00366) of a gas, has suggested that the resistance of such metals would be zero at the absolute zero of temperature (−273° C.), and that this would be indicated in the curve by the parabola intersecting the axis of temperature at the point −273°. But recent measurements of the resistance of several pure metals at and below the temperature of boiling air (−182°.5 C.) have shown that the parabolic relationship between resistance and temperature does not hold good at extremely low temperatures—in fact the curve bends round in the opposite direction, and becomes convex to the axis of temperature; hence the above formula can be relied upon only for a very limited range of temperature.

It is important to notice that carbon has a *negative* temperature coefficient. The same phenomenon is shown by most non-metals, including indiarubber, mica, ebonite, etc. In the case of electrolytes the specific resistance falls rapidly with rise of temperature, and the coefficient varies from 0.5% to 2% per degree Centigrade. The specific resistance and the temperature-coefficient of the principal metals, alloys, and electrolytes are given in the following table :

SPECIFIC RESISTANCES.

(In microhms[1] per centimetre cube, at 0° C.).

Metals.	Specific Resistance.	Temperature Coefficient.
Silver (annealed), - - - -	1.521	0.00377
„ (hard drawn), - - - -	1.652	
Copper (annealed), - - - -	1.590	0.004284
„ (hard drawn), - - -	1.622	
Platinum, - - - - -	11 (at 18° C.)	0.0034
Iron, - - - - - -	9–15	0.007
Tin (pure), - - - - -	9.565	0.004
„ (pressed commercial), - -	13.36	
Lead (pressed), - - - - -	19.85	0.00387
Bismuth (hard), - - - -	132.6	0.0054
Mercury, - - - - - -	94.07	0.00089
Alloys.		
German Silver (4Cu+2Ni+1Zn), -	20.2	0.000273
Mànganin (84Cu+12Mn+4Ni),	42–44 (at 18° C.)	0.000025
Constantan, or Eureka (60Cu+40Ni),	49	−0.0004 to +0.00001
Carbon (arc lamp), - - - -	7×10^3	−0.0005
Liquids (at 15° to 20° C.).		
Water[2] (ordinary distilled), - -	72000×10^3	
Sulphuric Acid, 10%, density, 1.07, -	850×10^3	
„ 25%, „ 1.17, -	1000×10^3	
„ 75%, „ 1.70, -	4250×10^3	
Copper Sulphate (saturated), - -	25000×10^3	
Zinc „ „ - -	21000×10^3	

[1] The microhm is the millionth part, or 10^{-6}, of an ohm.

[2] The specific resistance of water is considerably modified by traces of impurity, thus, Kohlrausch found that the value at 18° C. for water purified by distillation in vacuo is 3.76×10^{15} C.G.S. units.

Effect of a Magnetic Field on the Resistance of Bismuth.—In 1881, Tomlinson observed that the resistance of a rod of bismuth is considerably modified when it is subjected to a longitudinal magnetic force. It was subsequently observed that the effect is far more marked when the force is transverse to the wire. The following data, due to Henderson, indicate the extent to which the resistance increases, the wire being kept at a constant temperature.

H (C.G.S. Units).	Resistance.
0	1
12500	1.63
27450	2.54
38900	3.34

The effect is influenced largely by changes in temperature; thus Dewar has determined that at the temperature of boiling air, – 182°.5 C., a magnetic field of 21800 C.G.S. units increased the resistance 150 times. The principle has been applied to the measurement of the intensity of a magnetic field. A fine bismuth wire is wound into the form of a flat spiral, 1.5 cm. diameter, and attached to a long ebonite handle which enables the spiral to be held in any part of a magnetic field.

The Action of Light on Resistance.—Mr. Willoughby Smith discovered that *selenium*, an element resembling sulphur in its chemical properties, is acted upon by light so that it becomes a better conductor of electricity than it is in the dark. This property was applied, in Prof. Graham Bell's *photophone*, to the transmission of speech by light. The transmitter consisted of a thin silvered glass diaphragm clamped round the circumference like the diaphragm of an ordinary telephone; and a beam of light was reflected from the diaphragm to a distant parabolic mirror, at the focus of which was fixed a *selenium cell* joined in series with a battery and telephone-receiver. On exposing the diaphragm to air-waves, the variation in the light reflected to the parabolic mirror caused instantaneous variation in the resistance of the selenium, thus giving rise to corresponding vibrations of the diaphragm in the telephone-receiver. Sensitiveness of the receiver demands a considerable and thin surface of selenium to be exposed to the light; at the same time, since the specific resistance of selenium is about 10^{10} times that of copper, the length of the element included in the circuit must be small. The selenium cell, proposed by Mr. Shelford Bidwell, consists of a cylinder of unglazed porcelain round which two separate wires are wound in parallel screw threads cut in the surface of the cylinder; the whole surface is then covered with a thin layer of selenium, which is 'crystallised' by melting the selenium and allowing it to cool slowly. The cell is permanently enclosed within an exhausted glass bulb. The action of light in altering the conductivity is supposed to arise from a modification of the crystalline condition of the metal when exposed to light.

Resistance Coils.—For the comparison of resistance it is necessary that other resistances of known magnitude are available. If only a moderate degree of accuracy is desired, sets of coils are generally used which are so arranged that they may be used in any desired combination; such an arrangement is known as a *resistance-box*. Each coil consists of silk-covered wire wound on a brass or copper cylinder previously covered with varnished paper, the wire being doubled-back upon itself before winding, so as to avoid the effects of self-induction (p. 417). The wire is usually of a material having a low temperature-coefficient, *e.g.* manganin. Each layer of wire is coated with melted paraffin-wax, so as to fix the turns, to insure good insulation, and to protect the coil from damp. Fig. 214 represents one method of arranging the coils; the free ends of each coil are soldered to two long brass screws, which are connected to adjacent brass-blocks *b* on the outside of the cover of the box, and the space between these blocks is bored conically and filled with a conical brass plug fitted with an ebonite handle *p*. When the plug is inserted any current passing along the brass blocks is conveyed through the plug, which offers a negligible resistance, and scarcely any current passes through the coil, but when the plug is removed all the current has to pass through the resistance coil beneath. The coils are adjusted so that their resistances are multiples of the *ohm*.

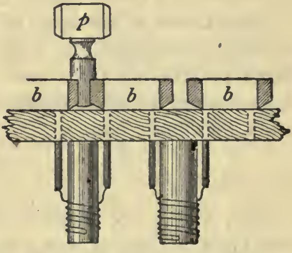

FIG. 214.—Construction of a Resistance-Box.

When great accuracy is required so-called 'standard' resistances are used. Fig. 215 represents the form of standard adopted by the German National Physical Laboratory. The coil consists of silk-covered manganin wire coated with shellac, and wound round a hollow brass cylinder which is supported from the vulcanite lid of a metal vessel perforated at the bottom. The ends of the wire are attached to stout copper rods, the other ends of which are amalgamated and dip into mercury cups. When in use the coil is immersed in an oil bath, which enables the temperature to be kept constant; the temperature is observed by inserting a thermometer through the vulcanite lid.

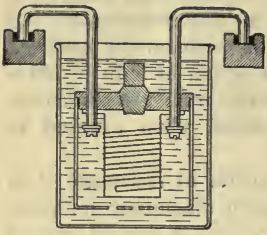

FIG. 215.

Resistance of Systems of Conductors.—(i) IN SERIES.—If several conductors are joined together in series so that the same current traverses each conductor, then if R and E are respectively the total resistance and P.D. between the extreme ends, by Ohm's Law,

$$R=\frac{E}{C}.$$

If r_1 and e_1 are respectively the resistance and P.D. between the ends of one of the conductors, then $r_1=e_1/C$; similarly $r_2=e_2/C$, etc. Hence

$$\frac{1}{C}(e_1+e_2+\ldots)=r_1+r_2+\ldots.$$

But $e_1+e_2+\ldots$ is equal to E, hence

$$R=\frac{E}{C}=r_1+r_2+\ldots,$$

or, **the total resistance of a number of conductors joined in series is equal to the sum of the resistances of the separate conductors.**

(ii) IN PARALLEL.—If several conductors connect together the same points (Fig. 216), the total current C divides between the conductors, so that

$$C=c_1+c_2+\ldots.$$

If E and R are respectively the P.D. and resistance between A and B, then $C = E/R$; also $c_1 = E/r_1$, $c_2 = E/r_2$, etc.

Hence $$\frac{E}{R} = E\left(\frac{1}{r_1} + \frac{1}{r_2} + \ldots\right),$$

$$\therefore \quad \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots,$$

or, the reciprocal of the total resistance is equal to the sum of the reciprocals of the resistances of the separate conductors.

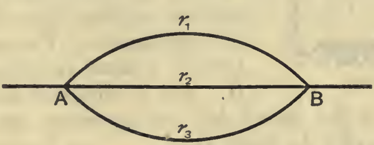

FIG. 216.

By means of this result we may readily prove that the resistance of a wire varies *inversely as the cross-section.* For, let two wires of the same length and cross-section be joined together in parallel; if the resistance of each is r, then $\frac{1}{R} = \frac{2}{r}$, or $R = \frac{r}{2}$, and the conditions would be exactly the same if the two wires were merged into a single wire of the same length, and having a cross-section *twice* as great as that of either of the original wires.

When conductors are connected in parallel, the strength of current in each branch may be expressed in terms of the total current C. Thus, if there are three conductors,

$$c_1 = \frac{E}{r_1},$$

and $$C = E\left(\frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3}\right).$$

Hence $$\frac{c_1}{C} = \frac{1}{r_1} \times \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2},$$

or $$c_1 = C \times \frac{r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}.$$

Similarly, $$c_2 = C \times \frac{r_1 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}, \text{ etc.}$$

If there are only two conductors in parallel, then

$$c_1 = \mathrm{C} \times \frac{r_2}{r_1 + r_2}, \text{ and } c_2 = \mathrm{C} \times \frac{r_1}{r_1 + r_2}.$$

Shunts.—In measuring strong currents with a sensitive galvanometer, it is often necessary to connect a resistance (s, Fig. 217) in parallel with the coils of the galvanometer, so as to allow only a known fraction of the current to traverse the instrument. The resistance s is termed a **shunt** to the galvanometer. If s and g represent respectively the resistances of the shunt and galvanometer, and if c is the current traversing the latter, then

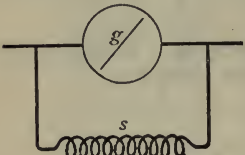

FIG. 217.

$$c = \mathrm{C} \times \frac{s}{g+s}.$$

In order that the ratio c/C may equal $1/n$, the values of s and g must be adjusted so that $s/(g+s) = 1/n$, or $ns = g + s$, or

$$s = \frac{1}{n-1} \cdot g.$$

Hence, when $\frac{1}{n} = \frac{1}{10}$, $s = \frac{1}{9} \times g$,

" $\frac{1}{n} = \frac{1}{100}$, $s = \frac{1}{99} \times g$, and so on.

EXAMPLE.—A galvanometer of 100 ohms resistance requires a shunt in order to reduce the current traversing the coils to one-twentieth of the original amount. What must be the resistance of the shunt? Also, what additional external resistance must be added so that the total resistance remains unchanged?

$$s = \frac{1}{n-1} \times g = \frac{100}{19} = 5.263 \text{ ohms}.$$

The combined resistance of the galvanometer and shunt is

$$\frac{gs}{g+s} = \frac{526.3}{105.263} = 5 \text{ ohms}.$$

Hence, additional resistance required is (100 − 5) or 95 ohms.

The Ayrton Mather Universal Shunt.—The ordinary type of shunt-box, used with a galvanometer for the comparison of currents, consists of a series of coils having resistances equal to $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th part of that of the galvanometer, and the shunt can only be used with a galvanometer of one resistance; also, in using such a shunt, the position of the mains conducting the current to and from the shunted galvanometer is fixed, and the resistance of the shunt is varied. The Ayrton Mather shunt may be used with *any* galvanometer for the purpose of comparing current strengths, and the important feature of the method is that the position of the mains is altered.

Fig. 218 represents the construction of the shunt-box. The galvanometer is connected permanently to the points K and L, and the main current is conveyed to and from the apparatus by the conductors M_1 and M_2. The ends of the shunt coil AE terminates in brass blocks on the top of the shunt-box; and points B, C, and D, of the shunt wire, are connected to other brass blocks and adjusted so that the resistances of the lengths AD, AC, and AB are equal to $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th part respectively of that of the total length AE.

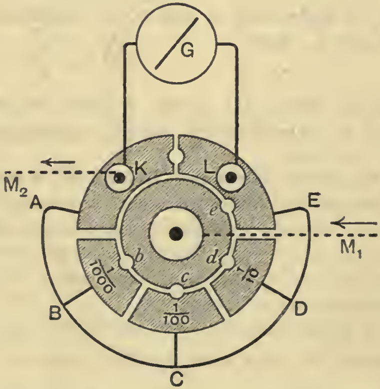

Fig. 218.—The Ayrton Mather Universal Shunt.

Suppose a current C_1 to traverse the main circuit; then, if the plug is inserted at *e*, the current g_1 traversing the galvanometer is

$$g_1 = \frac{r}{G+r} C_1$$

where G and r are the resistances of the galvanometer and of the shunt AE respectively.

If the plug is inserted at *d*, and if the resistance AD is equal to $\frac{r}{n}$, the main current divides at D between two branches—the resistance of one branch being $\frac{(n-1)r}{n}+G$, and that of the other being $\frac{r}{n}$. Hence, if the main current is now increased to C_2, the current g_2 traversing the galvanometer is now

$$g_2 = \frac{\frac{r}{n}}{\frac{r}{n}+\frac{(n-1)}{n}r+G}C_2 = \frac{\frac{r}{n}}{r+G}C_2;$$

$$\therefore \quad \frac{C_2}{C_1} = \frac{G+r}{\frac{r}{n}} \cdot g_2 \times \frac{r}{G+r} \cdot \frac{1}{g_1}$$

$$= n \cdot \frac{g_2}{g_1}.$$

If the deflections, d_1 and d_2, of the galvanometer are proportional to g_1 and g_2, then

$$\frac{C_2}{C_1} = n \cdot \frac{d_2}{d_1},$$

or, *the ratio of the current strengths are proportional to the deflections, and independent of the relative resistance of the galvanometer to that of the shunt.*

It is also evident that, in any case, if the resistance of the shunt is $1/n$th part of the total shunt resistance, the current traversing the galvanometer is $1/n$th part of that which traversed it when the total shunt was used, *providing that the same current is maintained in the main circuit*; and this fraction is independent of the actual resistances of the galvanometer and shunt.

A simple calculation will prove that when $r = 10G$, the total current remains the same when the shunt resistance is changed from r to $\frac{r}{10}$.

Kirchhoff's Laws.—The distribution of current through a network of conductors may be determined by the aid of Kirchhoff's Laws.

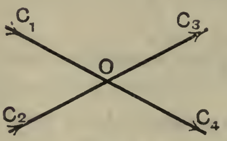

FIG. 219.

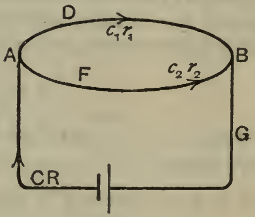

FIG. 220.

LAW 1.—**The algebraic sum of the currents meeting at any point of a network is zero.** This is a direct deduction from the fact that at any point O (Fig. 219) there is no tendency for electricity to accumulate, and that the sum of the currents approaching O is equal to the sum of the currents leaving O, or

$$c_1 + c_2 - c_3 - c_4 = 0.$$

LAW 2.—**In any mesh of a network the sum of the electromotive forces is equal to the sum of the products of the resistances into the respective currents of the separate parts.** As an application of this law consider the case of a divided circuit (Fig. 220) in which the currents are C, c_1 and c_2, and the resistances are R, r_1 and r_2. Then

at point A, $C - c_1 - c_2 = 0$ (by Law 1) ....(1)
in mesh BGD, $E = CR + c_1 r_1$ (by Law 2)..................(2)
in mesh BGF, $E = CR + c_2 r_2$ (by Law 2)..................(3)

From (2) and (3), $c_1 r_1 = c_2 r_2$.

Hence, from (1), $C - c_1 - \dfrac{c_1 r_1}{r_2} = 0$,

or $c_1 = C \times \dfrac{r_2}{r_1 + r_2}$.

This agrees with the result obtained on p. 299.

Fall of Potential along a Conductor.—During the maintenance of a steady current along a homogeneous conductor of uniform cross-section, one end of the conductor is at a higher potential than the other, and intermediate points are at intermediate potentials. Since each unit length offers the same resistance, the fall of potential for each unit length will be the same, or, in other words, the fall of potential will be uniform, and can be represented graphically by a straight line. Thus, the sloping line VB (Fig. 221) will represent the

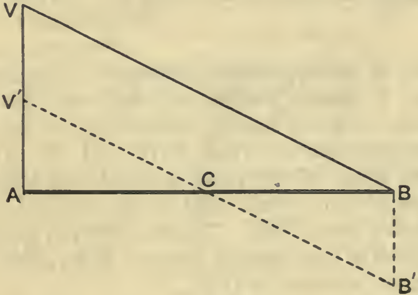

FIG. 221.

fall of potential along the homogeneous conductor AB, assuming that the potential of the point B is zero; the potential at any intermediate point is represented by the vertical distance of the line VB above the point. If a point C midway along AB is earth-connected, while A and B are insulated, then the potential-slope is represented by the line V′B′, and all points between C and B will have *negative* potential. At the same time the *total* P.D. between A and B has not been altered, and therefore the current remains the same, the only alteration being in the absolute values of the potentials at individual points.

Since the resistance of a conductor of uniform cross-section is proportional to its length, the diagram can be drawn so that the length of the line AB is a measure of the resistance. If the vertical lines represent the P.D. to the same scale, then, since

$$C = E/R = \tan\theta,$$

the current-strength is represented by the tangent of the angle of slope.

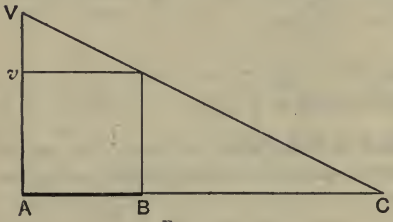

FIG. 222.

In the case of heterogeneous conductors connected in-series the diagram is equally simple, providing that the lengths on the horizontal axis are strictly proportional to the resistances of the component parts. Thus, if AB and BC (Fig. 222) represent two wires of the same material and length, but the cross-section of AB is *twice* that of BC, the resistance of AB will be one-half that of BC. The P.D. between the ends of AB will be one-half of that between the ends of BC; this is indicated in the diagram, where $Av = 2Vv$.

Finally, when the diagram is required to indicate the fall of potential along the entire circuit, including one or more voltaic cells, allowance has to be made for the resistance of the liquids in the cells. In Fig. 223 the lines AB and BCC′A represent the resistances respectively of a simple voltaic cell and of a wire connecting the terminals. The thick line represents the potential diagram for the cell on open circuit, and the total E.M.F. of the cell is therefore represented by the height of the line BV. When the circuit is closed, and if we assume that the zinc plate is at zero potential, the dotted line will represent the potential at various points in the circuit, and a current will flow in the direction of the arrow. It is important to notice that the P.D. at the terminals is only equal to the height of the line BV when the cell is on open circuit, and that when the circuit is closed this P.D. is reduced to BV′—the diminution VV′ representing that portion of the total E.M.F. which is required to transmit the current through the cell. If the lines representing resistance and P.D. are drawn to the same scale, the current strength is then represented by tan θ. And

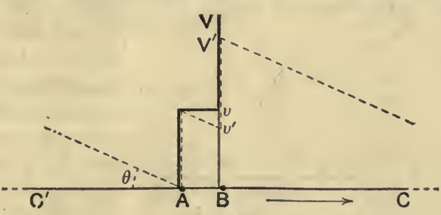

FIG. 223.

$$VV' = vv' = r \tan \theta = rC,$$

where r is the internal resistance of the cell. This result agrees with the statements of the following paragraph.

In the simplest expression for Ohm's Law, $C = E/R$, the symbol R is used to denote the *total* resistance in the circuit. But this equation may be modified so as to distinguish between the internal resistance of the battery and the external resistance; and if these are denoted by the symbols r and R respectively, then $C = E/(R + r)$, or

$$E = CR + Cr.$$

By means of this equation the total E.M.F. is divided into two portions, a part equal to CR being used in the external circuit, and the remainder Cr in transmitting the current through the battery.

EXAMPLE.—The E.M.F. of a battery is 20 volts. When the poles are connected by a wire, a current of 2 amperes is produced and the P.D. between the battery terminals falls to 15 volts. Find the resistance of the wire and the internal resistance of the battery.

Since $E=20$, and $C=2$,

$$(R+r)=\frac{20}{2}=10.$$

Also $CR=15.$

Hence $$R=\frac{15}{2}=7.5 \text{ ohms},$$

and $$r=10-7.5=2.5 \text{ ohms}.$$

The fall of potential along two wires B_1C_1 and B_2C_2, joined in-parallel, is represented in Fig. 224. The current at B_1 and B_2 divides into two parts which rejoin at C_1 and C_2; let c_1 and c_2 represent the current-strengths along B_1C_1 and B_2C_2 respectively. The potentials at B_1 and B_2 are equal; so also at C_1 and C_2. If a terminal of a galvanometer be connected to any point A on B_1C_1, and the other terminal to a point D on B_2C_2, then no currents will traverse the galvanometer if $Av=Dv$.

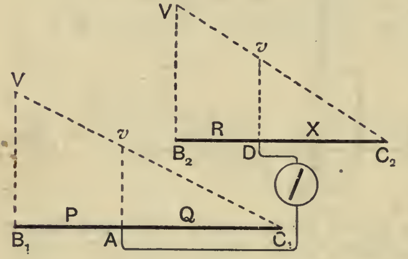

FIG. 224.

Let V_B, V_C, V_A and V_D be the potentials at the points B, C, A and D respectively; and P, Q, R and X the resistances of B_1A, AC_1, B_2D and DC_2 respectively; then, by Ohm's law,

$$V_{B_1}-V_A=c_1P,$$

$$V_{B_2}-V_D=c_2R.$$

But $$V_{B_1}-V_A=V_{B_2}-V_D;$$

therefore $$c_1P=c_2R; \text{ or } c_1/c_2=R/P.$$

Similarly $$c_1Q=c_2X; \text{ or } c_1/c_2=X/Q.$$

Hence $$X/Q=R/P,$$

or $$X=R\times Q/P.$$

This shows that when we know the magnitude of the resistance R, and the ratio of the resistances Q and P, we can calculate that of X. This principle, which is applied in the **Wheatstone**

Bridge, is represented more clearly in Fig. 225. If the ratio P/Q is adjusted so that there is no deflection in the galvanometer, then $P/Q=R/X$. As this equation may also be written $P/R=Q/X$, the positions of the battery and galvanometer may be interchanged, and there will still be no deflection ; *in such a case the battery and galvanometer are said to be* **conjugate** *to each other.*

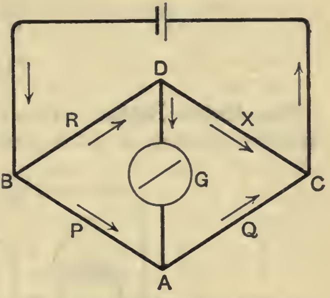

FIG. 225.—The Wheatstone Bridge.

Application of Kirchhoff's Laws to the Wheatstone Net.—The current traversing the galvanometer circuit of a Wheatstone net may be calculated with the aid of Kirchhoff's laws. If P, Q, R, X, and G represent the resistances of the several branches of the net, and if p, q, r, x, and g represent the currents traversing the individual branches, then, if we assume the direction of the currents to be represented by the arrows,

in mesh BDA, $0=rR+gG-pP$,
and, in mesh CDA, $0=xX-qQ-gG$.
But, at point D, $x=r-g$,
and, at point A, $q=p+g$.
Hence, $0=rR+gG-pP$, ...(i)
and $0=(r-g)X-(p+g)Q-gG$
$=rX-pQ-g(X+Q+G)$. ......(ii)

From equations (i) and (ii),

$$\frac{g}{p+r}=\frac{PX-QR}{G(P+Q+R+X)+(P+R)(Q+X)}.$$

This is the fraction of the total current which passes through the galvanometer ; and it is evidently equal to zero when $PX=QR$, or when $P/Q=R/X$.

If we disregard the small effect of G upon the total resistance of the net, then the total current may be written

$$p+r=E\Big/\left(B+\frac{(P+Q)(R+X)}{P+Q+R+X}\right)$$

where E and B are the E.M.F. of the battery and the total resistance between B and C through the battery. Hence, the current through the galvanometer is

$$g=\frac{PX-QR}{G(P+Q+R+X)+(P+R)(Q+X)}\times\frac{E(P+Q+R+X)}{B(P+Q+R+X)+(P+Q)(R+X)}.$$

The Metre Bridge.—The metre bridge (Fig. 226) is the simplest application of the previous paragraph to the measurement of an unknown resistance. It consists of a uniform German-silver[1] wire, one metre long, stretched alongside a metre scale, and with its ends soldered to stout copper strips E and F. There are four gaps between copper strips fixed along the other edge of the board, but when the bridge is used in the simplest manner the gaps U and V are closed by copper strips held in position by binding screws. The resistances R and X which are to be compared are attached to binding screws as shown. The galvanometer circuit is closed by depressing the knob A, which makes contact between a knife-edge and the bridge wire. Having found by trial the position of A which gives no deflection, then, since the wire is uniform and the resistance of any portion is proportional to its length, the ratio R/X will be equal to the ratio of the *lengths* P and Q.

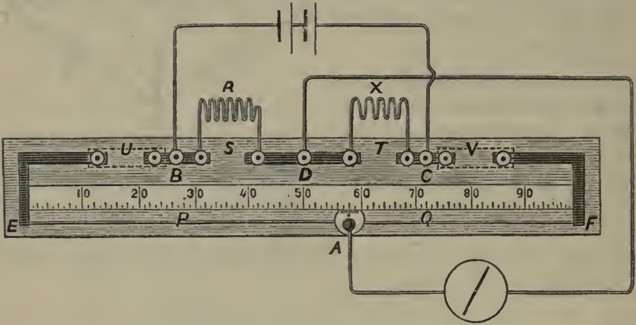

FIG. 226.—The Metre Bridge.

It is usual to insert a key or commutator in the battery circuit; and the current should be established before closing the galvanometer circuit, so as to prevent deflection by the transient current due to the self-induction of the coils.

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[1] Iridio-platinum is preferable, and is now frequently used.

The relative positions of the battery and of the galvanometer in Fig. 226 are reversed sometimes so as to avoid disturbance of the galvanometer by thermo-electric effects at the contact A. But, if the apparatus is arranged as in Fig. 226, an obvious method of eliminating thermo-electric effects is as follows: determine the approximate position of A, open the battery circuit, keeping A depressed, and observe the deflection; then adjust the position of A until there is no permanent change of deflection when the battery circuit is closed. Thermo-electric effects which may be produced at the ends of the bridge-wire are minimised by protecting the ends with cotton-wool.

The accuracy of the comparison is much increased by inserting resistances r_1 and r_2 in the gaps U and V, the magnitudes of these resistances in terms of a division of the bridge-wire being known. When a balance is obtained then $R/X=(r_1+P)/(r_2+Q)$. In Carey Foster's method of using the bridge the resistances R and X to be compared, and which are assumed to be nearly equal, are inserted in the gaps U and V, and two other resistances r_1 and r_2 approximately equal to R or X are placed in the gaps S and T. If x_1 is the bridge reading, then

$$\frac{r_1}{r_2}=\frac{R+x_1\rho}{X+(100-x_1)\rho},$$

where ρ is the resistance of one division of the bridge wire.

R and X are now reversed and a balance again obtained, then

$$\frac{r_1}{r_2}=\frac{X+x_2\rho}{R+(100-x_2)\rho}.$$

From these equations, $R-X=\rho(x_2-x_1)$.

A more compact form of the Wheatstone Bridge is known as the **Post Office Box** (Fig. 227), which consists of a number of resistance coils arranged so as to form the three resistance arms P, Q, and R of Fig. 225. (Corresponding points in Figs. 225 and 227 are denoted by the same letters.) Each of the arms BA and BD, known as 'ratio arms,' consist of three coils the resistances of which are 10, 100, and 1000 ohms respectively. The coils between *a′* and C constitute the third arm of the bridge, and their resistances are arranged so that any whole number of ohms, from 1 up to 10,000, is available. The point A is permanently connected to *a′* by a thick wire passing under the cover of the box; another wire connects A to the stud of the key *a*, to which one terminal of the galvanometer is connected. The point B is similarly connected to the stud of the battery key *b*. These permanent connecting wires are indicated by the white lines drawn on the cover of the box. The battery terminals are connected to *b* and C; and the galvanometer to *a* and D. The resistance X to be measured is joined to D and C. In making the measurement, equal resistances are inserted in the ratio arms, and the resistance Q is adjusted until a minimum deflection of the galvanometer is observed; this gives the resistance of X to the nearest whole number. The ratio P/R is now made equal to 10, and the resistance of Q again adjusted as nearly as possible; Q is now approximately 10 times as great as X. Finally the ratio P/R is increased to 100, and Q again adjusted; X is then equal to Q/100.

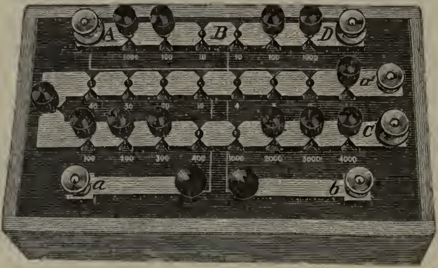

Fig. 227.—A Post-Office Box.

Platinum Resistance Thermometers.—The accuracy with which variations in the resistance of a wire can be measured is applied now successfully to the measurement of both high and low temperatures for which a mercury thermometer is inapplicable and for which an air thermometer[1] is inconvenient. The methods of applying this principle are largely due to Prof. H. L. Callendar.[2] The resistance consists of a coil X (Fig. 228) of pure platinum

[1] See Edser's *Heat* (Macmillan), p. 111.

[2] *Phil. Trans.*, clxxviii., 1887.

wire, wound on a mica frame and enclosed in a porcelain tube. For extremely high temperatures it is found necessary to use a frame of biscuit-porcelain instead of mica. The ends of the coil are welded to two thick platinum wires joined to the terminals T, which are connected by long copper wires to the points A and B in the Wheatstone bridge. Alterations in the resistance of the leads due to changes in temperature are compensated by inserting a duplicate pair of leads C, of the same resistance, in the opposite arm of the bridge at EF; these duplicate leads are termed the *compensator*. The two arms of the bridge are separated by a straight bridge-wire FB, 50 cms. long, and adjusted by means of a shunt so that the *equivalent* resistance of 1 cm. length of FB is some exact submultiple—usually $\frac{1}{10}$, $\frac{1}{25}$, or $\frac{1}{50}$—of the lowest resistance in the set of coils R.

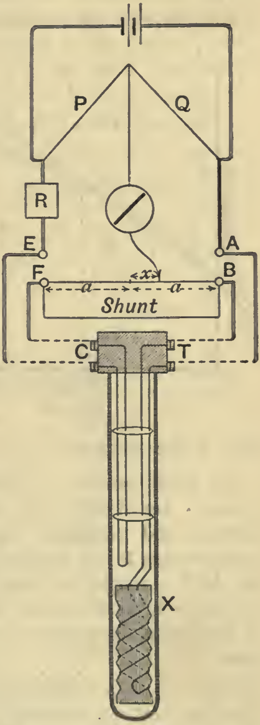

FIG. 228.—A Platinum Resistance Thermometer.

In taking an observation, suppose that a balance is obtained at a distance x from the centre of the bridge-wire. Then, if x represents the resistance of the length of wire between the centre of the bridge and the point of contact, if $2a$ represents the total resistance of the bridge-wire, and if P = Q, then

$$R+C+a+x=X+T+a-x.$$

But
$$C=T$$

$$\therefore X=R+2x,$$

where x is positive or negative according as it is to the right or left of the centre. The quantity $2x$ represents the *equivalent* resistance, mentioned above, and is equal to twice the *actual* resistance.

The constants of any platinum thermometer are determined by measuring its resistance at three different temperatures, viz.: (i) the boiling point of water, (ii) the melting point of ice, and (iii) the boiling point of sulphur (which has been accurately determined by means of an air thermometer). From these three observations the constants in the parabolic formula

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

are obtained. Any other temperature can then be calculated from this equation if the resistance R_t is measured.

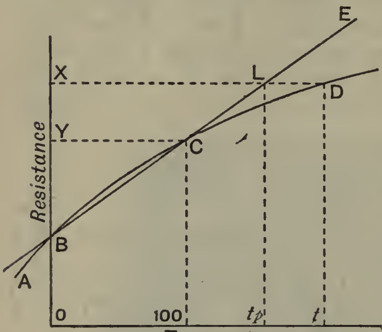

FIG. 229.

The usual method of determining an unknown temperature is as follows. Let ABCD (Fig. 229) represent the relation between temperature and resistance for pure platinum, the points B and C corresponding to the temperatures 0° C. and 100° C. Draw the straight line BCE through the points B and C. Let OX be the resistance of the thermometer at an unknown temperature t° C. Draw XL parallel to the axis of temperature and Lt_p parallel to the axis of resistance. Then

$$XL/YC = XB/YB,$$

or

$$t_p/100 = (R_t - R_0)/(R_{100} - R_0),$$

or

$$t_p = \frac{R_t - R_0}{R_{100} - R_0} \times 100. \qquad \text{.............................(1)}$$

But this incorrectly assumes that the straight line BLE represents the relation between resistance and temperature. The observed temperature is really represented by the line XD and is slightly higher than t_p. The difference LD is calculated thus: Since $R_t = R_0(1 + \alpha t + \beta t^2)$ and $R_{100} = R_0(1 + 100\alpha + 100^2\beta)$, then, by substituting in equation (1)

$$t_p = \frac{R_0(\alpha t + \beta t^2)}{R_0(100\alpha + 100^2\beta)} \times 100 = \frac{\alpha t + \beta t^2}{\alpha + 10^2\beta}.$$

If the correct temperature is denoted by t, then

$$t - t_p = t - \frac{\alpha t + \beta t^2}{\alpha + 10^2\beta} = \frac{-\beta t(t - 100)}{\alpha + 10^2\beta} = \mathrm{D}t(t - 100).$$

The value of the constant D is calculated from the known values of α and β; in the case of pure platinum it is equal to $+1.5 \times 10^{-4}$, but it varies slightly according to the past history of the wire. Sir James Dewar[1] has recently shown that the constant D has a value $+2.5 \times 10^{-4}$ if calculated from the resistance at 100° C., 0° C., and $-182°.5$ C.

Grouping of Cells.—In any experiment in which voltaic cells are used, the strength of current depends upon the total E.M.F. and upon the external and internal resistances in the circuit, and the best arrangement in any given case will be determined by the relative magnitudes of these quantities and by the strength of current required. Cells may be connected either (i) *in series*, (ii) *in parallel*, or (iii) *in series-parallel.*

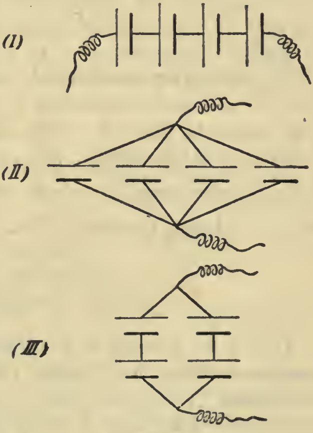

FIG. 230.

(i) **Cells in series.**—Fig. 230 (i) represents four cells connected in series, the negative terminal of one cell being connected to the positive of the next, and so on. If each cell has an E.M.F. of E volts and an internal resistance r ohms, the E.M.F. and resistance of a battery of n cells will be nE volts and nr ohms respectively. If the circuit is completed through an external resistance of R ohms, then, by Ohm's Law,

$$\mathrm{C} = \frac{n\mathrm{E}}{\mathrm{R} + nr}.$$

(ii) **Cells in parallel** (Fig. 230, ii).—The E.M.F. of the battery will now be the same as that of a single cell, but the resistance of the battery will be $\frac{1}{4}$th of that of one cell, for the battery is really equivalent to one large cell with plates four times as large as those of a single cell. If n cells are connected in parallel, then, by Ohm's Law,

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[1] *Proc. Roy. Soc.*, 1904.

$$C = \frac{E}{R + \frac{r}{n}}.$$

From the above formulae it is evident that, *when r is great compared with R*, no advantage is derived by increasing the number of cells in series, but that it is an advantage to increase the number of cells in parallel. On the other hand, *when R is great compared with r*, an increase in the number of cells in parallel does not appreciably increase the current, but the current is approximately proportional to the number of cells connected in series.

(iii) **Cells in series-parallel** (Fig. 230, iii).—If N cells are divided into m sets, each containing n cells in series, the E.M.F. will be equivalent to that of n cells in series, each having an internal resistance of r/m ohms. The E.M.F. and the internal resistance of the battery will therefore be nE volts and $\frac{nr}{m}$ ohms respectively. Hence, by Ohm's Law,

$$C = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr}.$$

It can be proved in a simple manner that **the current is a maximum when the internal resistance is equal to the external resistance.** For, since mnE is a constant, C will be a maximum when $(mR + nr)$ has a minimum value. Now $(mR + nr)$ may be written

$$(\sqrt{m}R - \sqrt{nr})^2 + 2\sqrt{mnRr},$$

and this has a minimum value when

$$\sqrt{m}R - \sqrt{nr} = 0,$$

or

$$mR = nr,$$

or

$$R = \frac{nr}{m}.$$

This arrangement of cells is however not the most *economical*, since, if the external and internal resistances are equal, just as much energy is wasted inside the cells as is utilised in the external circuit. (See *Efficiency of Cells*, p. 351.)

SUMMARY

Ohm's Law.—*In any wire at uniform temperature the current is directly proportional to the potential-difference between its ends.*

Resistance.—The *resistance* of any conductor is the ratio of the P.D. between its ends and the resultant current, or $R = E/C$. The reciprocal of the resistance is termed the *conductance.*

A conductor has a resistance of one *ohm* when a P.D. of one *volt* between its ends creates in it a current of one *ampere.*

The ohm, volt, and ampere are the 'practical' units of resistance, potential-difference, and current respectively. Also, 1 ohm $= 10^9$ C.G.S. units, 1 volt $= 10^8$ C.G.S. units, and 1 ampere $= 10^{-1}$ C.G.S. units.

The **Specific Resistance** of a conductor is *the resistance of a uniform mass of the material* 1 *cm. long and* 1 *sq. cm. cross-section.* The specific resistance varies with the temperature, and its value at any temperature $t°$ C. is expressed by the equation

$$R_t = R_0(1 + \alpha t),$$

where α is the *temperature coefficient* of the material. This equation is only approximately correct, and the effect of changes of temperature is more accurately expressed by the equation

$$R_t = R_0(1 + \alpha t + \beta t^2),$$

where α and β are constants for the given material.

Resistance of Systems of Conductors.—(i) *In series.* $R = r_1 + r_2 + \dots.$

(ii) *In parallel.* $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \dots.$

Shunts.—If in any circuit only a portion of the current is required to pass through an instrument included in the circuit, the terminals of the instrument may be joined by a wire, which is termed a *shunt* to the instrument. In order to reduce the current through the instrument to $1/n$th part of the total current, the relationship between the resistances of the shunt s and that of the instrument g is given by the equation

$$s = \frac{1}{n - 1} \cdot g.$$

Internal and external resistance in a circuit.—In the case of a simple circuit, Ohm's Law may be expressed $C = E/(R + r)$ or

$$E = CR + Cr,$$

where R and r are the external and internal resistances respectively. The product Cr represents the E.M.F. used in sending a current C through the internal resistance r.

The **Wheatstone Bridge** is a method of measuring resistance based upon the fall of potential along two conductors connected in parallel. The *metre bridge* and the *Post Office box* are special types of the Wheatstone Bridge.

Platinum resistance thermometers are applications of the principle that the resistance of a conductor increases when its temperature is raised.

Grouping of Cells.—Cells, N in number and each having an E.M.F. and internal resistance equal to E and r respectively, may be arranged either

(i) *in series*, when $C=NE/(R+Nr)$,

(ii) *in parallel*, when $C=E\Big/\left(R+\dfrac{r}{N}\right)$, or

(iii) *in m rows of n cells each*, when $C=nE\Big/\left(R+\dfrac{nr}{m}\right)$.

QUESTIONS ON CHAPTER XVII

1. State Ohm's law, and explain its meaning as carefully as you can. Apply the law to prove that the conductivity of any number of coils placed in parallel is equal to the sum of the conductivities of the separate coils.

2. State the law of Ohm and apply it to calculate how many Grove cells, each having an electromotive force of 1.8 volts and an internal resistance of 0.07 ohm, will be required to send a current of 10 amperes through a resistance of 2.2 ohms.

3. Twelve wires of equal length and electrical resistance are arranged to form the edges of a cube, and a current of 4 centiamperes is led into the cube at one corner and out at the opposite corner. Find the difference of potential between these two corners and the effective resistance of the framework between them, the resistance of each wire being 6 ohms.

4. Four points A, B, C, D, are connected together as follows: A to B, B to C, C to D, D to A, each by a wire of 1 ohm resistance; A to C, B to D, each by a cell of 1 volt E.M.F. and 2 ohms resistance. Determine the current flowing through each of the cells.

5. Suppose the electric resistance of a human body, measured through the arms from hand to hand was 7,500 ohms, and that the hands grasped the terminals of 30 Daniell's cells joined in series, how much current would pass through the body? (1 Daniell cell = 1.1 volt, and internal

resistance = 0.3 ohm). If the various tissues have different conducting powers describe in general terms how the current would distribute itself between them.

6. A battery of 6 cells in series, an ammeter and a key are arranged in circuit. The poles of the battery are also connected to the terminals of a high resistance voltmeter. The voltmeter registers 12.4 volts *before*, and 10.6 volts *after*, the ammeter circuit is completed, the ammeter indicating 15 amperes. What is the resistance of each cell of the battery?

7. A circuit is formed of six similar cells in series and a wire of 10 ohms resistance. The E.M.F. of each cell is 1 volt and its internal resistance 5 ohms. Determine the difference of potential between the positive and negative poles of any one of the cells.

8. The terminals of a battery of E.M.F. 4 volts and resistance 3 ohms are connected by a wire of resistance 9 ohms. By how much is their difference of potential altered thereby?

9. The positive poles A and B of a Grove and a Daniell cell are joined by a wire of 0.3 ohm resistance, and the negative poles C and D by a wire of 0.5 ohm. What is the difference of potential between the middle points of AB and CD?

Grove cell:—Int. resistance = 0.2 ohm, E.M.F. = 1.8 volt.

Daniell cell:—Int. resistance = 0.4 ohm, E.M.F. = 1.1 volt.

10. The terminals of a battery of E.M.F. 4 volts and resistance $1\frac{1}{2}$ ohms are connected to those of a battery, of E.M.F. 3 volts and resistance $\frac{1}{4}$ ohm, by wires of the resistances 1 and 6 ohms respectively, so that both batteries act in the same direction. Show that if a third wire be placed so as to join the middle points of these wires no current passes through it.

11. A battery of 12 equal cells in series screwed up in a box, being suspected of having some of the cells wrongly connected, is put into circuit with a galvanometer and two cells similar to the others. Currents in the ratio of 3 to 2 are obtained according as the introduced cells are arranged so as to work with or against the battery. What is the state of the battery? Give reasons for your answer.

12. Two cells, A and B (E.M.F. and internal resistance of each are one volt and one ohm respectively), are arranged in series. The positive and negative poles of this battery are connected with the positive and negative poles respectively of a third cell C, exactly like A and B, the connecting wires having negligible resistance. What is the current in the circuit, and what is the potential difference between the positive and negative poles of the cell C?

13. A circuit is made up of (1) a battery with terminals A, B, its resistance being 3 ohms, and its E.M.F. 2.7 volts; (2) a wire BC, of resistance 1.5 ohms; (3) two wires in parallel circuit, CDF, CEF, with respective resistances 3 and 7 ohms; (4) a wire FA, of resistance 1.5 ohms. The middle point of the last wire is put to earth; find the potential at the points A, B, C, F.

14. A compass needle is placed at the centre of two concentric circles which are in the same vertical plane and are made of wires similar in all respects except that the outer is copper, the inner German silver. The wires are connected in multiple arc, but so that the currents which flow through them circulate in opposite directions. What must be the ratio of the diameters of the circles that no effect may be produced on the needle? (*N.B.*—Assume the conductivity of copper to be twelve times that of German silver.)

15. It is found that a current in a given circuit is lowered by the inclusion in it of a certain cell. Show that even if the poles of the cell were connected by a wire of no resistance it would be unable to generate a current as large as that originally in the circuit.

16. The electrodes of a quadrant electrometer are joined to the terminals of a battery of five cells in series. In what ratio will the deflection of the needle be altered if the electrodes are *also* joined to the terminals of a battery of three cells in series similarly arranged, all the cells being alike and the connecting wires thick?

17. A galvanometer, the resistance of which is $\frac{1}{3}$ ohm, being joined up in circuit with a cell by thick copper wires, the resulting current is noted; and it is found that the current in the galvanometer is halved if, without any other change being made, the terminals of the galvanometer are joined by a wire of resistance 0.1 ohm. What is the resistance of the cell?

18. A wire is uniformly stretched until its length is trebled. Compare its resistance before and after stretching.

19. The resistance of 100 metres of copper wire (No. 24 S.W.G.; diameter = 0.0559 cm.) is 6.63 ohms at 0° C. What is the specific resistance of the copper?

20. A column of mercury 106.3 cms. long and 1 sq. millimetre cross-section has a resistance of 1 ohm at 0° C. What is its specific resistance?

21. If the specific resistance of platinum at 0° C. is 8.96 microhms, and its temperature coefficient is 0.0032, how long must a wire of No. 32 S.W.G. (diameter, 0.0274 cm.) platinum be to have a resistance of 4 ohms at 50° C.? What would be its resistance at 100° C.?

22. What length of manganin wire, cross-section 0.4 sq. millimetre, is required for a coil of 10 ohms resistance at 15° C.? (specific resistance

at 0° C. = 41 microhms, temp. coeff. = 0.000025). What percentage error is introduced by assuming that manganin has no temperature coefficient?

23. An accumulator cell consists of 5 positive and 6 negative plates, each 30 cms. long and 20 cms. broad, placed alternately positive and negative, and separated so that the surfaces are at a uniform distance of 2 cms. apart. If the acid has a resistivity of 10^6 microhms, calculate the internal resistance of the cell.

24. An E.M.F. of 110 volts is maintained at the ends of a circuit which consists of 50 ohms of iron and 60 ohms of copper (both measured at 0° C.) connected in series. If the temperature coefficients of iron and copper are 0.007 and 0.0043 respectively, plot a curve of current as the temperature of the circuit rises to 200° C.

25. If a shunt is to be applied to a galvanometer of 20 ohms resistance so that only 1% of the total current passes through the galvanometer, what must be the resistance of the shunt? If the resistance of the remainder of the circuit is 20 ohms, to what extent is the main current affected? What additional resistance must be inserted in the main circuit so as to keep the main current constant?

26. The 'flow' and 'return' wires of a lighting circuit are laid separately underground. If each wire has an insulation resistance of 5 megohms, and if the voltage of the supply is 440 volts, what is the leakage current?

27. A telegraph line, including instruments, has a total resistance of 2000 ohms and is to be worked by Daniell cells. If the internal resistance and E.M.F. of each cell is 8 ohms and 1.07 volt respectively, how many cells will be required in order to transmit a current of 25 milliamperes?

28. The coils of a Post Office box are made of German silver and are correct at 15° C. The box is used to measure the resistance of an iron wire which is found to be 16 ohms. If the temperature of the room is 20° C., and if the temperature coefficients of German silver and of iron are 0.00027 and 0.007 respectively, calculate the resistance of the iron at 0° C.

29. A reflecting galvanometer of 250 ohms resistance is shunted with 25 ohms. A cell of negligible resistance and an E.M.F. of 1.5 volts is connected in series with the galvanometer through a resistance of 10,000 ohms. A deflection of 200 scale divisions is observed. What is the sensibility of the galvanometer?

30. If there are 18 cells in a battery, each of resistance 1.5 ohm, how can they best be arranged so as to send a strong current through an external circuit of 3.5 ohms resistance?

CHAPTER XVIII

ELECTRICAL MEASUREMENTS

The Potentiometer.—The principle of the potentiometer has been described previously in the experimental verification of Ohm's Law; but a more complete statement may be given here with advantage, since the method is applied very frequently in electric measurements.

In Fig. 230 AB represents a long wire, of uniform cross-section, joined in series with a battery E of constant E.M.F., an ammeter or galvanometer G_1, and an adjustable resistance R. If the positive terminal of another cell E_2 is connected to the point A, the positive pole of E_2 will have the same potential as A, and the P.D. between A and the free end k of a wire attached to the negative pole of E_2 will be equal to the P.D. between the terminals of E_2. If the E.M.F. of the battery E is sufficiently great, a point C may be found on AB which has the same potential as k; and if k is brought into contact with this point, the potential conditions of the circuit AR_2k remain unaltered,

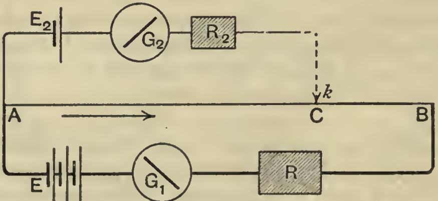

FIG. 231.—The Potentiometer.

and no current traverses this circuit. The P.D. between A and C is then equal to the P.D. between the poles of E_2; and, if the wire AB is uniform, the length AC is a measure of the E.M.F. of E_2. If contact is made to the left of C, a current will pass from *k* through E_2 to A; and if contact is made to the right of C, a current will pass through E_2 in the reverse direction. The point C can therefore be located by inserting a galvanometer G_2 in the branch circuit. A high resistance R_2 included in the same circuit serves to prevent any disturbance of the current, and therefore of the uniform fall of potential, in AB; it also protects the cell E_2 from a current of strength sufficient to cause polarisation.

Greater accuracy is obtained by increasing the length of the wire AB, which should be constructed of material having high specific resistance and low temperature coefficient (*e.g.* manganin). In the Crompton potentiometer, the wire is divided into fifteen equal lengths, of which the fifteenth only is stretched over a scale, the remaining fourteen lengths being coiled up and joined to the brass pieces of a disc contact maker. By connecting one terminal of E_2 to the rotating arm of the contact maker and the other to the sliding contact on the stretched wire any required length of the wire may be included between the points joined to E_2.

RESISTANCE OF A GALVANOMETER

Direct Method.—If a second galvanometer is available, the galvanometer of unknown resistance may be inserted in the arm X (Fig. 225) of the Wheatstone bridge, and its resistance then measured in the usual manner.

Thomson's Method.—The galvanometer is included in the arm X (Fig. 225), and a simple key is placed between A and D. If the conjugate condition is satisfied, so that $P/R = Q/X$, no current will traverse the wire AD when the key is closed, and the currents in other branches will not be altered by closing the key. The resistance Q is therefore modified until the deflection of the galvanometer remains the same whether the key is open or closed, and the resistance is then calculated from the equation $X = Q \times R/P$.

RESISTANCE OF A BATTERY.

Mance's Method.—Mance made use of the conjugate condition of the Wheatstone bridge in order to measure the resistance of a battery. The battery is inserted in the arm X (Fig. 225, p. 307), and a key is placed in the position usually occupied by the battery. The galvanometer is traversed by a current which, if Q is adjusted until $Q/X=P/R$, is the same whether the key is closed or open. The disadvantage of this method is that a current is always traversing the battery, and this tends to modify its resistance, and this current is greater when the key is closed than when open ; also the current may be so strong as to injure the galvanometer.

Sir Oliver Lodge has introduced a modification of the method, in which a condenser is joined in series with the galvanometer. The condenser is charged to a P.D. equal to that between the points A and D (Fig. 225), and the needle will only be deflected by charging or discharging currents produced by differences of potential at A and D. Hence if Q is modified until the galvanometer is unaffected by opening or closing the key, the condition $P/R=Q/X$ must hold good.

Ammeter and Voltmeter Method.—The battery B (Fig. 232) is connected in series with a variable resistance R, an ammeter A, and a key K. A voltmeter V is connected as a shunt to the battery. If E_1 is the voltmeter reading when the key is open, and if E_2 and C are the readings of the voltmeter and ammeter when the key is closed, then

$$E_1=CR+Cr,$$
$$E_2=CR,$$
$$\therefore\ r=\frac{E_1-E_2}{C}.$$

This method is specially applicable to the measurement of the internal resistance of a secondary battery.

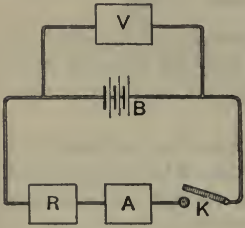

FIG. 232.

Potentiometer and Standard Resistance Method.—The values of E_1 and E_2, as defined in the previous paragraph, may be determined by means of a potentiometer, in which the fall of potential in each centimetre length is known. Also, since

$$r=\frac{E_1-E_2}{C}=\frac{E_1-E_2}{E_2}\,.\,R,$$

the ammeter may be dispensed with, if the resistance R is known ; nor is it necessary, in this case, to know the rate of fall of potential in the potentiometer wire. The wires leading to the potentiometer should be connected as near as possible to the terminals of R.

MEASUREMENT OF E.M.F. OF VOLTAIC CELLS

Comparison by the Method of Sum and Difference.—If E_1 and E_2 are the E.M.F.'s of two cells which are connected in series, and if the circuit is completed through a tangent galvanometer and a resistance R, then

$$\frac{E_1+E_2}{R+r_1+r_2}=k\tan\theta_1.$$

If one of the cells is now reversed, then

$$\frac{E_1-E_2}{R+r_1+r_2}=k\tan\theta_2.$$

Hence,

$$\frac{E_1+E_2}{E_1-E_2}=\frac{\tan\theta_1}{\tan\theta_2},$$

or,

$$\frac{E_1}{E_2}=\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_1-\tan\theta_2}.$$

Comparison by the Method of Direct Deflection.—If a sensitive mirror galvanometer of high resistance is connected in series with a voltaic cell of E.M.F. equal to E_1 and with a resistance so large that the angular deflection of the mirror is small, the deflection δ_1 is proportional to E_1. If the cell is now replaced by another cell of which the E.M.F. is E_2, the deflection δ_2 will be proportional to E_2. And we may assume that the internal resistance is so small compared with the external resistance that it may be neglected. Hence,

$$\frac{E_1}{E_2}=\frac{\delta_1}{\delta_2}.$$

Comparison by Clark's Potentiometer Method.—Since the passage of a current through a cell is liable to cause slight variation in its E.M.F., it is desirable to adopt a method in which no current is passing through the cell. This advantage is obtained in the *Potentiometer Method*, of which Fig. 231 may be taken as a diagram. One of the cells under comparison E_2 is connected in series with a high resistance R_2 and a mirror galvanometer G_2. The point of contact C is adjusted so that no current traverses G_2; the E.M.F. of E_2 is then balanced by the P.D. between A and C. E_2 is then replaced by the cell with which it is to be compared, and the observation is repeated. The E.M.F.'s are directly proportional to the observed lengths of AC.

In **Poggendorff's** method, which is a modification of the above, the wire AB is replaced by two resistance boxes, the terminals of *one* of them being joined to those of the cell E_2. While the total resistance between A and B is always kept equal to 10,000 ohms, the portion of this obtained from the box connected to E_2 is varied until no deflection is obtained in the mirror galvanometer.

The potentiometer method is particularly applicable to the measurement of small E.M.F.'s, such as are obtained by means of thermo-couples.

Method of Determining the absolute E.M.F. of a Cell.—The potentiometer method of comparing E.M.F.'s explains how the absolute E.M.F. may be determined; for, if the resistance AC (Fig. 231) is known, and if the current traversing it is measured, then, since $E = C \times R$, the P.D. between the points A and C can be calculated. A coil of manganin wire S (Fig. 233) is immersed in a beaker of oil, and its ends are joined to thick copper wires which dip into mercury cups. The cell E, of which the E.M.F. is required, is joined in series with a mirror galvanometer G and a high resistance R_2. The current in the main circuit,

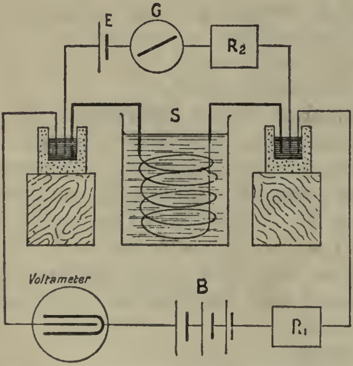

FIG. 233.—The Determination of the Absolute E.M.F. of a Cell.

which is adjusted by the variable resistance R_1 until there is no deflection in G, is measured by means of a copper- or silver-voltameter (p. 457). In accurate experiments S is a *standard* resistance, but for approximate results it is sufficient to use an ordinary coil, the resistance of which is measured by means of a Post-office Box.

MEASUREMENT OF CURRENT

Potentiometer Method.—This method is frequently adopted for the measurement of strong currents. The current traverses a low resistance R (Fig. 234) of known magnitude. The end A is connected to one end *a* of the potentiometer *ab*, and the other end B is connected through a galvanometer g_1 to a sliding contact P_1 ; when the position of P_1 is adjusted so that there is no deflection in g_1, the length aP_1 is a measure of the P.D. between A and B, E is a standard cell of known E.M.F., and is connected to the potentiometer through a galvanometer g_2 ; if there is no deflection in g_2 when contact is made at P_2, then the length aP_2 is a measure of the E.M.F. of the cell. If C is the current strength through R, and if E is the E.M.F. of the standard cell, then

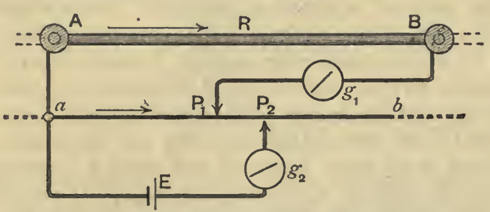

FIG. 234.—Potentiometer Method of Measuring a Current.

$$RC/E = aP_1/aP_2$$

or,

$$C = \frac{E}{R} \cdot \frac{aP_1}{aP_2}.$$

MEASUREMENT OF HIGH AND LOW RESISTANCES

High Resistances.—The method frequently adopted for the determination of high resistances involves the direct measurement of a current transmitted through the resistance by means of a large E.M.F. Thus the resistance of a china telegraph insulator is measured by pouring mercury into the inverted insulator, which is partially immersed in a mercury bath ; one terminal of the battery is connected to the mercury in the cup, and the other is joined, through a mirror galvanometer of known constant, to the mercury bath.

In the case of a telegraph cable, a long length of the cable is coiled within a tank of water, both ends being kept well above the water. One terminal of a battery of 100–1000 Leclanché cells is joined through a galvanometer to the copper core, and the other terminal is joined to the tank. Surface leakage between the core of the cable and the surrounding water is prevented by Price's *guard-ring* method, in which several turns of bare copper wire are wound round the insulation, near to both bared ends, and connected to that terminal of the battery which is joined to the core. Thus the core and the guard-ring are maintained at the same potential, and there is consequently no tendency for leakage across the bared ends of the insulation. Since the resistance of the insulating material usually changes slowly under an applied E.M.F., the current is only observed after a definite interval of time, say one minute. The insulation resistance of unit length of the cable is found by *multiplying* the total resistance by the length of the cable : the reason for this is obvious when we consider that the current leaking through the insulation on two units of length will be twice as great as that leaking through the insulation on unit length.

The *specific resistance* of the insulating material may be derived from the insulation resistance of a length of cable in the following manner: Consider the resistance of a thin circular lamina of the insulation, of radius r, thickness dr, and length l; then, since

$$\mathrm{R}=\rho\times\frac{l}{s},$$

the resistance of the lamina is

$$\rho\times\frac{dr}{2\pi rl}$$

The insulation may be regarded as a series of concentric laminae, similar to the above, the radii varying from r_1 (the radius of the core) to r_2 (the external radius of the insulation). Hence the total resistance R of the insulation is

$$\mathrm{R}=\frac{\rho}{2\pi l}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$=\frac{\rho}{2\pi l}\cdot\log_e\frac{r_2}{r_1}$$

or,

$$\rho=2\pi l\mathrm{R}\Big/\log_e\frac{r_2}{r_1}=2\pi l\mathrm{R}\Big/\left(2.3026\times\log_{10}\frac{r_2}{r_1}\right).$$

EXAMPLE.—The radius of the core of a cable is 2 mms., and the external radius of the insulation is 4 mms. If the insulation resistance of 1 kilometre of the cable is found to be 400 megohms, calculate the specific resistance of the insulating material.

Since

$$l=10^5 \text{ cms.},$$

$$\mathrm{R}=400\times10^6 \text{ ohms.},$$

$$r_2=4 \text{ mms.},$$

$$r_1=2 \text{ mms.},$$

$$\rho=\frac{2\pi\times10^5\times(400\times10^6)}{2.3026\times\log_{10}2}=3.626\times10^{14}\text{ ohms.}$$

The specific resistance of *lead glass* has been measured by filling a thin flask of the material up to the neck with mercury and immersing it to the same depth in a mercury bath. The terminals of a large battery joined in-series with a galvanometer were connected to the mercury inside and outside the flask. The specific resistance was found to be 8.4×10^{13} ohms.

The measurement of a very high resistance, such as that of a small piece of insulating material, can only be made by an *electrostatic method*, in which the rate of loss of charge of an air-condenser, the plates of which are connected through the substance under examination, is measured by means of a quadrant electrometer. If the substance is in the form of a thin sheet, opposite faces are covered with foil and connected to the opposite plates of the condenser and to the quadrants of the electrometer. If the capacity of the condenser is C C.G.S. electrostatic units, and if Q is the charge, then initially Q=CV. The rate of loss of charge, or the instantaneous current due to leakage, may be written $-dQ/dt=-C\,.\,aV/dt$. But, by Ohm's Law, the current at any instant is equal to V/R; hence

$$\frac{\mathrm{V}}{\mathrm{R}}=-\mathrm{C}\frac{d\mathrm{V}}{dt},\quad\text{or }\mathrm{C}\,.\,\frac{d\mathrm{V}}{\mathrm{V}}+\frac{dt}{\mathrm{R}}=0.$$

Integrating this expression, $\mathrm{C}\,.\,\log_e\mathrm{V}+\frac{t}{\mathrm{R}}=k$, where k is a constant.

If the initial P.D. is V_1, when $t=0$, then $k = C \log_e V_1$.

Hence
$$C \log_e V + \frac{t}{R} = C \log_e V_1$$

or
$$C \log_e \frac{V_1}{V} = \frac{t}{R},$$

or
$$R = t/C \,.\, \log_e \frac{V_1}{V}.$$

This indicates how the resistance is calculated by observing the time required for the P.D. to fall from V_1 to V. The logarithms are in the Napierian system and equal to the common logarithm to the base 10 multiplied by 2.3026. R is given in C.G.S. electrostatic units, and may be converted into electromagnetic units by multiplying by 9×10^{20} (p. 521), or into ohms by multiplying by 9×10^{11}.

Owing to the practical difficulties of working a quadrant electrometer, the following modification of the above method is frequently adopted: One plate of the condenser is connected to earth through a highly insulated ballistic galvanometer, and the other plate is connected to earth through a key and charging battery. On closing the key, the first throw θ_1 of the needle is proportional to the quantity of electricity acquired by either plate of the condenser. The key is now opened and, after an observed interval of time t, again closed; the throw θ_2 is proportional to the quantity which has leaked away during the interval t. Hence $\theta_1 - \theta_2$ is proportional to the quantity which has remained in the condenser, and
$$R = t/C \,.\, \log_e \frac{\theta_1}{\theta_1 - \theta_2}.$$

Measurement of Low Resistances.—The Wheatstone bridge is not suitable for the measurement of low resistances; in such cases the method of **Comparison of Potentials** (Fig. 235) may be adopted. Let AC and CB represent two low resistances connected in series; if the lengths are proportional to the resistances, the slope of the line EB, which represents the fall of potential when the ends A and B are connected to a battery, will be uniform. The resistance *ab* is proportional to the P.D. between its ends, viz. V_1; also the resistance *cd* is proportional to V_2. The ratio of V_1 to V_2 may be determined by connecting each pair of points consecutively to the terminals of a high resistance galvanometer. If the deflections are θ_1 and θ_2 respectively, then
$$\frac{ab}{cd} = \frac{V_1}{V_2} = \frac{\theta_1}{\theta_2}.$$

In practice the resistance *ab* may consist of ten coils, each of 0.1 ohm resistance, connected together in parallel; its resistance is therefore 0.01 ohm.

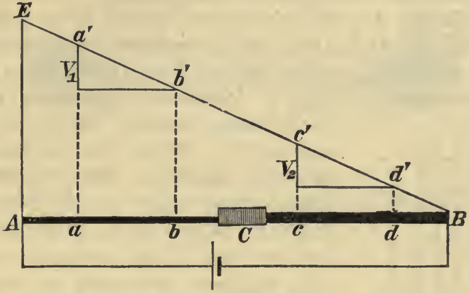

FIG. 235.—Measurement of a Low Resistance.

Resistance of Electrolytes.—The measurement of the resistance of electrolytes is complicated by the *polarisation* (p. 239) at the surfaces of the electrodes through which the current enters and leaves the liquid. The effect is minimised if the solution is that of a salt of the metal of which the electrodes are made, *e.g.* when amalgamated zinc plates are used with a solution of zinc sulphate, and trustworthy results may then be obtained by the Wheatstone bridge method.

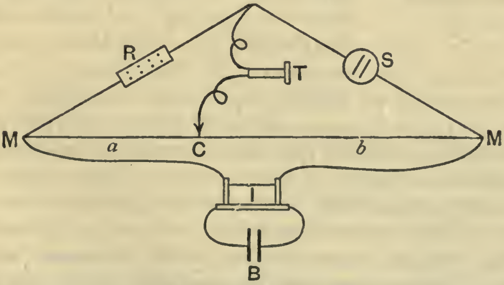

Fig. 236.—Kohlrausch's Method of Measuring Electrolytic Resistance.

Another method, which resembles the Comparison of Potential method of measuring low resistances, consists in connecting opposite quadrants of an electrometer to platinum electrodes in a column of the electrolyte, through which a constant current is passing, and comparing the P.D. with that between the ends of a known resistance included in the same circuit.

The difficulty due to polarisation is still more completely obviated in a method (Fig. 236), introduced by Kohlrausch, in which an alternating current derived from a small induction coil I is transmitted through a Wheatstone net, the galvanometer being replaced by a telephone-receiver T. When the resistances are correctly balanced, the telephone is silent. The type of electrolytic cell (Fig. 237) suggested by Arrhenius is the best for very dilute solutions; two circular platinum electrodes are supported horizontally in a glass vessel from the lower ends of sealed glass tubes which are supported by the ebonite cover of the vessel; the thick platinum wires attached to the electrodes project well within the glass tubes, and contact is made through mercury to copper wires inserted down the tubes from the outside. Even with rapidly alternating currents there may be sufficient polarisation to disturb the accuracy of the experiment, but the effect is diminished by increasing the size of the electrodes; for this reason the surfaces of the electrodes are previously covered with finely divided platinum by electrolysing a solution of platinic chloride between the electrodes by means of a direct current. For solutions of low specific resistance it is better to use a long straight glass tube of small circular bore and terminating in two wider tubes (Fig. 238). The mean diameter of the tube is measured by filling the dry tube with mercury, which is poured out and weighed. Flat amalgamated copper plates placed close to the ends of the tube may be used as electrodes, but platinised platinum is preferable. After filling the apparatus with the electrolyte the resistance of the electrodes and connecting wires

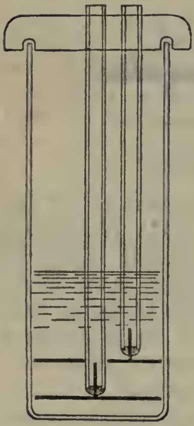

FIG. 237.

must first be determined by placing them together, and this must be deducted from the total resistance in order to give that of the narrow column of electrolyte.

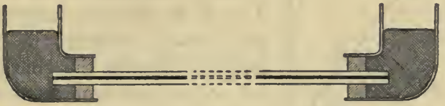

FIG. 238.

If l and s are the length and cross-section of the column of liquid, and if R is its resistance, the *specific resistance* is equal to $(R \times s)/l$ and the *specific conductivity* is $l/(R \times s)$.

MEASUREMENT OF CAPACITY

The measurement of the capacity of condensers of *small* capacity by electrostatic methods has been described in Chap. X. The following paragraphs refer to electromagnetic measurements with condensers of large capacity. The **Farad** is the practical unit of capacity, and a condenser has this capacity when a P.D. of one volt between its plates charges each of them with one coulomb. If V is the P.D. between the plates, C is the capacity, and Q is the charge on each plate, then $C = Q/V$. Hence,

$$1 \text{ Farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} = \frac{10^{-1}}{10^{8}} = 10^{-9} \text{ C.G.S. units of capacity.}$$

The *microfarad*, or millionth part of the farad, is a more convenient unit for practical purposes.

Comparison of Capacities by the Ballistic Galvanometer.—One of the condensers is charged by connecting its terminals to the poles of a constant battery. It is then discharged through a ballistic galvanometer (p. 281) and the amplitude of the first throw is observed. The observations are repeated with the other condenser. The throw in each case is proportional to the charge, and, therefore, to the capacity of the condenser.

Comparison of Capacities by the Wheatstone Bridge.—The two condensers are placed in adjacent arms of a Wheatstone bridge (Fig. 239). The resistances P and Q are adjusted until the galvanometer needle is not deflected when contact is made at *a*. When so adjusted, the potentials at A and B during charging must be the same. Hence, in any short interval of time, the charge received by each condenser must be proportioned to its capacity. But the charges passing through P and Q during any such interval must be proportional to $\frac{1}{P}$ and $\frac{1}{Q}$; hence,

$$C_1 : C_2 :: \frac{1}{P} : \frac{1}{Q},$$

or
$$C_1/C_2 = Q/P.$$

The condensers are discharged by raising the key so as to make contact at *b*.

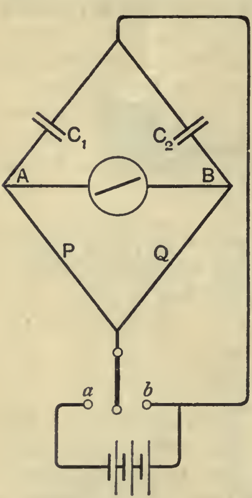

FIG. 239.

Comparison of Capacities by the Method of Mixtures.—This method, which is particularly useful when the condensers differ widely in capacity, consists in the measurement of the potentials required to give equal charges to the two condensers. Equality of charge is detected by mixing the charges and observing the presence or absence of any resultant charge by discharging through a ballistic galvanometer.

In Fig. 240 a battery of several cells is connected in series with two high resistances R_1 and R_2 (where $R_1 + R_2$ is not less than 10,000 ohms). The insulated plates of the condensers C_1 and C_2 are joined through a Pohl

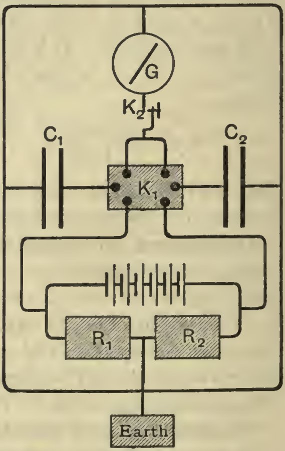

FIG. 240.

commutator K_1 (from which the diagonal connections have been removed) to the ends of the resistance. With the commutator in one position the condensers are charged to potentials proportional to R_1 and R_2. On reversing the commutator, the opposite charges in the condensers are mixed, and the resultant charge is equal to their difference; this resultant charge is observed by completing the circuit through the galvanometer G by means of the key K_2. The resistances R_1 and R_2 are altered until there is no resultant charge left, when

$$\frac{C_1}{C_2}=\frac{R_2}{R_1}.$$

The absolute Capacity of a Condenser.—When a condenser is discharged through a ballistic galvanometer the quantity of electricity, in C.G.S. units, is given by the equation (p. 283),

$$Q=\frac{H}{G}\cdot\frac{T}{\pi}\sin\frac{\theta}{2}.$$

If this quantity has been imparted to a condenser of capacity C by an applied potential difference V, then,

$$C=\frac{1}{V}\cdot\frac{H}{G}\cdot\frac{T}{\pi}\cdot\sin\frac{\theta}{2}.$$

In Fig. 241, $(R+r)$ represents a high resistance of at least 10,000 ohms, r being small compared with R; and K is a highly insulated key by which connection may be made between either *a* and *c*, *a* and *b*, or *b* and *d*. When *a* and *c* are connected, the condenser C is charged to a potential-difference V corresponding to that between the ends of $R+r$. The connection *ac* is broken, and the condenser is discharged through the galvanometer by connecting *a* and *b*. The time of oscillation T is determined by observing the total time required for about 30 complete oscillations.

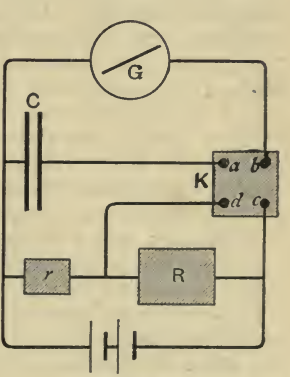

FIG. 241.

The quantity H/VG is determined by connecting *b* to *d*, so that a small steady current traverses the galvanometer.

Then, if v=potential-difference between the ends of r, R_g= resistance of galvanometer, c=current traversing G, and α= deflection,

$$v=\frac{r}{R+r}\cdot V,$$

and
$$c=\frac{r}{R_g\,(R+r)}\cdot V=\frac{H}{G}\cdot\tan\alpha,$$

or
$$\frac{H}{VG}=\frac{r}{R_g\,(R+r)\tan\alpha}.$$

Hence
$$C=\frac{r}{R_g\,(R+r)\tan\alpha}\cdot\frac{T}{\pi}\cdot\sin\frac{\theta}{2}.$$

When accurate results are required corrections must be made for the *logarithmic decrement* (p. 283).

The capacity C will be expressed in farads or in C.G.S. units according as R is expressed in ohms or in C.G.S. units.

QUESTIONS ON CHAPTER XVIII.

1. Two cells, the E.M.F.'s of which are as 2 : 1, are joined up in series with their E.M.F.'s acting in the same direction, and the circuit is completed through a tangent galvanometer, the needle of which is deflected through 60°. If one of the cells is reversed, no other change being made, what will be the deflection of the galvanometer?

2. Explain fully a method of comparing the E.M.F.'s of two cells in which an electrometer is not employed.

3. A wire AB of 0.33 ohm resistance, forms part of a circuit through which an electric current flows in the direction from A to B. The points A and B are also connected by another conducting path, in which is included a cell of E.M.F. 1.287 volts and a galvanometer, the positive pole of the cell being that joined to A. If the galvanometer is not deflected, what is the strength of the current in the wire AB?

4. Describe how you would compare a small E.M.F., say that of a thermo-couple, with the E.M.F. of a Clark cell.

5. Describe some practical form of standard of electromotive force, and explain carefully the method by which such a standard, in conjunction with a set of resistance coils and a galvanometer, may be utilized for current measurement.

6. Show how to compare by the aid of a galvanometer the electromotive forces of two cells whose internal resistances are large but unknown.

7. Describe a simple method of measuring the specific resistance of an electrolyte, and state the chief precautions necessary in the experiment.

8. Given an ammeter and a galvanometer of high resistance and known reduction factor, show how to determine the internal resistance of a given cell.

9. How would you ascertain the electromotive force of a cell while it is giving out a current? (Lond. Univ. Int. B.Sc. Hons., 1899.)

10. A battery sends a current of 1 ampere through a circuit when the P.D. of the terminals is 16 volts, and 2 amperes when the resistance is changed so that the P.D. is 10 volts. What is the internal resistance and the E.M.F. of the battery?

11. Explain why an electromagnetic voltmeter should have as high a resistance as is practicable. Given a voltmeter of this kind reading from 0 to 5 volts with a resistance of 500 ohms, how may the same instrument be adapted to read from 0 to 50 volts with the same scale?

12. Explain how the capacity of a condenser may be measured with the aid of a ballistic galvanometer.

13. Describe De Sauty's (Wheatstone Bridge) method of comparing capacities of two condensers, and prove the formula employed.

14. Calculate the capacity of a condenser formed (*a*) of two concentric spheres with a concentric shell of dielectric between them, (*b*) of two long coaxial cylinders.

Describe an accurate method of comparing two small capacities.

15. Describe how you would determine a very small resistance accurately.

CHAPTER XIX

THERMAL EFFECTS OF THE ELECTRIC CURRENT. ELECTRIC ENERGY AND POWER. EFFICIENCY

The Mechanical Equivalent of Heat.—One form of energy can be transformed into another ; and, as a general rule, during such transformation at least a portion of the energy is converted into heat. The conversion of the energy of an electric current into heat is a familiar case. Thus, in generating electric light, the heat due to the combustion of the coal is, in the first instance, converted into mechanical energy; this is converted into electric energy, which is again reconverted—in the incandescent or arc lamp—partly into heat energy and partly into radiant light energy.

Based upon the fact that heat and mechanical energy are mutually transformable, and that there is undoubtedly an equivalence between them, we may define the **unit of heat** as *that which is equivalent to the unit of work* : in the C.G.S. system this is the **erg**. Another unit of heat, which is universally recognised as a practical unit, is defined as *the heat required to raise the temperature of unit mass of cold*[1] *water through one degree of the thermometer scale used* ; in the C.G.S. system the centigrade scale is employed, and the unit is called the **therm** or the **calorie**. The erg and the calorie may be termed the *primary* and *secondary* units of heat.

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[1] Since the specific heat of water is not constant, it is more accurate to state the temperature of the cold water. For this reason a temperature of 17° or 18° C. is usually assumed in this definition.

The ratio of the secondary to the primary unit is termed the **mechanical equivalent of the unit of heat, or Joule's equivalent**; in the C.G.S. system it is therefore **the number of ergs equivalent to one therm** (or **calorie.**).

The earliest determinations of the mechanical equivalent of heat were made by Joule in 1843 and subsequent years. His method consisted in measuring the amount of mechanical work expended in stirring a known weight of water in a calorimeter, and in measuring the resultant rise in temperature of the water. His result may be stated thus: *the quantity of work required to raise the temperature of one gram of water at* 16°.5 *C. through* 1° *C. is* 4.167×10^7 *ergs.* In simple calculations the value 4.2×10^7 is sufficiently accurate.

Conversion of Electric Energy into Heat.—The definition of unit potential difference states that *unit work is done when unit quantity of electricity is conveyed between two points of a conductor, the points differing in potential by unity.* Hence, if Q units are conveyed between two points which differ in potential by E units, both being measured in C.G.S. electromagnetic units,[1] the work done is equal to

$$EQ = ECt \text{ ergs.}$$

In any simple circuit in which no mechanical work is done, this work appears as heat. If the work done is expressed in *therms* the above expression must be divided by the mechanical equivalent of heat, which is denoted usually by the symbol J. Hence, if H represents the number of heat units generated,

$$H = ECt/J = C^2Rt/J = E^2t/RJ \text{ therms} \ldots\ldots\ldots\ldots\ldots (1)$$

If C and R are expressed in practical units then, since C amperes $= C/10$ C.G.S. units, R ohms $= R \times 10^9$ C.G.S. units, and $J = 4.2 \times 10^7$ ergs.

$$H = C^2Rt \times 10^7/4.2 \times 10^7 = C^2Rt/4.2 \text{ therms.}$$

Joule's Law.—The result expressed in equation (1) is known as Joule's Law, which may be written thus: **The heat generated**

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[1] The student will remember that the same definition of unit P.D. has been used already in the case of the *electrostatic* system of units, although the units of quantity and of P.D. are not the same in the two systems. By reference to p. 521 it is seen that 1 E.M. unit of quantity is equal to 3×10^{10} E.S. units, and 1 E.M. unit of P.D. is equal to $\frac{1}{3 \times 10^{10}}$ E.S. unit, hence the numerical value of the product $Q \times E$ will be the same in the two systems.

in a simple circuit is proportional (i) **to the square of the current,** (ii) **to the resistance,** (iii) **to the time during which the current continues.**

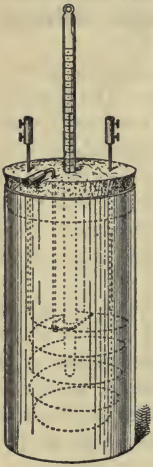

FIG. 242.

The general principle of the apparatus used by Joule in the experiments upon which this law is based may be understood by reference to Fig. 242. The ends of an open coil of thin manganin wire are connected to thick copper wires passing through the cork of a calorimeter. The coil is immersed in water, and all the heat generated in the wire is utilised in raising the temperature of the water and calorimeter.[1]

The relation between *the heat generated* and (i) *the current strength* and (ii) *the time* may be determined by connecting the coil, of known resistance, in series with a battery of constant E.M.F., a rheostat, and a tangent galvanometer. The relation between *the heat generated* and *the resistance* is observed by connecting in series two coils, R_1 and R_2 (Fig. 243) immersed in equal weights of water or turpentine contained in separate calorimeters. In the diagram C represents a commutator, TG a tangent galvanometer, and R a rheostat.

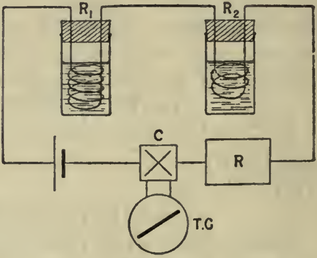

FIG. 243.

An instructive application of Joule's Law may be shown by connecting to the battery terminals a length of thin platinum wire in series with a rheostat and so adjusting the current that the wire glows *feebly*. On cooling a portion of the wire by immersion in cold water, its resistance is diminished and the resulting increase of the current causes the remainder of the wire to glow more brightly than before. Also, if the wire consists of alternate lengths of silver and platinum wire of the same thickness, the current may be adjusted so that the platinum portions glow brightly while the silver portions remain dull. The effect is due to the greater specific resistance of platinum.

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[1] In the experimental verification of Joule's Law, any possibility of the current short-circuiting through the water may be prevented by coating the wire with a thin layer of hardened shellac; this may be done by dipping the coil into a weak solution of shellac in methylated spirit and afterwards drying the coil in an air oven at 130° C. Or the difficulty may be surmounted by using turpentine instead of water; a further advantage being that turpentine has a much lower specific heat (0.462) than water.

EXAMPLE.—A current from a secondary cell is sent through a coil of wire, resistance 1 ohm, immersed in 100 gms. of water. In 50 minutes a rise in temperature of 16° C. is measured. Assuming no loss of heat due to radiation, what is the current strength?

$$H=(100\times 16) \text{ calories}, \quad R=10^9 \text{ C.G.S. units}, \quad t=3000 \text{ secs.};$$

$$\therefore \; C^2=\frac{4.2\times 10^7\times 1600}{10^9\times 3000}=0.0224;$$

$$C=0.15 \text{ C.G.S. units}=1.5 \text{ amperes.}$$

Determination of Joule's Equivalent by Electrical Methods.—If a coil of wire is immersed in water, one of three methods of determining the energy spent in raising the temperature of the water can be used: this is evident from the equation on p. 337, for the energy may be expressed either (i) in terms of E, C, and t, or (ii) in terms of C, R, and t, or (iii) in terms of E, R, and t.

Principal E. H. Griffiths conducted an elaborate series of determinations[1] initially with a view to ascertain the accuracy of our system of electrical units, assuming the accuracy of previous determinations of J. On the other hand, the same experiments serve to determine the value of J by electrical methods, assuming that the definitions of the electrical units are trustworthy. The energy was measured in terms of E, R, and t; and the apparatus resembled in principle the experiment represented in Fig. 233. The apparatus consisted of a platinum wire coil, the resistance of which was measured accurately, immersed in a calorimeter containing water. The current traversing the coil was derived from a storage battery, and the P.D. between the terminals was kept constant by balancing against a set of standard Clark cells, the E.M.F. of these cells being determined by comparison with a standard Clark cell of special construction.

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[1] *Phil. Trans. Roy. Soc.* 1893; *Proc. Roy. Soc.* 1894.

The value of J, for water at a temperature of 15° C., was found to be 4.198×10^7 ergs.

Schuster and Gannon[1] have measured the energy in terms of E, C, and t. The current was measured by connecting a silver voltameter in series with the coil, and the electro-chemical equivalent (p. 455) of silver was assumed to be 0.001118. The wire coil was of platinoid, and had a resistance of about 31 ohms. The P.D. at the terminals was measured by balancing against a series of 20 Clark cells, the E.M.F. of which had been determined previously. The value of J, when the temperature of the water was 19°.1 C., was found to be 4.1905×10^7 ergs.

Heat generated in a Battery, and at a junction of two Metals.—Joule's Law is applicable to every portion of a circuit, including the battery. Thus, if C is the current, R and r the external and internal resistances, all expressed in C.G.S. units, then the *total* heat generated is equal to $C^2(R+r)t/J$, and

$$\textit{heat generated in the battery} = C^2rt/J, \text{ and}$$
$$\text{,, \quad ,, \quad ,, } \textit{ external circuit} = C^2Rt/J.$$

When a current traverses the junction of two metals, the heat generated at the junction usually varies according to the direction of the current. This is due to the existence at the junction of a potential difference which is permanent and independent of the P.D. due to the battery (see Peltier effect, p. 371). Hence, in addition to the heat generated owing to the *ohmic* resistance, a small quantity of heat is also generated or absorbed at the junction, according to the direction of the current. If this local P.D. is denoted by the symbol P, then

$$H = (C^2Rt \pm PCt)/J.$$

As a general rule the magnitude of P is small, amounting to not more than a few thousandths of a volt.

Electric Energy or Work (in Practical Units).—As previously stated, if Q and E are expressed in C.G.S. units, the product of these gives the work done in *ergs*. But it is usually more convenient to express Q and E in *practical* units, *i.e.* in coulombs and volts, and the unit of energy in this system is termed the **Joule.** Or, **the Joule is the work done when 1 coulomb is conveyed between two points which differ in potential by 1 volt.**

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[1] *Phil. Trans. Roy. Soc.*, 1895.

Since 1 coulomb $= 10^{-1}$ C.G.S. unit, and 1 volt $= 10^8$ C.G.S. units, then

$$1 \textbf{ Joule} = 10^{-1} \times 10^8 = 10^7 \text{ C.G.S. units of work (or } \textit{ergs}\text{).}$$

EXAMPLE.—A current of 15 amperes passes through a wire for 45 minutes. If the P.D. between the ends of the wire is 110 volts, calculate the work (in Joules) done by the current.

Quantity (Q) of electricity conveyed $= (15 \times 45 \times 60)$ coulombs.

Work done $= (15 \times 45 \times 60) \times 110 = 4{,}455{,}000$ joules.

Electric Power.—**Power** may be defined as **the rate at which work is done,** and is measured by the number of units of work done per second. If *in t seconds* a quantity of electricity Q is conveyed through a potential difference E, the work done per second is QE/t. But $Q/t = C$, hence

$$\text{Power} = C \times E.$$

In C.G.S. units, power is measured in ergs per second. In practical units, **unit power is obtained when a current of 1 ampere is conveyed through a potential difference of 1 volt.** This unit is called the **Watt.** Hence,

$$1 \textbf{ Watt} = 10^{-1} \times 10^8 = 10^7 \text{ ergs per second} = 1 \text{ Joule per second.}$$

The **Board of Trade Unit** (B.T.U.) is usually adopted for purposes of public supply. It is called the **Kilowatt-hour,** and is evidently equal to the product of a rate of doing work and a period of time. Since the kilowatt is equal to 1000 watts, the number of units supplied is obtained by dividing the product of the volts, amperes, and hours by 1000.

The **Horse-power** (H.P.) is adopted frequently by engineers as a unit of power, and is defined as 550 ft. lbs. of work per second. The relationship between the Horse-power and the Watt may be derived in the following manner: Since 1 lb. weight $= 453.6$ grams weight $= (453.6 \times 981)$ dynes, and 1 ft. $= 30.49$ cms., then

$$1 \textit{ ft. lb. per sec.} = (453.6 \times 981 \times 30.49)/10^7 = 1.357 \textit{ watts.}$$

$$1 \textit{ Horse-power} = (550 \times 1.357) = 746 \textit{ watts} = 0.746 \textit{ kilowatt.}$$

Hence 1 *kilowatt* is approximately equal to $1\frac{1}{3}$ *Horse-power.*

EXAMPLE.—What horse-power is required to drive a current of 120 amperes through a resistance of 12 ohms?

$$E = C \times R = (120 \times 12) \text{ volts.}$$

$$\text{Power} = C \times E = 120 \times (120 \times 12) \text{ watts.}$$

$$\text{H.P.} = (120 \times 1440)/746 = 231.6.$$

The Electric Heating of Wires.—On passing an electric current through a wire the temperature increases, and it would continue to increase if heat were not dissipated from the wire either by conduction through the metal to other parts of the circuit, or by radiation and convection from the surface. In the case of long wires the loss by conduction may be neglected. The rate of loss of heat by radiation and convection, and the relative importance of these sources of loss, depend upon the physical condition of the surface of the wire, upon the medium surrounding the wire, and upon the difference of temperature between the wire and the surrounding medium ; even if the wire is bare, the rate of loss is by no means the same with all gases—thus, the rate of loss is much greater in hydrogen than in air—and it also depends upon the pressure of the gas.

In considering this phenomenon the **emissivity** of a surface is an important physical quantity. Emissivity may be defined as *the quantity of energy given up by unit area of the surface in one second per unit difference of temperature between the surface and surrounding bodies.* The emissivity of polished platinum has been measured by heating, by means of a steady electric current, a platinum wire stretched inside a copper cylinder. The energy dissipated per second is equal to the product of the P.D. between the ends of the wire and the current strength. The resistance and therefore the temperature can be calculated from the same data. The emissivity of platinum in ergs per sq. cm. per degree centigrade, when the wire is surrounded by dry air at 76 cms. pressure, is 0.002 when the difference of temperature is 1000° C., and 0.0007 when the difference is 500° C. In practice the emissivity is often expressed in calories, instead of in ergs ; in these units the emissivity of polished copper, when the temperature difference is 60° C., is 0.00023.

In any given case the temperature of the wire becomes stationary when the rate of loss of heat is equal to the rate of generation of heat by the current. With a wire of length l, radius r, emissivity e, and excess of temperature T, the heat lost per second is $e \,.\, 2\pi r l \,.\, \mathrm{T}$ calories. The heat generated by the current is $\mathrm{C}^2\mathrm{R}/4.2$ calories, C and R being expressed in practical units. Hence

$$e \,.\, 2\pi r l \,.\, \mathrm{T} = \frac{\mathrm{C}^2\mathrm{R}}{4.2}.$$

But, if ρ is the specific resistance of the material, $R = \rho l / \pi r^2$,

$$\therefore\ T = \frac{C^2 \rho}{4.2 \times 2\pi^2 r^3 e}.$$

From this it is evident that the heating of wires, of the same material but of different diameters, will be the same if r varies as $C^{\frac{2}{3}}$.

Example.—Calculate the strength of current necessary to fuse a tin wire of radius 0.5 mm. if the temperature of the surrounding air is (i) 15° C., (ii) 30° C. The melting point of tin is 230° C., its specific resistance is 13×10^{-6} ohms. Assume that the emissivity is 0.00023.

Heat dissipated per second at 230° C. $= e \,.\, 2\pi r l \,.\, (230 - 15)$

$$= 1.553 \times 10^{-2} \times l \text{ calories.}$$

Heat generated per second by current $= \dfrac{C^2 R}{4.2} = \dfrac{C^2 \rho l}{4\,.\,2 \times \pi r^2}$

$$= 3.941 \times 10^{-4} \times C^2 l,$$

$$\therefore\ C = \sqrt{\frac{1.553 \times 10^{-2}}{3.941 \times 10^{-4}}} = 6.28 \text{ amperes.}$$

If the temperature of the surrounding air is 30° C., the maximum current is found, in the same manner, to be 6.05 amperes.

The Incandescent Electric Lamp.—The incandescent lamp is an application of the principle that a conductor is heated when traversed by an electric current, and that it is rendered incandescent when the material of the conductor has a high melting point and a high resistance. The first lamp, introduced by Edison in 1878, consisted of a fine platinum wire, but the liability of the wire to fuse prevented its commercial success. After numerous materials had been tried, the *carbon filament* was found to be the most satisfactory. Since carbon readily burns in air, the filament must be enclosed in an exhausted glass vessel; the current passes into the filament by means of platinum wires fused into the end of the glass vessel.

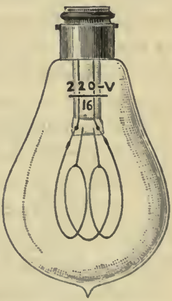

Fig. 244.

The filament is prepared from thin strips of vegetable or carbonaceous material which are wrapped round *formers*, to give the desired shape to the filament, and surrounded by finely divided carbon contained in a crucible, which is then heated to a high temperature in a furnace. The first carbon filaments were made from bamboo, but a form of soluble cellulose is now used. Pure cotton-wool is dissolved in zinc chloride, and the thick liquid is forced through a die, thus giving a uniform thread which, when dry, resembles catgut; this is cut into suitable lengths and then carbonised. During the process of exhaustion of the glass bulb, the filament is heated to redness by means of a current in order to drive out any occluded gases from the carbon and also from the inner surface of the glass vessel.

The electric power consumed by a lamp is expressed, in *watts*, by the product of the voltage at the terminals and the current. The energy expended in the lamp appears partly as heat and partly as light[1]: the portion converted into heat only is wasted, so far as the function of the lamp is concerned, and it is necessary to render the filament as bright as possible if the lamp is to be satisfactory. On the other hand, when the lamp is in use, the carbon slowly volatilises and is deposited on the surface of the glass, thus diminishing the candle-power; this effect increases with the temperature of the filament. Hence the life of a lamp is longer if the voltage applied to the filament is diminished, but much energy is wasted; and if the voltage is increased, within certain limits the light is produced by less expenditure of energy, but the life of the lamp is comparatively short. It is customary to mark on the lamps the voltage which is necessary in order to transmit the strength of current required to make the filament glow brightly.

The consumption of energy in a carbon filament lamp is slightly less than *four watts for each candle-power*.[2] If the temperature of the filament is raised the consumption of energy per candle-power is less; thus, at 1600° C., the efficiency is raised to 3-3½ watts per c.p.; but at this temperature the filament begins to disintegrate, and the life of the lamp is very short. An important disadvantage of the carbon filament is that its resistance *diminishes* with rise of temperature, and therefore it is very sensitive to fluctuations in the voltage. In 1905, Dr. von Bolton produced pure tantalum from the mineral tantalite. This metal has a very high melting-point, about 2300° C.; it is very suitable therefore for making lamp filaments. When consuming 1.5 watts per candle-power its temperature is only 1850° C. Another great advantage is that it is less sensitive to fluctuations of the voltage, since its resistance increases with rise of temperature. As the conductivity is fairly high, the filament must be very long and narrow: the standard type of lamp has a filament 65 cms. long and 0.05 mm. diameter.

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[1] The *luminous* radiation constitutes only about 5 °/₀ of the total energy. This may be proved by immersing a lamp in water contained in (i) an opaque calorimeter, which absorbs all the radiation, and (ii) in a transparent calorimeter which retains only the heat radiation.

[2] The *candle-power* of a lamp is measured in terms of what is called a *standard candle*, *i.e.* a No. 8 sperm candle burning 120 grains of spermaceti wax per hour.

The "Osram" lamp, which has a fine filament of tungsten, is probably the most successful of metallic filament incandescent lamps. Very fine, yet strong, filaments of tungsten can now be produced; and it is possible to obtain lamps of low candle-power suitable for use with currents of usual town voltages. A 16 candle-power, 220-volt, 28-watt Osram lamp, with a filament only about 0.015 mm. in diameter, is now an article of commerce.

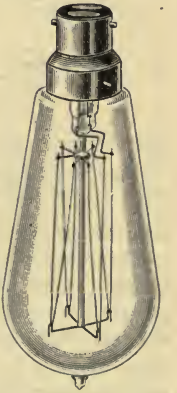
FIG. 245.—The Osram Lamp.

"Tungsten, being more refractory than carbon, can be kept continuously at a higher temperature without volatilising, and a tungsten filament would give three times the light of a carbon filament for the same consumption of power and the same life. But as the specific resistance of tungsten is lower than that of carbon it is necessary to produce a finer filament than the carbon one, and to find a means of supporting a greater length of this fine filament in the lamp. These difficulties were at first only partly overcome. The filament of a 25 candle-power 220-volt carbon lamp has a diameter of about 0.16 mm., and a length of about 350 mm., while the filament of a tungsten lamp of the same candle-power and voltage is about 0.02 mm. in diameter and 850 mm. long." (*Nature*, Oct. 19, 1911.)

Efficiency of Lamps.—The **efficiency** of an incandescent lamp may be defined as **the ratio of the rate of expenditure of energy**

expressed in watts, to the candle-power developed. Fig. 246 represents a simple method of measuring the efficiency of a low-voltage (7-10 volts) lamp. The candle-power is measured by a Rumford photometer ***a***. The current supplied to the lamp is measured by means of a tangent galvanometer TG, and the potentiometer P and galvanometer G serve to measure the potential difference at the lamp terminals. In ordinary cases it would of course be simpler to measure the current and potential-difference by means of an ammeter and voltmeter.

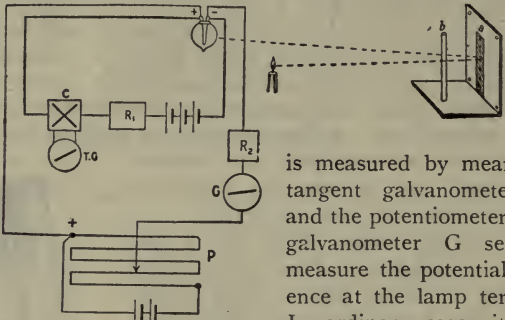

Fig. 246.—Method of determining the efficiency of a lamp.

The Vernon-Harcourt *pentane* lamp, instead of the sperm candle, is now used generally as a light standard. The Carcel *colza-oil* lamp and the Hefner-Alteneck *amyl-acetate* lamp are the legal light standards in France and Germany respectively. Prof. Fleming[1] has found that the carbon filament of an incandescent lamp acquires a constant resistance after being run at normal voltage for 50 hours. The candle-power is diminished by deposition of carbon on the inner surface of the glass bulb; but if the filament, *aged* in the above manner, is re-mounted in a much larger glass bulb and compared with the pentane standard, it may be used as a trustworthy standard for the comparison of other lamps. The voltage is measured by means of a Crompton potentiometer, standardised by means of a Clark standard cell, and the current is determined by measuring the voltage at the ends of a standard resistance connected in series with the lamp.

In accurate measurements of the candle-power of an incandescent lamp, other types of photometer than the *Rumford* are used. In the Lummer-Brodhun photometer, a white slab of magnesia is illuminated on two opposite sides by the two lights to be compared ; by means of totally reflecting glass prisms the diffused light from the two surfaces are reflected down the axis of a telescope, the light diffused from one surface only occupying one-half of the field of view. The difficulty of comparing sources of light of different spectral character is minimised by using Prof. Rood's *flicker* photometer.[1]

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[1] A complete resumé of the photometry of electric light has been given by Prof. Fleming (*Journ. Inst. Elec. Eng.*, vol. 32, p. 119).

EXAMPLES.—1.—How many 16 candle-power lamps, taking 4.08 watts per candle, can be run off a dynamo giving 7 electric horse-power?

$$7 \text{ horse-power} = 7 \times 746 \text{ watts.}$$

If N is the number of lamps, the energy consumed is

$$N \times 16 \times 4.08 \text{ watts};$$

$$\therefore \; N = (7 \times 746)/(16 \times 4.08) = 80.$$

2.—The current passing through a lamp is 0.8 ampere, and the lamp absorbs $\frac{1}{10}$ H.P. What is the E.M.F. required to work the lamp? What will be the candle-power of the lamp, if 3.73 watts are consumed for each c.p.?

Since $\text{Current} \times \text{E.M.F.} = \text{watts},$

$$0.8 \times E = \frac{746}{10} = 74.6;$$

$$\therefore \; E = \frac{74.6}{0.8} = 93.25 \text{ volts.}$$

Also $$\text{candle-power} = \frac{74.6}{3.73} = 20.$$

The Arc Lamp.—This, like the incandescent lamp, is an application of the conversion of electric energy into heat and light. If two rods of carbon are connected to the terminals of a battery or dynamo, so that there is a potential difference of at least 30 volts, then, when the ends of the rods are brought into contact and slightly separated, the current continues to pass as an *arc* between the carbon terminals. The arc has considerable resistance, and much heat is therefore generated locally. The maintenance of the arc is due to the fact that carbon volatilises at a very high temperature, and the carbon vapour thus formed gives the necessary conductivity to the arc.

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[1] Modified forms of this photometer are described in the paper in footnote 1, p. 346.

The vapour seems to be conveyed by the current from the positive rod to the negative rod ; the former is therefore eaten away into the form of a crater, and the latter assumes the form of a point (Fig. 247). The crater is the region of highest temperature—Violle has estimated it to be 3500° C. ; consequently, if the lamp is to be used for lighting purposes, the positive carbon should be uppermost. The temperature of the crater remains constant even when the current strength varies, the effect of an increase in the current is only to increase the luminous area of the crater.

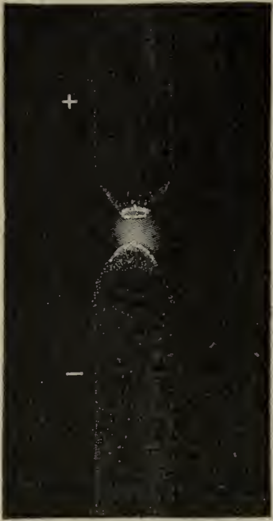

FIG. 247.—The Electric Arc.

The carbon rods gradually wear away, partly owing to the transfer of carbon from the positive to the negative rod, and partly to the oxidation of the carbon ; if a *direct* current is used the positive rod wears away twice as rapidly as the negative, but they wear away equally when an *alternate* current is used.

The efficiency of an arc lamp is far greater than that of an incandescent lamp : thus, about 1 watt is required for each candle power, and fully 10% of the total energy is converted into light.

The potential difference required by an arc lamp varies from 30 to 70 volts, and the current varies from 5 to 10 amperes.

In the direct current arc, a gradual increase of current is accompanied by a decrease of the P.D. between the carbons. If the current is increased beyond a certain critical value the arc commences to *hiss* ; and at this stage the area of incandescence extends beyond the edge of the crater. Mrs. Ayrton[1] has observed that the hissing is prevented by surrounding the arc within an enclosed vessel, and attributes the hissing to combustion of the incandescent carbon on that portion of the anode which is not protected by carbon vapour.

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[1] *Jour. Inst. Elec. Eng.*, xxviii., p. 200.

According to Frölich, the relationship between the P.D. of the carbons and the length of the arc may be expressed by the formula $V=m+nL$, where m and n are numerical constants. Mrs. Ayrton has proved that both m and n depend upon the strength of the current, and that, in the case of the direct current arc between solid carbon rods, the relationship may be represented by the expression

$$V=a+\frac{c}{C}+\left(b+\frac{d}{C}\right)L.$$

If V, C, and L are measured in volts, amperes, and millimetres respectively, then the constants have the following values: $a=38.9$, $b=2.07$, $c=11.7$, $d=10.5$. It is evident that the P.D. between the carbons consists of two parts—one part is fixed and never less than 30 volts, and the other part depends upon L and C. The distribution of potential along the arc is by no means uniform: there is a considerable fall close to the anode, a much smaller one near to the kathode, and a gradual gradient in the intervening space. Until recently, the high value of the constant a has been attributed to the presence of a *back* E.M.F., which closely resembles the phenomenon of polarisation in electrolysis; but more recent experiments have shown that this effect, if present at all, is practically negligible. Experiments conducted by Prof. Fleming suggest that the arc discharge consists of a stream of *negatively* charged particles of carbon which are shot off from the kathode and strike against the anode.

The arc is deflected by a transverse magnetic field, just like a metallic conductor. The longer curved path has the same effect as lengthening the arc by separating the carbons: hence the arc may be *blown out* by applying a strong magnetic field.

Space does not permit a description to be given of the various automatic devices for *striking* the arc and for regulating the *feed* of the carbon rods.

The Electric Furnace.—Much higher temperatures may be obtained by means of the electric current than by any chemical process. Much of our present knowledge of this application is due to M. Moissan, whose initial type of furnace is represented in Fig. 248; it consists of two superposed blocks of lime or of limestone, with a cavity in the lower block, and with a shallow inverted cavity in the upper block, thus forming a lid. Thick carbon electrodes are inserted through holes in the opposite walls. In order to avoid the formation of calcium carbide by the action of carbon on the limestone, the cavity is lined with alternate layers of magnesia and carbon. In this arrangement the heat, derived from the arc which is formed between the ends of the electrodes, is reverberated downwards from the cover, and subjects the crucible to a 'toasting action.' This type of furnace is too costly for operations on a large scale, and in such cases the *resistance* type is used. In the resistance furnace the carbon electrodes are buried in the substance to be fused, the ends of the electrodes being connected initially by fragments of badly conducting material, such as gas-carbon; on passing a current the gas-carbon fuses, and the electrodes are then joined by a semi-fluid mass, the resistance of which is sufficiently high to maintain the temperature required in the furnace. In the resistance furnace, therefore, an arc is never formed between the electrodes.

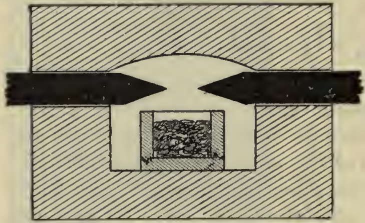

FIG. 248.—Moissan's Electric Furnace.

Calcium Carbide.—Calcium carbide is manufactured by subjecting a mixture of pure lime and coke dust to an extremely high temperature. The mixture is placed in a carbon-lined vessel. The current, which may be either direct or alternating, is transmitted between two vertical carbon rods immersed in the mixture. The reaction is as follows: $CaO + 3C = CaC_2 + CO$. As the carbide is formed, it fuses and sinks to the bottom of the vessel.

Carborundum.—Carborundum is now largely used as an abrasive. Chemically it is a silicide of carbon (SiC). The mixture used consists chiefly of coke and sand; but salt is added to serve as a flux, and sawdust to keep the mass porous. A strong current is passed between carbon electrodes inserted into the mixture. The carbon monoxide which is set free burns at the top of the furnace.

A General Statement of Current Energy.—If between any two points in a battery circuit there is an appliance which absorbs energy in some form other than heat, then the total energy consumed between these points will be transformed partly into heat and partly into energy of another form. If the P.D. between the points is e volts, and if the current strength is C amperes, then

$$\text{Work done per sec.} = eC \text{ joules.}$$

If the resistance between the two points is r ohms, then

$$\text{heat generated per sec.} = C^2r.$$

If the energy consumed in some other form than heat is w watts, then

$$eC = C^2r + w.$$

If the total E.M.F. of the battery is E volts, and if the total resistance of the circuit is R ohms, then

$$EC = C^2R + w.$$

But, by Ohm's Law, CR is the effective E.M.F. of the circuit and if this is denoted by E′, then

$$E = E' + \frac{w}{C},$$

or

$$E' = E - \frac{w}{C}.$$

Hence, any work, other than the generation of heat, introduces into the circuit a *negative* E.M.F., of which the magnitude is w/C.

Efficiency.—The **efficiency** of a battery or generator is **the ratio of the power absorbed in the external circuit to the total power generated.** If the sole function of the generator is to develop heat in the external circuit, the efficiency is equal to the ratio

$$\frac{\text{heat developed in external circuit,}}{\text{total heat developed.}}$$

In the case of a battery, of internal resistance r and connected to an external resistance R, the efficiency is

$$\frac{C^2R}{C^2(R+r)} = \frac{R}{R+r}.$$

If the circuit is adjusted for *maximum electric activity* by making the internal and external resistances equal (p. 314), the *efficiency* is only 0.5 (or 50%); and the efficiency has a greater value when r is negligibly small compared with R.

On the other hand, if the function of the generator is to do external mechanical work, the efficiency is defined as *the ratio of the energy spent in doing such work to the total energy expended.* Thus, if a motor is included in the circuit, and if η is the back E.M.F. (p. 351), then

$$C=\frac{E-\eta}{R},$$

or

$$EC=C^2R+\eta C,$$

and the efficiency is the ratio of ηC, the energy spent per second in the circuit otherwise than in heating the conductors, to the whole energy EC spent in the circuit; or

$$\text{Efficiency}=\frac{\eta}{E}=\frac{E-CR}{E}=1-\frac{CR}{E}.$$

It is evident from this equation that the efficiency is more nearly equal to *unity* (*i.e.* 100%) as C is made smaller. But when arranged for *greatest electric activity*, ηC must have a maximum value, and (p. 351)

$$C^2R-EC+w=0,$$

or

$$C=\frac{E\pm\sqrt{E^2-4Rw}}{2R}.$$

From this equation C can have a real value only when $4Rw$ is not greater than E^2; hence the maximum value which w can have is $E^2/4R$; in which case $C=E/2R$, and

$$\text{Efficiency}=1-\frac{CR}{E}=1-\tfrac{1}{2}=0.5.$$

Hence, as in the case of a battery with simple circuit, the efficiency is only about 50% when arranged for maximum activity, and it approximates to 100% when arranged so that the energy is given out very slowly.

EXAMPLE.—A storage battery of 60 cells, arranged in two rows of 30 cells each, is charged by a dynamo of resistance 0.2 ohm. Each cell has an internal resistance of 0.005 ohm and a back E.M.F. of 1.95 volt. The resistance of the connecting wires is 0.1 ohm. What E.M.F.

must be set up in the dynamo to give a charging current of 40 amperes? Also, what is the electrical efficiency of the system?

$$\text{Back E.M.F.} = 1.95 \times 30 = 58.5 \text{ volts.}$$

$$\text{Total resistance} = \frac{0.005 \times 30}{2} + 0.2 + 0.1 = 0.375 \text{ ohm.}$$

$$\text{Current} = 40 = \frac{E - 58.5}{0.375} \text{ amperes,}$$

$$\therefore\ E = 73.5 \text{ volts.}$$

$$\text{Efficiency} = \frac{\eta}{E} = \frac{58.5}{73.5} = 0.796 = 79.6\ \%.$$

If each cell, when *fully* charged, has a back E.M.F. of 2.2 volts, what E.M.F. must be generated to maintain the same charging current, and what will then be the efficiency of the system?

SUMMARY

The Mechanical Equivalent of Heat.—The *primary unit of heat is the quantity of heat equivalent to one erg, and the secondary unit of heat is the quantity of heat required to raise one gram of cold water through 1° C.* The ratio of the secondary to the primary unit is termed the *mechanical equivalent of heat* or *Joule's equivalent*; its value is approximately 4.2×10^7.

Relation between Electric Energy and Heat.—If a current of C C.G.S. units traverses a conductor, between the ends of which there is a potential difference equal to E C.G.S. units, the energy liberated is EC ergs per second; and, during a time t seconds, the energy liberated is ECt ergs. If the whole of this energy is converted into heat, the number of secondary heat units generated is

$$H = ECt/J = C^2Rt/J = E^2t/RJ \text{ therms (or calories).}$$

If the current and potential-difference are measured in amperes and volts, then

$$H = C^2Rt/4.2 \text{ therms.}$$

Joule's Law is expressed by the above equation, and may be written thus: *The heat generated in a simple circuit is proportional* (i) *to the square of the current,* (ii) *to the resistance, and* (iii) *to the time during which the current flows.*

The **heat generated in a battery** is C^2rt/J, where r is the internal resistance of the battery.

The **heat generated at a junction of two metals** is $(C^2Rt \pm PCt)/J$, where R is the ohmic resistance of the junction, and P is the potential-difference at the surfaces of contact due to the Peltier effect.

The **practical unit of work** is the **Joule**, and is equal to *the work done when one coulomb is conveyed between two points which differ in potential by one volt.* It is equal to 10^7 ergs.

The **practical unit of power** (or *rate of doing work*) is the *Watt.* It is equal to one Joule per second.

The **Board of Trade Unit** is the *kilowatt-hour.* The number of units supplied by any circuit is obtained by dividing the product of the volts, amperes, and hours by 1000.

The **Horse-power** is equal to *550 foot-pounds of work per second.* It is equivalent to 746 watts. Hence 1 kilowatt = $1\frac{1}{3}$ H.P.

The Electrical Heating of Wires.—The temperature of a wire traversed by a current becomes constant when the rate of generation of heat in the wire is equal to the rate at which heat is lost by *radiation* and *convection* from the surface. The rate of loss by radiation is proportional to the **emissivity,** which is defined as *the quantity of energy given up by unit area of the surface in one second per unit difference of temperature between the surface and surrounding bodies.*

The Incandescent Lamp.—The consumption of energy is between 3 and 4 watts per candle-power.

The *efficiency* may be defined as *the ratio of the candle-power developed to the rate of expenditure of energy* (in watts).

In the **Nernst Lamp** a rod consisting of a mixture of the oxides of zirconium, yttrium, and erbium is rendered incandescent by means of an electric current. The efficiency is about 1.8 watts per candle-power.

The Arc Lamp.—When the ends of two carbon rods which differ in potential by at least 30 volts are brought together and then separated a current continues to pass through an *arc* which forms between the carbon points. The positive rod assumes the form of a crater and wears away twice as rapidly as the negative rod, which remains pointed. The temperature of the crater is about 3500° C. About 1 watt is required for each candle-power.

The Electric Furnace.—With the high temperature of the arc various chemical processes, *e.g.* the manufacture of *calcium carbide* and *carborundum*, are possible.

Current Energy.—If E is the E.M.F. applied to any circuit in which energy is consumed, at the rate of w watts, in work other than the generation of heat, then $E' = E - \frac{w}{C}$, where E' is the *effective* E.M.F. of the circuit.

Efficiency.—The efficiency of a battery or generator is *the ratio of the power absorbed in the external circuit to the total power generated.*

But, if the function of the generator is to do external mechanical work, the efficiency is *the ratio of the energy spent in doing such work to the total energy expended.*

QUESTIONS ON CHAPTER XIX

1. State the laws relating to the production of heat by an electric current. What will be the ratio of the currents which will produce in one second the same amount of heat in two wires of the same material and length, if the radius of one wire is twice that of the other?

2. Two wires of the same material but of different lengths and diameters, are joined in parallel and connected to a battery so that they are heated to a high temperature. What must be the relation between the lengths and diameters of the two wires in order that they may have the same temperature?

3. A wire of 5.23 ohms resistance was placed in a calorimeter containing 1000 grammes of water, and a current of 5 amperes was sent through the coil for 10 minutes. If the initial temperature of the water is 10° C., what will be its final temperature?

4. A current is passed through a wire of 5 ohms resistance placed in a calorimeter. A steady stream of water is kept flowing through the calorimeter at the rate of 15 c.c. per minute, and the heating effect was such that the water was 4° C. warmer on leaving the calorimeter than it was on entering. Find the strength of the current.

5. (i) Two circuits whose resistances are respectively 1 ohm and 10 ohms are arranged in parallel. Compare the amount of current passing through each of these circuits with that through the battery. Compare also the amount of heat developed in the same time in the two circuits.

(ii) If the cell has an E.M.F. of 1.08 volts and 0.5 ohm internal resistance, and if the terminals are connected by two wires in parallel of 1 ohm and 2 ohms resistance respectively, what is the current in each, and what is the ratio of the heats developed in each?

6. A wire 0.1 cm. in diameter carrying a current of 10 amperes is found to reach a steady temperature at 100° C. Assuming that the specific resistance of the material is 2.1×10^{-4} ohm per cm.^{3}, and $J = 4.2 \times 10^7$ ergs, determine the amount of heat emitted in unit time from a square centimetre of the surface.

7. Part of the circuit of an electric current consists of a bare wire which passes through a vertical glass tube corked at the lower end. Explain the effect on the temperature of the wire and on the current in the circuit of gradually filling the tube with mercury.

8. A wire of resistance r connects A and B, two points in a circuit, the resistance of the remainder of which is R. If without any other change being made A and B are also connected by $n-1$ other wires, the resistance of each of which is r, show that the heat produced in the n wires together will be greater or less than that produced originally in the first wire according as r is greater or less than $R\sqrt{n}$.

9. Explain why a battery capable of producing an arc light which would rapidly burn up your finger may nevertheless be scarcely able to produce any sensible irritation of the nerves of your fingers when its terminals are held in your hands.

10. The poles of a storage cell are connected successively (*a*) to two feet of copper wire, (*b*) to two feet of iron wire of the same size, (*c*) to one foot of each wire in series. Supposing the wires to be very fine, and neglecting the resistance of the cell, compare the heat produced in each wire with that in (*a*), if the resistance per foot of the iron wire is ten times that of the copper wire.

11. A current was sent through the thick coils of a galvanometer and through a 1-ohm resistance immersed in 100 grams of water. In 40 minutes the rise of temperature was 15.8°, and the mean deflection was 32°. Calculate the strength of the current, and the reduction-factor of the galvanometer.

12. Twelve Grove's cells, each of which has an E.M.F. of 1.9 volts and a resistance of 0.28 ohm, are to be coupled up so as to generate the maximum amount of heat in a resistance of 0.21 ohm. How must the cells be arranged?

13. A current of one ampere flowing for one second through a resistance of 1 ohm produces 0.239 gram-centigrade units of heat. What current would have to flow for an hour through a resistance of 41.84 ohms in order that the heat produced might suffice to raise a kilogram of water from 0° C. to the boiling point?

14. Heat is generated in a wire of resistance 10 ohms which forms part of a circuit containing a battery and having a total resistance of 25 ohms. Find the resistance of a shunt which when applied to the extremities of the wire will cause the heat generated in it per second to diminish in the ratio 4 to 1; assuming that the resistance of the wire does not alter sensibly with its temperature.

15. A jacketed vessel contains a liquid in which a spiral of wire is immersed. An E.M.F. of 20 volts is applied to the ends of the spiral, and a current of 2 amperes passes through it. Five grammes of the liquid is boiled away every minute after steady boiling has begun. What is the latent heat of vaporization of the liquid?

16. When a current divides between two wires arranged in parallel, the current in each branch is inversely proportional to its resistance. Prove that the total amount of heat generated in the two wires is less than it would be if the current were to divide between them in any other proportion.

17. How many foot-pounds are equivalent to 1 calorie?

18. How many candle-power units are developed per calorie in a 16 c.p. 200-volt lamp taking 0.32 ampere?

19. How many calories are equivalent to 1 kilowatt-hour?

20. How many candle-power units per horse-power are obtained from an arc-lamp taking 12 amperes from a 50-volt circuit and giving a light equal to 600 candle-power?

21. At 6d. per B.T.U., how much does it cost for electric energy to keep ten lamps, each giving 15 candles, glowing for 5 hours, if 3.5 watts are required for each candle-power?

22. A charging current of 10 amperes is sent through a secondary cell for 12 hours, during which the voltage rises uniformly from 2 to 2.2 volts. A discharge current of 11 amperes is taken from the cell for 9 hours, and the voltage drops uniformly from 2.2 to 1.8 volt. Calculate the energy in watt-hours put in and given out.

23. A dynamo feeds 1000 16-candle-power lamps. What current must the dynamo supply if the difference of potential at its terminals is 200 volts and each lamp absorbs 3.6 watts per candle?

24. You are given an ammeter, a voltmeter, and a Bunsen photometer. Explain how you would determine the watts per candle absorbed by a glow lamp.

25. Show the conditions under which the electric arc can be formed and maintained between two carbon electrodes, and describe the differences in shape, temperature, and rate of consumption of the two electrodes.

26. An engine supplies 150 horse-power to a dynamo delivering 230 amperes at a voltage of 440. What is the commercial efficiency of the generator?

27. A battery of 12 cells, each with 2 volts E.M.F. and 1 ohm resistance, may be connected in 6 different ways: all in series; 2 rows

of 6 each; 3 rows of 4 each; 4 rows of 3 each; 6 rows of 2 each; all in parallel. If the external resistance is 3 ohms, what is the current strength and the efficiency in each case?

28. If electric energy for heating purposes be supplied at 2d. per unit, find the cost of boiling a quart of water in 5 minutes, and the mean power required. Initial temperature of water 15° C. Efficiency of kettle 90% (1 lb. = 453.6 grams).

29. An electric battery of constant E.M.F. having an internal resistance of 5 ohms, is connected to resistance coils of 10 ohms and 20 ohms respectively, arranged (1) in series, (2) in parallel. Neglecting the resistance of the connecting wires, compare the amounts of heat produced (*a*) in the whole circuit, (*b*) in the two coils.

30. On passing a current of 1 ampere through a piece of platinum wire it is found that its temperature rises 10° C. above that of surrounding objects which are at 0° C. Assuming that the rate of loss of heat is proportional to the difference of temperature, calculate the temperature of the wire when a current of two amperes passes through it. The temperature coefficient of the resistance of the wire may be taken as 0.004 of the resistance at 0° C.

31. How would you prove experimentally that the heat produced by a current in a wire is proportional to the square of the current?

Explain how to determine the resistance of a wire in E.M. units with the aid of a standard ammeter or tangent galvanometer.

32. Explain how the mechanical equivalent of heat may be determined by measuring the electric energy spent in heating a resistance. What instruments would you require, and how would you perform the experiment?

33. Describe what arrangements you would make if you wanted to run a single 50-volt incandescent lamp off a 110-volt circuit. If the lamp takes 0.5 ampere, how much power is taken from the mains and how much power is absorbed by the lamp?

34. A current of water of 0.8 gm. per second is passed through a glass tube of 50 cm. in length, along the axis of which a platinum wire is stretched. The wire is heated by an electric current. The strength of the current is 2 amperes, and the difference in potential between the ends of the wire is 16 volts. Calculate the rise in temperature of the water as it passes through the tube. The mechanical equivalent of the thermal unit is 4.2×10^7 C.G.S. units.

CHAPTER XX

THERMO-ELECTRICITY

A Thermo-Element.—A closed chain of metallic conductors, with all junctions at the *same* temperature, cannot give rise to a current. Seebeck (of Berlin) observed, in 1821, that this is no longer true if the temperatures of the junctions are not the same. Fig. 249 represents Seebeck's initial experiment, in which the ends of a bent copper bar are soldered to the ends of a bar of bismuth, and a magnetised needle (*n*) is supported horizontally between the bars; when one of the junctions is heated a deflection of the needle indicates a current proceeding *through the cold junction from copper to bismuth*, or the current proceeds *along the copper from the hot to the cold junction.* A pair of metals arranged in this manner is called a **Thermo-Element** or **Thermo-couple.**

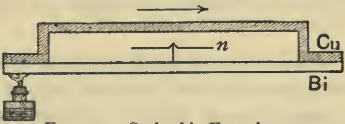

FIG. 249.—Seebeck's Experiment.

Seebeck arranged various metals in a **Thermo-Electric Series,** such that when any pair of metals is used the current always proceeds through the cold junction from the metal which is higher to that which is lower in the series. Seebeck's series is as follows:

Antimony,
Iron,
Zinc,
Silver,
Gold,
Tin.
Lead.
Copper.
Platinum.
Bismuth.

Thermo-Electric Inversion.—The electro-motive force generated by any pair of metals depends both on the *difference* of temperature of the junctions and on the *actual* temperatures of the junctions.

EXPT. 74.—Attach two copper wires to the terminals of a low-resistance mirror galvanometer G (Fig. 250). Determine the direction of current required to cause a deflection to the right or left by momentarily including a cell in circuit. Connect the free ends of the copper wires by means of a piece of thin soft-iron wire. Gradually raise the temperature of the junction T by holding it over a spirit flame. Notice that the current proceeds through the cool junction from iron to copper. As the temperature of T rises the deflection slowly increases to a maximum value, it then diminishes to zero. Finally, at a still higher temperature, *the* E.M.F. *is reversed*, showing that copper is now thermo-electrically positive to iron. Allow the junction to cool, and notice how the deflection undergoes the same cycle of changes in a reverse order.

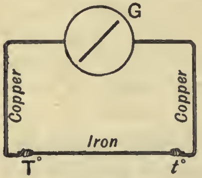

FIG. 250.

Cumming, in 1823, first observed this *inversion of* E.M.F. when the hot junction is raised above a certain temperature. *The temperature of the hotter junction when the* E.M.F. *is a maximum is termed the Neutral Temperature.* The neutral temperature for copper/iron is 275° C.

Law of Intermediate Metals.—If a junction of a thermo-element **A/B** at a temperature T° is opened, and a piece of a third metal C is inserted (Fig. 251), the E.M.F. remains the same, providing that both junctions of the metal **C** are at the same temperature T°. This may be expressed by the equation

$$E_t^T(A/B)=E_t^T(A/C)+E_t^T(C/B).$$

Fig. 252 represents the method by which Becquerel verified the truth of this law. He observed the deflections when any one junction was heated to 20° C., while the others were cooled to 0° C. The following deflections were observed :

27.96 if the Iron/Copper junction was heated.
3.5 „ Copper/Tin „ „
31.24 „ Iron/Tin „ „

The E.M.F. for iron/copper + copper/tin = 27.96 + 3.5 = 31.46, which is approximately the same as for the iron/tin element.

From this law it is evidently unnecessary to determine the E.M.F. for all pairs of metals, since that of a given pair can be deduced from the E.M.F. of each metal when combined with a third metal. For reasons which will be subsequently explained (p. 375), lead is taken usually as the standard metal; some observers have used mercury owing to the high degree of purity in which it can be obtained, but it cannot be used with some metals, *e.g.* lead, owing to the readiness with which it dissolves them.

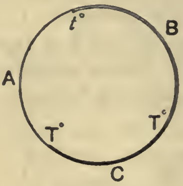

FIG. 251.

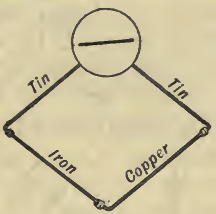

FIG. 252.

Law of Successive Temperatures.—The E.M.F. of any couple between the temperatures T and t is equal to the sum of the E.M.F. between T and any intermediate temperature θ, and the E.M.F. between the temperatures θ and t, or

$$E_t^T = E_\theta^T + E_t^\theta.$$

Thermo-Electric Curves.—The relationship between the E.M.F. of any couple and the temperatures of the junctions may be represented graphically by a curve in which the ordinates and abscissae represent the E.M.F. and the temperature of the hotter junction respectively, the cold junction being kept uniformly at 0° C. This method of representation was introduced by Gaugain. Fig. 253 represents a hypothetical case, which serves as a type of the curve which would be obtained with any couple. If the temperature of the hot junction is t_1° C., the E.M.F. is represented by t_1e_1. If t_1 is nearly zero, the line Oe_1 is practically straight, and the tangent of the angle e_1Ot_1 may be represented by dE/dt, where dE is the small E.M.F. caused by the small temperature-difference dt; this ratio may be termed *the rate of increase of* E.M.F. *with temperature.* As the temperature of the hot junction increases the angle of inclination of the curve at that temperature becomes less, *i.e. the rate of increase of* E.M.F. *becomes less*, until the E.M.F. has a maximum value at a temperature $t_n°$ C., which is the *neutral temperature.* At higher temperatures the E M.F. diminishes, and becomes zero at $t_4°$ C.; at $t_5°$ C. it is reversed. If the temperatures of the junctions are t_1 and t_2, the E.M.F. is represented by e_2b. If the temperatures are equally distant on either side of the neutral temperature, *e.g.* t_2 and t_3, the E.M.F. is zero.

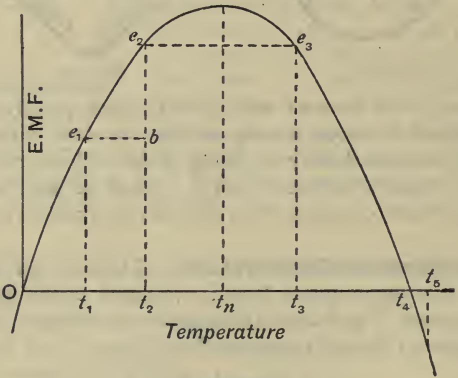

FIG. 253.—Thermo-Electric Curve.

Gaugain considered these curves to be hyperbolas, but Avenarius and Lord Kelvin proved by experiment that they were parabolas, and that the curves were symmetrical on either side of the vertical line drawn through the neutral temperature.

Since the curves are parabolic, they may be represented empirically by an equation of the type $y=bx+cx^2$. Avenarius introduced the formula $E=bt+ct^2$, where E and t represent the E.M.F. and temperature of the hot junction, b and c are constants for the given pair of metals. In order to find the E.M.F. when the junctions are at any temperatures t_1 and t_2, then, by the *law of successive temperatures*,

$$E_0^{t_2}=E_{t_1}^{t_2}+E_0^{t_1},$$

or $$E_{t_1}^{t_2}=E_0^{t_2}-E_0^{t_1}$$
$$=bt_2+ct_2^2-(bt_1+ct_1^2)$$
$$=(t_2-t_1)\left[b+2c\left(\frac{t_2+t_1}{2}\right)\right]\text{....................}(1)$$

Hence the E.M.F. is zero either (i) when $t_2=t_1$, or (ii) when $(t_2+t_1)/2=-b/2c$. In the latter case, $(t_2+t_1)/2$ must be the neutral temperature, the numerical value of which is given by the ratio $-b/2c$.

The values of the constants b and $2c$ for any metal with lead may be determined by measuring the E.M.F. of the couple at two different pairs of temperatures, thus giving two equations similar to (i) above; the values of b and $2c$ are derived by solving these equations. The earliest determinations were made by Tait, who measured the E.M.F. by means of a mirror galvanometer of known constant, and the temperature by mercury thermometers. Later experimenters have measured the E.M.F. by a potentiometer method. The earlier values of the constants are untrustworthy, owing to the difficulty of obtaining perfectly pure specimens of the metals.

Thermo-Electric Power.—Imagine a couple consisting of lead and another metal, with the junctions at $t°$ C. and 0° C., then
$$E_0^t=bt+ct^2\text{................................}(1)$$

If the temperature of the hotter junction is raised to a slightly higher temperature $=(t+dt)$, then
$$E_0^{t+dt}=b(t+dt)+c(t+dt)^2$$
$$=bt+b\,.\,dt+ct^2+2ct\,.\,dt+c\,.\,dt^2.$$

Hence, if the two junctions are at temperatures t and $(t+dt)$ respectively, the small E.M.F. generated in the circuit may be expressed thus
$$dE=E_t^{t+dt}=E_0^{t+dt}-E_0^t$$
$$=b\,.\,dt+2ct\,.\,dt.$$

(Since dt is very small, the term involving dt^2 may be neglected.)

Hence, $$\frac{dE}{dt}=b+2ct^{1}\text{..............................}(2)$$

---

[1] This equation may be directly obtained from equation (1) by simple differentiation.

The ratio dE/dt is termed the Thermo-Electric Power of the metal with respect to lead at the temperature t° C. If the values of this ratio at various temperatures are plotted on squared paper, a straight line is obtained, for any equation of the type $y = b + 2cx$ represents a straight line, and the tangent of the angle which the line makes with the axis of X is $2c$.

Since lead is taken as the standard metal, its thermo-electric line may be represented by the horizontal OX (Fig. 254), and the vertical distance of the line AB above OX may represent the thermo-electric power of another metal with respect to lead at various temperatures measured along the axis OX. Oy represents the value of the constant b, and $\tan\theta = 2c$. If $2c$ is negative, the line AB will slope in the opposite direction, and if b is negative then AB will be drawn below the axis OX.

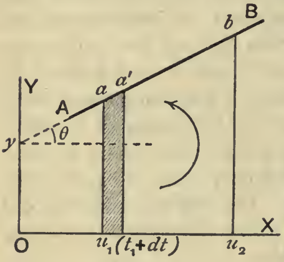

FIG. 254.

In order to calculate the E.M.F. due to the couple when the junctions are at temperatures t_1 and t_2, consider, in the first case, the E.M.F. if the temperature of the hotter junction is $(t_1 + dt)$, then, if the points u_1 and u_2 correspond to temperatures $t_1°$ and $t_2°$,

$$\text{Area of shaded strip } a'u_1 = \frac{dE}{dt} \times dt = dE$$

(since the area of the strip is equal to the average thermo-electric power, between the temperatures t_1 and $(t_1 + dt)$, multiplied by the width of the strip). If the temperatures of the junctions were $(t_1 + dt)$ and $(t_1 + 2dt)$, the E.M.F. would be represented by the next strip. If the temperatures are t_1 and t_2, then, by the Law of Successive Temperatures, the total E.M.F. is equal to the sum of all the shaded areas between the limiting temperatures. The area au_1u_2b is equal to the product of half the sum of the vertical ends and the distance apart, hence:

$$\text{Total E.M.F.} = \left(\frac{au_1 + bu_2}{2}\right)(t_2 - t_1).$$

Or, **the total E.M.F. is equal to the product of the average thermo-electric power and the difference in temperature.**

Since the metal in question is thermo-electrically positive to lead, the current will traverse the cold junction from the metal to lead, and the direction of the current round the circuit would be represented in Fig. 254 as *counter-clockwise.*

If the couple consists of two metals **AB** and **CD** (Fig. 255), with junctions at t_1 and t_2, the area at_2 represents the E.M.F. due to a couple **AB/lead**, and the area ct_2 represents the E.M.F. due to a couple **CD/lead**. Hence, by the Law of Intermediate Metals, the area *cabd* represents the E.M.F. due to the couple **AB/CD**. The E.M.F. will increase as the temperature of the hotter junction is raised to t_n° C., which is the neutral temperature for the two metals. At a still higher temperature the total E.M.F. is equal to the *difference* of the shaded areas on the left and right of N. If the temperatures of the junctions are equidistant from N, the E.M.F. is zero.

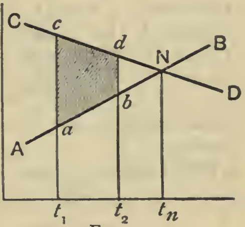

FIG. 255.

Thermo-Electric Diagram.—Fig. 256[1] represents the thermo-electric powers of several metals compared with lead, measured in *micro-volts per* 1° C.

If the thermo-electric power of a metal, at any standard temperature t° C., is represented by the symbol p, then

$$p=\frac{dE}{dt}=b+2ct.$$

The thermo-electric power at t_1° C. $=b+2ct_1=b+2ct+2c(t_1-t)$

$$=p+2c(t_1-t)\ldots\ldots\ldots\ldots\ldots(3)$$

But the E.M.F. of any couple with junctions at t_1 and t_2 is equal to the product of the thermo-electric power at the *mean* temperature into the difference of temperature. Hence

$$E_{t_1}^{t_2}=\left[p+2c\left(\frac{t_2+t_1}{2}-t\right)\right](t_2-t_1)\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

---

[1] This diagram is based upon the experimental results obtained by Noll, and the table of constants is taken from Prof. Callendar's article on Thermo-Electricity (*Encyclo. Brit.*, 1902).

Table of thermo-electric powers, in micro-volts at 50° C., of pure metals with respect to lead. (Noll.)

Metal.	p.	$2c$.	Metal.	p.	$2c$.
Aluminium,	− 0.41	+0.00174	Mercury, -	− 4.03	−0.0086
Antimony, -	+ 3.21	+0.02817	Nickel, -	−20.58	−0.0302
Bismuth, -	−76.87	−0.0848	Platinum, -	− 4.09	−0.0211
Cadmium, -	+ 4.71	+0.0339	Silver, -	+ 2.68	+0.0076
Copper, -	+ 3.22	+0.008	Tin, - -	− 0.067	+0.0019
Cobalt, -	−19.252	−0.0734	Zinc, -	+ 3.318	+0.0172
Iron, - -	+11.835	−0.0306			

The constants for Antimony and Bismuth are due to Dewar and Fleming.

The constants given in the above table are correct at 50° C., hence the formulae (3) and (4) will become

$$p_{t_1}=p+2c(t_1-50),$$

and

$$E_{t_1}^{t_2}=\left[p+2c\left(\frac{t_2+t_1}{2}-50\right)\right](t_2-t_1).$$

If it is required to calculate the E.M.F. obtained by two metals $\mathbf{m}_a$ and $\mathbf{m}_b$, with junctions at t_1 and t_2, then

$$\text{For couple } \mathbf{m}_a/\textbf{Lead},\ E_a=(t_2-t_1)\left[p_a+2c_a\left(\frac{t_2+t_1}{2}-50\right)\right],$$

$$\text{,,}\quad \mathbf{m}_b/\textbf{Lead},\ E_b=(t_2-t_1)\left[p_b+2c_b\left(\frac{t_2+t_1}{2}-50\right)\right].$$

Hence

$$E=E_a-E_b=(t_2-t_1)\left[(p_a-p_b)+(2c_a-2c_b)\left(\frac{t_2+t_1}{2}-50\right)\right]\text{.......(5)}$$

EXAMPLE.—Calculate the E.M.F. obtained with an iron-copper couple when the junctions are at 0° C. and 200° C.

From the table of constants, $p_a=+11.835$, $p_b=+3.22$, $2c_a=-0.0306$, $2c_b=+0.008$. Hence, from equation (5), $E=200[8.615-(0.0386\times 50)]$ $=200[8.615-1.93]=1337$ *microvolts.*

If the fundamental equation of Avenarius (p. 362), $E=bt+ct^2$, is used, the values of the constants b and c for any metal m may be derived by calculating the E.M.F. for the couple **m/Lead** at two pairs of temperatures by means of equation (5), and substituting the values of E and t in both cases in the equation of Avenarius, thus giving two equations from which the values of the constants may be determined. Thus, to find the constants for Iron, consider an **Iron/Lead** couple. From equation (5), if junctions are at 0° C. and 100° C., $E=11.835\times 100$

$=1183.5$ microvolts; if the junctions are at 0° C. and 200° C., $E=[11.835-(.0306\times 50)]200=2061$ microvolts. Substituting in the equation of Avenarius,

$$\left.\begin{aligned}1183.5&=100a+10^4b,\\ 2061&=200a+(4\times 10^4)b.\end{aligned}\right\}$$

From which $a=+13.365$, and $b=-0.0153$.

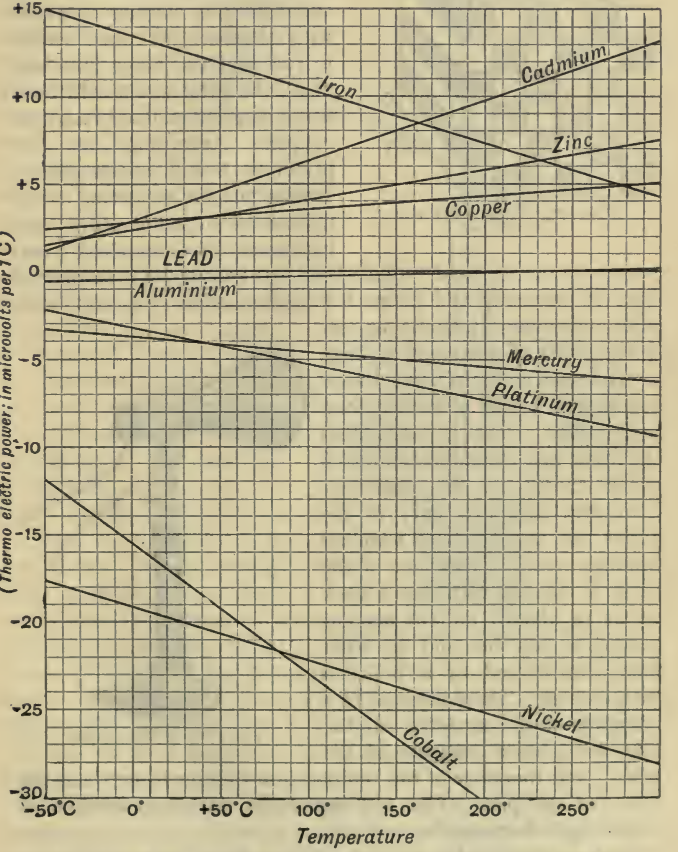

FIG. 256.—The thermo-electric powers of several metals compared with lead.

The Thermo-pile and Radio-micrometer.—The E.M.F. obtained by means of a single thermo-couple is extremely small; thus, by formula (5), the E.M.F. due to an **Antimony/Bismuth** couple, with junctions at 0°C. and 100° C., is only 0.008 *volt.* By connecting n couples in series with alternate junctions heated to 100° and the other junctions at 0°, the total E.M.F. will be n times as great as that of a single couple. According to this principle Nobili constructed a **Thermo-pile** for observing small differences of temperature: the instrument has rendered great service in the study of radiant heat. It consists of a number of bars of antimony and bismuth joined together (Fig. 257) with odd and even junctions on opposite sides of a column of bars, several such columns being joined together in series to form a compact cube. Care is taken that the bars are insulated from each other except at the junctions. The first and last bars in the series are connected to terminal binding screws. One face of the cube is coated with lamp-black so as to absorb radiant heat as readily as possible, and the other face is protected by a polished metal cover. The deflection of a galvanometer connected to the pile will give a measure of the E.M.F., and therefore of the temperature difference between the faces of the pile. Fig. 258 represents the usual method of mounting a thermo-pile.

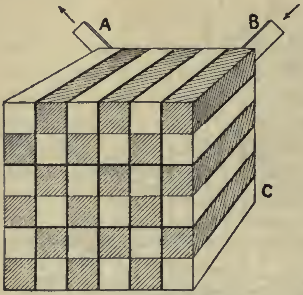

FIG. 257.—Construction of a Thermo-pile.

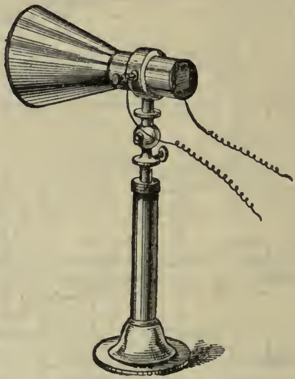

FIG. 258.—A Thermo-pile.

The sensibility of a thermo-pile is reduced by the fact that the bars are good conductors of heat, which tends to equalise the temperatures of the junctions; moreover, the bars have considerable heat-capacity.

The **Radio-micrometer** (Fig. 259), devised by Mr. Vernon Boys, is a far more sensitive appliance for detecting radiant heat. It consists of a vertical loop of thin copper wire, the circuit of which is completed by two extremely small bars of antimony and bismuth separated by a small square (2 mms.) of blackened copper foil. The circuit is suspended by a quartz fibre between the poles of a strong electro-magnet. When the temperature of the copper-foil is raised, a small current is generated round the circuit, and the loop tends to set perpendicularly to the lines of force. The deflection is observed by means of a beam of light reflected from a mirror *m*, attached to the copper loop. Since the magnetic qualities of antimony and bismuth may render the instrument untrustworthy, the couple is screened magnetically by a thick jacket of soft iron (shown by the darker shading). The instrument will give an appreciable deflection with a quantity of heat equal to that radiated on to a halfpenny by a candle-flame 1530 ft. away from it.

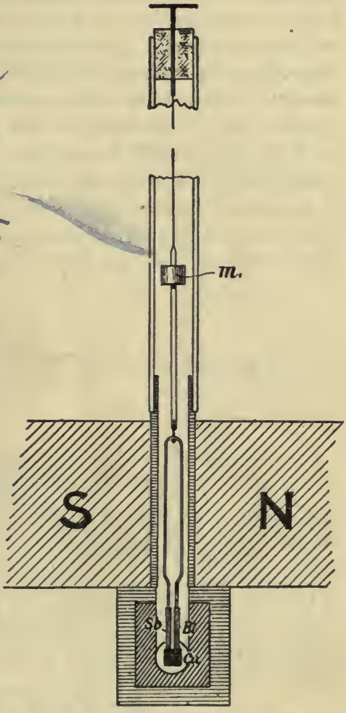

FIG. 259.—Boys's Radio-micrometer.

The Thermo-couple Pyrometer.—The application of the thermo-electric phenomenon to the measurement of high temperature was first suggested by Becquerel in 1826, and the method is now extensively used. The thermo-couple which has proved most satisfactory consists of two wires, one of pure platinum and the other of platinum alloyed with 10% of rhodium; iridium is sometimes used instead of rhodium. The two wires are twisted together and the junction soldered with gold. Fig. 260 represents how the thermo-couple is applied in the Roberts-Austen pyrometer to the measurement of the temperature of molten metal. The wire ACB is composed of the alloy, and the wires ADE and BF are of pure platinum. The junction A is protected by a fire-clay tube, and the junction B is maintained at a constant known temperature by surrounding it with steam. The E.M.F. generated by the couple is proportional to the deflection observed with a sensitive galvanometer G, of which the resistance is at least 200 ohms. The chief advantage of the thermo-couple, as compared with the *platinum resistance thermometer* (p. 310) or a *mercury thermometer*, is that it is capable of recording the temperature at a *point*, for there is scarcely any limit to the possible smallness of the junction.

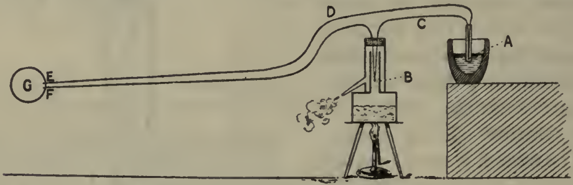

FIG. 260.—The Roberts-Austen Pyrometer.

The relationship between E.M.F. and temperature is expressed by the formula, due to Tait,

$$E = b(t_2 - t_1) + c(t_2^2 - t_1^2).$$

The pyrometer may be calibrated therefore by measuring the E.M.F. at two different pairs of temperatures, and calculating the values of the constants b and c from the two equations so obtained. It is evident that t_2 can be calculated if the values of b, c, and t_1 are known, and if E is measured by means of the galvanometer. The accuracy of the pyrometer at high temperatures has been verified by taking simultaneous readings with the pyrometer and with an air-thermometer.[1]

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[1] Edser's *Heat*, p. 111 (Macmillan).

The Peltier Effect.—When one junction of an **antimony/bismuth** couple is heated, the current flows through the hot junction from bismuth to antimony (Fig. 261). Peltier observed, in 1834, that if both junctions are at the same temperature, and if a battery is included in the circuit so as to send a current in the same direction as before, then heat is *absorbed* at the junction T, and *generated* at the junction *t*. Also, these heat effects are reversed when the direction of the current is reversed. This is termed the **Peltier Effect.** It is quite independent of the heat generated according to Joule's Law, which is equal to C^2Rt: it can therefore be most readily observed by using *thick* bars of metal in which the Joule effect will be a minimum. Fig. 262 represents the method adopted by Peltier to observe the effect, in which two rods of antimony and bismuth (11 cms. long and 16 sq. mms. cross-section) are soldered together to form a cross. If a current is sent across the junction in the direction of the arrow, the deflection of the galvanometer needle will indicate a thermo-current across the junction from antimony to bismuth, *showing that the junction is cooled.*

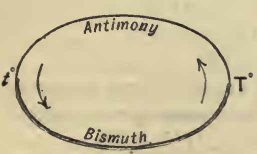

FIG. 261.

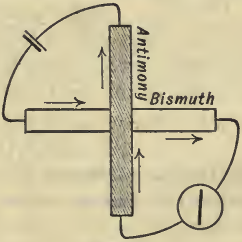

FIG. 262.

EXPT. 75.—Connect together, as shown in Fig. 263, a battery B, commutator C (with diagonal wires removed), thermo-pile TP, and a low-resistance galvanometer G. Pass a current through TP for a short time, and quickly reverse the commutator so as to cut out the battery and complete the circuit through G and TP. The observed deflection will indicate a current through TP in a reverse

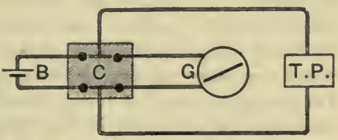

FIG. 263.—Method of showing the Peltier effect.

direction. By varying the strength of current from the battery it may be shown that the thermo-current is approximately proportional to the battery current.

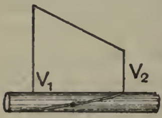

FIG. 264.

The Peltier effect may be explained if we assume that there is, at a junction of two metals, a sudden change of potential which is independent of the potential changes due to the battery. Thus, if a current traverses the junction represented in Fig. 264 from left to right, heat will be generated at the junction since the current will do work in traversing a fall of potential. If the direction of the current is reversed heat will be absorbed to an amount equivalent to the work done in overcoming the rise in potential. The mechanical equivalent of the heat generated or absorbed by a current of strength C in t seconds is given by the equation

$$JH=(V_1-V_2)Ct=PCt.$$

The Peltier coefficient P is defined as the energy generated or absorbed at a junction of two metals traversed by unit current for unit time.

In any portion of a circuit, of resistance R, which includes a single thermo-junction, the total energy generated

$$(JH)=C^2Rt+PCt$$

when the current is in one direction; but if the current is reversed the total energy $(JH')=C^2Rt-PCt$. The value of the coefficient P is determined by measuring the difference between the above quantities, thus

$$P=\frac{J(H-H')}{2Ct}.$$

Le Roux adopted the method of Fig. 265 in order to measure the Peltier coefficient. A and B are two *antimony/bismuth* couples, the shaded elements being antimony, immersed in water contained in two calorimeters.

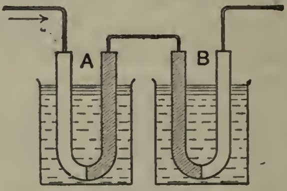

FIG. 265.

On passing a current of known strength through the circuit for a given time, the heat developed in the calorimeters is given by the equations

$$H_1=w_1t_1, \text{ and } H_2=w_2t_2$$

(where w_1 and w_2 are the water-equivalents of the calorimeters and contents, t_1 and t_2 the observed rise of temperature).

If Q_1 and Q_2 represent the heat due to the Joule effect in the two cases, also P_1 and P_2 the Peltier heat effect, then

$$H_1 = Q_1 + P_1, \text{ and } H_2 = Q_2 - P_2.$$

If a current of the same strength is now passed in the reverse direction for the same time, then

$$H_1' = Q_1 - P_1, \text{ and } H_2' = Q_2 + P_2.$$

Hence $P_1 = (H_1 - H_1')/2$, and $P_2 = (H_2' - H_2)/2$.

But $P_1 = P_2$, therefore $P = (H_1 - H_1' + H_2' - H_2)/4$.

Le Roux found that the value of P for any couple is proportional to the thermo-electric power of the couple, and also that the value increased with increase of temperature.

More accurate results have been obtained by Jahn (1888) by measuring the heat developed when the junction is enclosed in a Bunsen ice-calorimeter. The Joule effect was eliminated by repeating the observation when the current was reversed.

Thermodynamics of the Peltier Effect.—Consider the case of a single **antimony/bismuth** couple, with the junctions at 100° C. and 0° C. Heat is absorbed at the hot junction, and is generated at the cold junction; moreover, *these effects are reversible*. The current which is generated will also develop in the circuit a quantity of heat proportional to C^2Rt: this is independent of the direction of the current and is therefore *irreversible*. The whole cycle of operations is therefore not reversible. But if we assume the circuit to have so low a resistance that the Joule effect is negligible, and that the circuit includes some mechanism, such as a perfect motor, which will convert all the electrical energy into mechanical work, then the cycle of operations may be regarded as completely reversible; for, if the junctions are initially at the same temperature, and if the motor is rotated in the opposite direction, a reverse current will be generated which will produce reversed heat effects at the junctions. With this assumption the circuit is comparable to a *perfect heat engine*,[1] in which the ratio of the heat absorbed to the heat given up to the condenser is equal to the ratio of the absolute temperature of the source of heat to that of the condenser. Hence, if P_1 and P_2 are the mechanical equivalents of the heat absorbed and liberated at the hot and cold junctions respectively when unit quantity traverses the circuit, $P_2/P_1 = T_2/T_1$, where T_2 and T_1 are the absolute temperatures of the junctions; or,

$$(P_2 - P_1)/P_1 = (T_2 - T_1)/T_1, \quad \text{or} \quad E = (T_2 - T_1)P_1/T_1.$$

According to this result, the E.M.F. would be directly proportional to the difference of temperature of the junctions, which is contrary to observed facts. This process of reasoning led Lord Kelvin, in 1851, to predict the existence of reversible heat effects in other parts of the circuit besides the junctions. He afterwards demonstrated, when a current flowed through unequally heated conductors, that heat was generated in some cases, but absorbed in others; thus, a current through iron from a hotter to a colder part is accompanied by *absorption* of heat, while the same conditions in copper are accompanied by *generation* of heat. This is known as the **Thomson Effect.**

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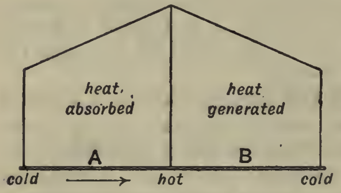
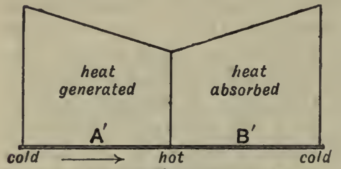
[1] See Edser's *Heat*, p. 347.

The Thomson Effect.—The Thomson effect may be represented as in Fig. 266, which represents a current traversing a copper or iron wire, the centres of which are maintained at a higher temperature than the ends. In the case of a copper wire, the hot portion is thermo-electrically positive to the cold ends, consequently heat is absorbed (and energy gained) in the first half of the wire, while in the second half heat is generated and energy lost. If we compare the temperatures of two points, A and B, which are equidistant from the centre, that of B will become higher than that of A. This is known as the **positive effect**, and is also observed in antimony, silver, cadmium, and zinc. In the case of iron, the cold ends are thermo-electrically positive to the hotter portions; heat is therefore generated in the first half and absorbed in the second half, also the temperature of a point A′ will become slightly higher than that of B′. This is termed the **negative effect**, and is also observed in platinum, bismuth, cobalt, nickel, and mercury.



POSITIVE EFFECT. (Cu, Sb, Ag, Cd, Zn,)



NEGATIVE EFFECT. (Fe, Pt, Bi, Co, Ni, Hg.)

FIG. 266.—The Thomson effect.

By reference to Fig. 256, it will be observed that the thermoelectric lines for all metals showing the *positive effect* slope upwards from left to right, while the lines for those showing the *negative effect* slope downwards. The Thomson effect has not been observed in lead, and its thermo-electric line is regarded as horizontal.

The apparent transfer of heat in a forward direction in those metals showing the positive effect has led to the expression that the *specific heat of electricity* in these metals is positive; in other metals the specific heat of electricity is negative. The phenomenon is sometimes referred to as a *convection of heat by a current of electricity*, which, in the case of copper, resembles that which would be caused by the flow of water along a pipe surrounded at intervals with jackets which are alternately hot and cold. In the case of iron, the 'convection' of heat is in the opposite direction to that of the current, and may be termed negative.

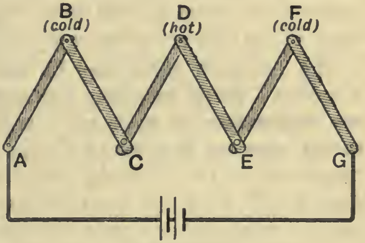


FIG. 267.

Lord Kelvin verified the *electric convection of heat* in the following manner: Fig. 267 represents broad strips of iron joined together, with cylindrical openings at C and E for two thermometers. The points B and F were maintained at a uniformly low temperature by means of cold water, and D was immersed in boiling water. The readings of the thermometers were compared when the current was sent from A to G and when in the reverse direction. The excess of E over C was always greatest when the current passed from G to A.

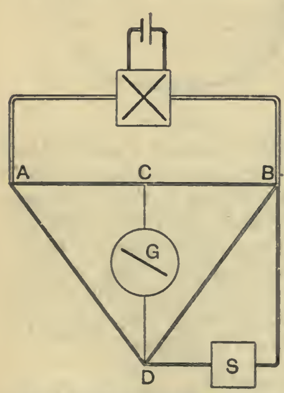


FIG. 268.—Method of showing the Thomson effect.

EXPT. 76.[1]—In Fig. 268 AB represents a piece of copper wire (No. 28, 4 cms. long) soldered at each end to thick copper wires

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[1] Described by Prof. Callendar (*Encycl. Brit.*, 1902).

(No. 12) and heated to 100°-150° C. by means of a strong current. AD and BD are each 1 metre long and of No. 22 copper wire. G is a low resistance galvanometer connected by *very* thin wires to the middle point of AB and to D. The shunt S serves to balance the adjustment when the current is passing through AB. If the metal is copper, and the current passes from A to B, then CB is hotter than AC; on reversing the current AC will become hotter than BC, and the galvanometer will be deflected owing to the balance being disturbed.

The Thomson effect may be defined quantitatively thus: **If a current C passes for a time t between two points in a copper wire, the temperatures of which are T and $T+dT$ (Fig. 269), then the energy absorbed is equal to** $Ct\sigma_{Cu}dT$, where σ_{Cu} represents the magnitude of the Thomson effect in copper. Hence the product $\sigma_{Cu}dT$ is of the same dimensions as an E.M.F., and σ_{Cu} may be expressed as a potential difference per 1° C.[1]

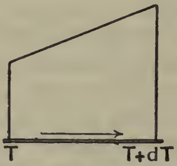

FIG. 269.

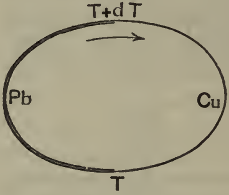

FIG. 270.

Thermodynamics of a Thermo-couple.—Consider a simple *Lead/Copper* couple (Fig. 270) with junctions at the absolute temperatures T and $(T+dT)$, and assume that the wires are so thick that the Joule effect may be neglected. Let $(P+dP)$ and P represent the Peltier coefficients at the hot and cold junctions. There will be a *gain* of energy at the hot junction, and a *loss* of energy at the cold junction; also, since the current flows through the copper from hot to cold, there will be a *loss* of energy in the copper due to the Thomson effect. The net gain of energy when a quantity Q traverses the circuit is therefore

$$[Q(P+dP) - QP - Q\sigma_{Cu}dT].$$

If $Q=1$, the net gain of energy will be numerically equal to the E.M.F. of the circuit, which may be represented by dE. Or,

$$dE = dP - \sigma_{Cu}dT. \qquad \text{......(6)}$$

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[1] If a current C passes for time t through a copper wire, the ends of which are at temperatures T_1 and T_2, the total energy gained $= Ct\int_{T_1}^{T_2}\sigma_{Cu}\cdot dT$.

By the reasoning on p. 373 we may regard the thermo-couple as a reversible heat engine. Hence

$$\frac{P+dP}{T+dT}=\frac{P}{T}+\frac{\sigma_{Cu}dT}{T}.$$

or, $$\frac{\sigma_{Cu}dT}{T}=\frac{P+dP}{T+dT}-\frac{P}{T}=\frac{T\,.\,dP-P\,.\,dT}{T(T+dT)},$$

or, $$\sigma_{Cu}\,.\,dT=dP-\frac{P\,.\,dT}{T}\text{ approximately. }\quad\text{.................(7)}$$

Therefore equation (6) may be written thus,

$$dE=dP-dP+\frac{P\,.\,dT}{T}=\frac{P}{T}\,.\,dT.$$

Hence, $$P=\frac{dE}{dT}\,.\,T.\quad\text{..................................(8)}$$

Or, **the Peltier effect is numerically equal to the product of the thermo-electric height into the absolute temperature.**

Also, from equation (7),

$$\frac{\sigma_{Cu}}{T}=\frac{1}{T}\,.\,\frac{dP}{dT}-\frac{P}{T^2}=\frac{d\left(\frac{P}{T}\right)}{dT}.$$

But, from equation (8), $$\frac{P}{T}=\frac{dE}{dT}.$$

Hence, $$\frac{\sigma_{Cu}}{T}=\frac{d^2E}{dT^2},\quad\text{or}\quad\sigma_{Cu}=T\,.\,\frac{d^2E}{dT^2}.\quad\text{.....................(9)}$$

Since d^2E/dT^2 is equal to the constant $2c$, by equation (2), **the Thomson coefficient is proportional to the constant $2c$ and to the absolute temperature.**

The thermo-electric diagram may be adapted so as to represent the magnitudes of the Peltier and Thomson effects if the scale of temperatures is in *absolute* units. Let Fig. 271 represent the diagram for a copper/lead couple, with the junctions at T_1 and T_2. The area $P_2OT_2A_2$ represents the energy gained at the hot junction, when unit quantity of electricity traverses the junction, for the area is equal to the product of the thermo-electric height A_2T_2 and the absolute temperature OT_2, and the same product is by definition equal to the Peltier effect. Also, if we consider the Thomson effect between two points, A_2 and A_3 which are very near together, and the temperatures of which differ by an amount dT, then, by p. 376, the energy lost between these two points$=\sigma_{Cu}dT$. But by equation (9), the Thomson coefficient $\sigma_{Cu}=2cT$.

Hence the energy lost $= 2c \,.\, T \,.\, dT = \tan\theta \,.\, T \,.\, dT$. In Fig. 271 the area of the narrow strip

$$P_2A_3 = (A_2a \times T_2) \text{ approx.} = dT \,.\, \tan\theta \times T_2 = dT \,.\, 2c \,.\, T_2 \,;$$

hence the area of the strip represents the energy which is lost owing to the Thomson effect. By adding together the small areas corresponding to each portion of the copper wire it is evident that the area $P_2P_1A_1A_2$ represents the total energy lost in the copper wire. Also, the area $P_1OT_1A_1$ represents the energy lost at the cold junction due to the Peltier effect. No energy is lost or gained in the lead wire, since the Thomson effect in lead is zero. The algebraical sum of these areas, $A_1T_1T_2A_2$, represents the total gain of energy when unit quantity of electricity traverses the circuit, and therefore represents the resultant E.M.F. of the couple. This result is identical with that obtained by the independent process of reasoning on p. 364.

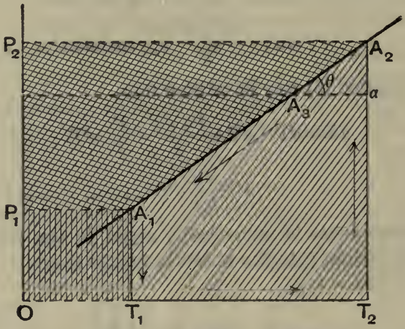

FIG. 271.—Thermo-electric diagram of a copper/lead couple.

EXAMPLES.—1. If the absolute unit of current traverses an iron/copper junction at 50° C., determine the number of heat units (calories) developed in one second.

The energy developed in one second by unit current is equal to the Peltier coefficient (P), and P = (thermo-electric height × absolute temp.). The thermo-electric height at 50° C. of iron above copper = (11.835 − 3.22) micro-volts = 861.5 C.G.S. units. Hence $P = 861.5(273 + 50) = 278{,}264$ ergs. Since the mechanical equivalent of 1 calorie is (4.2×10^7) ergs, the heat developed $= 278{,}264/(4.2 \times 10^7) = 0.0066$ calorie.

2. Calculate the Thomson Effect, in ergs per second, when the absolute unit of current traverses a copper wire from the cold end at 0° C. to the hot end at 100° C.

From Fig. 271 it is readily seen that the area which graphically represents the magnitude of the Thomson Effect is equal to the product of the mean absolute temperature of the ends and the difference in the thermo-electric heights of the ends.

The thermo-electric height of copper at 0° C. $= 3.22 + (.008 \times -50) = 2.82$ microvolts.

The thermo-electric height of copper at 100° C. $= 3.22 + (.008 \times 50) = 3.62$ microvolts.

The difference in thermo-electric heights $= 0.8$ microvolts $= 80$ C.G.S. units.

The mean absolute temperature $= (273 + 50) = 323°$ C.

The energy, which is *gained*, $= (323 \times 80) = 25{,}840$ ergs.

3. By calculating the gain and loss of energy in the parts of an iron/copper couple, with junctions at 0° C. and 200° C., when the absolute unit of current traverses the circuit for 1 second, determine the E.M.F. of the couple.

(i) *The Peltier Effect* (P_2) *at the hot junction.*—By equation (3), the thermo-electric height of copper, at 200° C., $= [p + 2c(200 - 50)] = 3.22 + (.008 \times 150) = 4.42$ microvolts $= 442$ C.G.S. units. Similarly, the thermo-electric height of iron at 200° C. $= 724.5$ C.G.S. units. Hence $P_2 = (724.5 - 442) \times 473 = 133622$ ergs.

(ii) *The Peltier Effect* (P_1) *at the cold junction.*—The thermo-electric height of copper at 0° C. $= 282$ C.G.S. units, and that of iron at 0° C. $= 1336.5$ C.G.S. units. Hence $P_1 = (1336.5 - 282) \times 273 = 287{,}878$ ergs.

(iii) *The Thomson Effect in the copper* = (mean absolute temp. × difference of thermo-electric heights) $= (373 \times 160) = 59{,}680$ ergs.

(iv) *The Thomson Effect in the iron* $= (373 \times 612) = 228{,}270$ ergs.

The Peltier Effect at the cold junction is the only case in which the energy is lost; therefore the net gain of energy $= (133622 - 287{,}878 + 59{,}680 + 228{,}270) = 133694$ ergs. Hence the E.M.F. of the couple $= 133694$ C.G.S. units $= 1336.94$ microvolts. (Cf. this result with that obtained on p. 366.)

SUMMARY

A **thermo-element** or **thermo-couple** consists of *a closed circuit of two different metal conductors with one junction at a higher temperature than the other.*

Metals may be arranged in a **thermo-electric series** such that, when any pair is used, the current passes through the cold junction from the metal which is higher to that which is lower in the series. The commoner metals may be arranged thus: *Antimony*, *iron*, *zinc*, *silver*, *gold*, *tin*, *lead*, *copper*, *platinum*, *bismuth.*

The neutral temperature.—If the temperature of one junction is gradually raised the E.M.F. increases to a maximum then diminishes to zero, and is finally reversed. This is known as *thermo-electric inversion.* The temperature of the hotter junction when the E.M.F. is a maximum is termed the *neutral temperature.*

Law of Intermediate Metals.—The relationship between pairs of three metals A, B, and C, is expressed thus:

$$E_t^T(A/B) = E_t^T(A/C) + E_t^T(C/B).$$

Law of Successive Temperatures: $E_t^T = E_\theta^T + E_t^\theta$.

Thermo-electric Curves.—If, with any thermo-couple, a series of readings of *temperature* and of *E.M.F.* are taken and plotted on squared paper, the resulting curve is a parabola and may be expressed by the equation

$$E = bt + ct^2,$$

where t is the temperature of the hotter junction, the other being kept at 0° C.

If the two junctions are at t_2° C. and t_1° C. respectively, then

$$E = (t_2 - t_1)\left[b + 2c\left(\frac{t_2 + t_1}{2}\right)\right].$$

Thermo-electric Power.—With a couple of which one metal is lead, *the rate of increase of E.M.F. with rise of temperature*, or dE/dt, *is termed the thermo-electric power of the metal with respect to lead at the temperature* t° C. It may be expressed by the equation

$$\frac{dE}{dt} = b + 2ct,$$

and this is *a straight line* when plotted on squared paper.

The total E.M.F. of any thermo-couple is equal to the product of the average thermo-electric power and the difference in temperature.

The **thermopile** and the **radiomicrometer** are applications of the thermo-couple to the detection of small differences of temperature. In the **thermo-couple pyrometer** the principle is applied to the measurement of high temperatures.

The Peltier Effect.—If the junctions of a thermo-couple are initially at the same temperature, and if a voltaic cell is inserted in the circuit so as to send a current in the same direction as would be caused by a difference in temperature of the junctions, *then heat is absorbed at the hotter junction and generated at the cooler junction.* The effect is reversed if the direction of the current is reversed, and it is quite independent of the heat generated according to Joule's Law. *The heat* *lost or gained due to the Peltier Effect is proportional to the current strength and to the time.* The **Peltier Coefficient** P is defined as *the energy generated or absorbed in one second at a junction traversed by unit current.* The total energy generated at any junction is expressed by the equation

$$JH = C^2Rt \pm PCt.$$

The Thomson Effect.—When a current traverses conductors which are heated unequally, heat is generated in some cases and absorbed in others. Thus, in iron, a current from a hotter to a colder part is accompanied by *absorption* of heat, but heat is generated when the current is reversed; the phenomena are reversed in the case of copper.

QUESTIONS ON CHAPTER XX

1. Two different metal wires are joined together at two points, and one of the junctions is kept at a constant temperature while the other is heated. Describe two typical cases of the change observed in the electromotive force as the difference of temperature increases.

2. Explain how the metals may be arranged in a thermo-electric series, and the conditions under which such a series has a definite meaning.

3. Two wires of different metals, A and B, are joined to a battery, their other ends being soldered together. If the current flows from A to B the junction is heated, if it flows from B to A the junction is cooled. How are these facts connected with the production of thermo-electric currents in a circuit made up of the metals A and B?

4. A thermopile is joined up in series with a Daniell cell, and the current allowed to flow for a short time. The thermopile is then removed from the circuit and connected to the terminals of a galvanometer, the needle of which is thereupon considerably deflected but gradually returns to its undisturbed position. Explain this.

5. AB, CD are two copper wires, and their extremities, A, C, are connected by a battery while the extremities, B, D, are joined by a wire. What occurs at the junctions B, D? and what happens if, after the connexions have been made for a considerable time, the battery is removed and the ends AC are joined by a thick copper wire?

6. Suppose that at some point in an electric circuit heat was being developed by the passage of the current. Describe how you would determine whether the heating was due to a resistance or to a thermo-electric (Peltier) effect.

7. Assuming the thermo-electric powers of iron and nickel with respect to lead to be +12 and −20 microvolts respectively, find the E.M.F. of an iron-nickel couple with junctions at 0° C. and 100° C.

8. By means of the thermo-electric diagram (p. 367) calculate the E.M.F. of a copper/iron junction when the junctions are at 20° C. and 120° C. What will be the E.M.F. if the temperature of the hotter junction is raised to 300° C.? In what direction does the current traverse the hotter junction?

9. By means of the diagram (p. 367) plot a curve showing the variation of the E.M.F. of an iron/cadmium couple when one junction is kept in ice and the other is gradually heated from 0° C. to 300° C.

10. One junction of a platinum/lead couple is inserted into an enclosed space, the temperature of which is required. The colder junction is immersed in a water bath at 20° C., and the circuit is completed through a galvanometer of high resistance. The terminals of the galvanometer are also connected to two points on a circuit which is traversed by a constant current of 0.01 ampere; the points of contact are adjusted until there is no deflection, and it is observed that the length of the circuit between the points of contact has a resistance of 0.1 ohm. What is the temperature of the hotter junction?

11. Give a short account of thermo-electric forces, including that due to the Thomson effect.

12. What is meant by the Peltier effect? Describe an experiment to illustrate this effect.

13. What is a thermo-electric diagram, and how are the measurements necessary to construct it made? Show on it in the case of a thermo-electric circuit (*a*) the Peltier effect at the hot junction, (*b*) the Kelvin (Thomson) effect in one of the metals.

CHAPTER XXI

MAGNETISATION OF IRON

Introductory.—The distinctions between *lines of force*, *lines of magnetisation*, and *lines of induction* have already been explained (p. 68). Also the *intensity of magnetisation* of iron or steel (p. 76), has been defined as the magnetic moment per cm.^{3}, or as the pole strength per cm.^{2} of a cross-section perpendicular to the lines of magnetisation : the former definition is more complete, since the latter is not applicable to a continuous iron ring which, though magnetised, does not necessarily exhibit any free poles.

If a mass of soft iron is placed in a magnetic field of intensity $\mathbf{H}$, the iron is magnetised, and the number of lines traversing each cm.^{2} of a cross-section drawn perpendicular to the field is increased from $\mathbf{H}$ to $(4\pi I+\mathbf{H})$, since $4\pi I$ is the number of lines due to the magnetisation acquired by the iron. The total group of lines, including those due to the original field and those due to the iron, is termed the *magnetic induction*, and is denoted by the symbol $\mathbf{B}$ (it is also sometimes termed the *flux-density*) ; hence

$$\mathbf{B}=4\pi I+\mathbf{H}.$$

In subsequent paragraphs it will be assumed that the iron is so placed in the magnetic field that the direction of magnetisation coincides with that of the original field, and that the quantities $4\pi I$ and $\mathbf{H}$ may be added together like simple algebraic quantities. Nevertheless, it is well to point out that $4\pi I$ and $\mathbf{H}$ are *vector* quantities, having direction as well as magnitude, and that their resultant, $\mathbf{B}$, is derived in the same manner as in the case of two mechanical forces. The separation of $\mathbf{B}$ into components is quite arbitrary, and somewhat difficult to realise; but the idea is of considerable utility in a further study of the subject.

Permeability and Susceptibility.—When a rod of soft iron is placed in a uniform magnetic field H, and with its axis coinciding with the direction of the lines of force, the rod becomes magnetised. The resultant magnetic force at any point is due partly to the original field and partly to the poles induced at the ends of the rod; but we may neglect the effect of the poles if we consider the rod to be sufficiently long, *e.g.* equal to 400-500 diameters, and only consider the magnitude of B at points near to the middle of the rod. **The ratio of B to H is termed the Permeability** (μ), or

$$\mu=\frac{B}{H}.$$

On the other hand, we may deduce the relationship between the magnetisation acquired and the magnetising force by determining the *intensity* (I) of magnetisation. **The ratio of I to H is termed the Susceptibility** (κ), or

$$\kappa=\frac{I}{H}.$$

Since $B=4\pi I+H$, we may write $\mu=(4\pi I+H)/H=1+4\pi\kappa$. Hence $\kappa=(\mu-1)/4\pi$.

With any paramagnetic substance, B is greater than H, hence μ is greater than unity. In air or empty space, $B=H$, hence *the permeability of the air is unity.* With diamagnetic substances μ is less than unity, but very slightly so compared with the high permeability of paramagnetic substances; thus, in the case of bismuth, $\mu=0.9998$. The susceptibility (κ) of diamagnetic substances is *negative.*

The manner in which lines of force appear to crowd into a rod of soft iron placed in a magnetic field is analogous to the concentration of lines of flow of an electric current into a long copper wire immersed in a tube of mercury along which a current is flowing—an effect due to the conductivity of copper being greater than that of mercury. This analogy has led to the expression that *iron has a higher conductivity for magnetic lines of force than the surrounding air.* But the analogy is only partly true, since the value of μ in iron varies according to the degree of magnetisation, while the conductivity of copper is independent of the current strength.

Measurement of Permeability.—In all the methods of measurement, the magnetising field is obtained by surrounding a rod or ring of the iron under examination with a solenoid of wire conveying a current of known strength; then (p. 258) if C = current in C.G.S. units and n = number of turns in unit length, $\mathbf{H}=4\pi nC$. The effect of the free poles induced at the ends of a rod may be neglected if it is sufficiently long.

The value of **B** may be determined by one of the following methods:

(i) **Magnetometer method.**—The pole-strength acquired by the iron is observed by means of a magnetometer; from this the value of **B** can be derived.

(ii) **Ballistic method.**—The current, generated in a secondary coil surrounding the magnetising coil when the induction is suddenly created or removed, is measured by the throw of a ballistic galvanometer.

(iii) **Traction method.**—This method depends upon the principle that $\mathbf{B}^2$ is proportional to the force necessary to separate two plane surfaces of the same substance, when magnetised to the same degree.

(i) MAGNETOMETER METHOD.—The deflections of a magnetometer placed at A (Fig. 272) are observed by means of a lamp and scale. The rod of iron BC is fixed vertically to the east or west of A so that the line AB is horizontal, and is surrounded by a closely wrapped spiral of insulated wire extending beyond each end of the rod. Let I = intensity of magnetisation, a = radius of the iron rod, then the pole-strength is equal to $\pi a^2 I$. Hence,

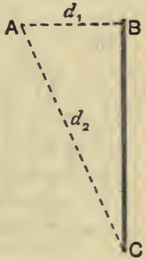

FIG. 272.

the intensity of field at A due to the pole at $B=\dfrac{\pi a^2 I}{d_1^{\,2}}$, and

„ „ „ „ at $C=-\dfrac{\pi a^2 I}{d_2^{\,2}}\times\dfrac{d_1}{d_2}$;

$$\therefore \text{ the } \textit{total intensity of field } (F_2)=\frac{\pi a^2 I}{d_1^{\,2}}\left\{1-\frac{d_1^{\,3}}{d_2^{\,3}}\right\}.$$

If F_1 = the intensity of field controlling the magnetometer, and θ = the deflection produced, then $F_2 = F_1 \tan\theta$, or

$$\frac{\pi a^2 I}{d_1^2}\left\{1 - \frac{d_1^3}{d_2^3}\right\} = F_1 \tan\theta;$$

$$\therefore\ I = \frac{d_1^2 F_1 \tan\theta}{\left\{1 - \frac{d_1^3}{d_2^3}\right\}\pi a^2}.$$

But, $\mathbf{B} = 4\pi I + \mathbf{H} = 4\pi I + 4\pi n C.$

Hence, **B** is determined if the values of I, n, and C are known.

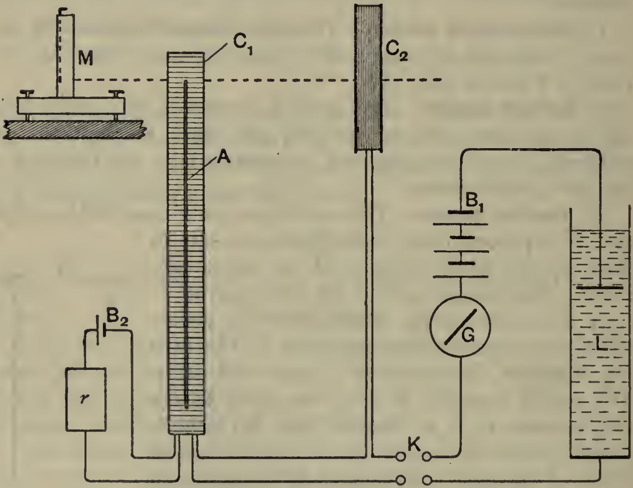

FIG. 273.—Measurement of Permeability (magnetometer method).

Fig. 273 represents the arrangement of apparatus. The magnetising solenoid C_1 extends well beyond each end of the rod A—so as to ensure uniformity of field in the space occupied by A—and is connected through a commutator K to an ammeter G, a battery B_1, and a liquid rheostat L. A second coil is wound over C_1 and is connected to a single cell B_2 and a resistance r, and the current is adjusted so that the effect of the coil exactly neutralises the magnetism induced in A by the earth's vertical component. The electromagnetic effect of the magnetising coil itself on the magnetometer is neutralised by including in the main circuit a *compensating coil* C_2 which is fixed vertically with its axis in line with the centre of the magnetometer needle and so placed that, with the iron rod removed, the magnetometer is not affected by the passage of a current through the coils.

(ii) BALLISTIC METHOD.—As explained in Chap. XXII, the E.M.F. generated in a coil by a *sudden* change in the number $\mathbf{N}$ of lines of force threaded through the coil is equal to the *rate of* change of $\mathbf{N}$, *i.e.* to $d\mathbf{N}/dt$. If R = total resistance of the circuit, C = current generated, and n_1 = number of turns of wire in the coil, then

$$C = n_1 \cdot \frac{d\mathbf{N}}{dt} \Big/ R.$$

The total quantity Q of electricity generated is equal to $C \cdot dt$; hence,

$$Q = n_1 \cdot d\mathbf{N}/R.$$

The *quantity* of electricity may be measured by joining a ballistic galvanometer in circuit with the coil, when the *throw* of the needle is proportional to the whole quantity of electricity generated. The galvanometer is standardised by observing the *throw* due to the rotation of a coil of known dimensions through 180° in a uniform field of known intensity, *e.g.* the earth's field; if rotated round a vertical axis, the effective field will be the earth's horizontal component. Let A_1 = area of this latter coil, n_2 = number of turns of wire, F = the component of the earth's field. A rotation through 180° will change the number of lines threading the coil by $2n_2A_1F$. The quantity of electricity generated $= 2n_2A_1F/R_2$, where R_2 is the total resistance. If θ_1 is the throw due to the change $d\mathbf{N}$, and θ_2 is the throw due to the standardising coil, then

$$\theta_1 : \theta_2 :: n_1 \cdot d\mathbf{N}/R : 2n_2A_1F/R_2;$$

$$\therefore\ d\mathbf{N} = \frac{\theta_1}{\theta_2} \times \frac{2n_2A_1F}{R_2} \times \frac{R}{n_1} = \theta_1 \times K \times \frac{R}{n_1}$$

(where K, the *constant* of the galvanometer, is equal to $2n_2A_1F/\theta_2R_2$, *i.e.* the quantity of electricity required to produce a throw of 1 scale division). Owing to uncertainty in the value of F, more trustworthy results are obtained if the galvanometer is standardised by means of a long straight solenoid round the middle of which a short secondary coil is wound. The latter coil is permanently connected in series with the galvanometer; in Fig. 273 it would take the place of the Earth-inductor E. If n_2 is the number of turns per unit length in the primary, a the area, and C the current, the change of induction on breaking the circuit is $4\pi n_2 aC$; and, if N is the total number of turns in the secondary, $K=4\pi n_2 aCN/\theta_2 R_2$.

In the ballistic method, a continuous iron ring is generally used and round it a magnetising coil is wound uniformly. The strength of the field is calculated from the formula $4\pi nC$ (p. 258). **B** is measured, according to the above principle, by winding over the magnetising coil a secondary coil connected to a ballistic galvanometer. It is advantageous to include the *earth coil* in the main circuit, in which case $R=R_2$ in the above formula. Since the magnetising force is not uniform across the whole section of an iron ring (p. 260), a *flat* iron ring similar in shape to a short piece of iron piping is generally used, so that the thickness of the iron is small compared with the radius of the ring.

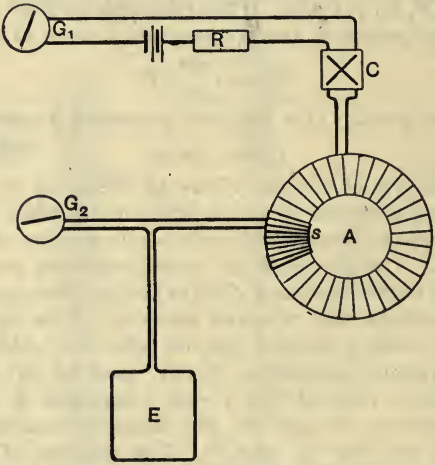

Fig. 274.—Measurement of Permeability (Ballistic Method).

Fig. 274 represents the arrangement of apparatus. The magnetising coil, wound round the iron ring A, is connected through a commutator C to a resistance-box R, a galvanometer G_1, and a battery. The secondary coil *s* is in series with the earth-coil E and a ballistic galvanometer G_2. The observations may be made by either of two methods, (i) *by steps*, or (ii) *by reversals*. In the former, the throw due to a sudden weak current is observed; the needle is then brought to rest, the current is suddenly increased and the throw again observed, and so on: the *total* induction is calculated by adding together the individual throws. In the latter method the induction is determined by taking one-half the throw observed when the current is suddenly *reversed*.

It has been observed by Prof. Ewing that the induction does not immediately acquire its final value when the magnetising current is altered, and that this is particularly evident when the field is weak and when the mass of material is large. This effect is termed **magnetic creeping** or **magnetic viscosity.** This time-lag is occasionally so marked that the ballistic method is not applicable. Except in this one detail, the ballistic method is generally preferred to the magnetometer method.

(iii) TRACTION METHOD.—Shelford Bidwell[1] has determined permeability by measuring the force required to separate the halves of a cut iron ring, each half of which is wound uniformly with wire traversed by the same current. It can be proved that the force, W*g* dynes, required to separate the surfaces of contact is equal to $\mathbf{B}^2S/8\pi$, where S is the area of surface of contact. Prof. S. P. Thompson has devised a simple appliance, termed a *permeameter* (Fig. 275) for measuring permeability. The rod of iron *a*, of which the permeability is required, passes freely through a hole bored in the upper side of a soft iron yoke *b*, and its lower end is in contact with the lower side of the yoke; both surfaces of contact are truly planed. The rod is surrounded with a magnetising coil *c*.

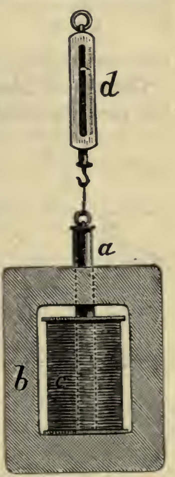

FIG. 275.—The Permeameter.

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[1] *Proceedings Roy. Soc.*, vol. xl., 1886.

A spring-balance *d* measures the pull required to separate the two surfaces of contact.

The force necessary to separate the two surfaces is determined by the tension along the lines of force which cross the surfaces. From this point of view we may apply the results previously obtained (p. 143) for the tension along electrostatic lines of force which is there proved to be equal to $2\pi\sigma^2$ dynes per cm.^{2}. In the magnetic problem under discussion σ is the magnetic density on each surface, and is equal to $\mathbf{B}/4\pi$. Hence, if S is the area of the surface of contact, the tension along the lines of force $=2\pi(\mathbf{B}/4\pi)^2S=\mathbf{B}^2S/8\pi$. Hence $Wg=\mathbf{B}^2S/8\pi$. The intensity of the magnetising field is given by the formula $4\pi n_1C$ (p. 258).

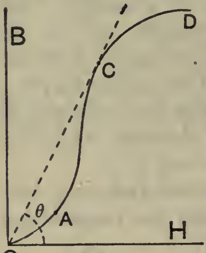

FIG. 276.

Curves of Magnetisation.—A convenient method of recording the relationship between the magnetic induction **B** and the magnetising force **H** is to plot the corresponding values of these quantities on squared paper, taking values of **H** as abscissae and values of **B** as ordinates. Fig. 276 represents a characteristic

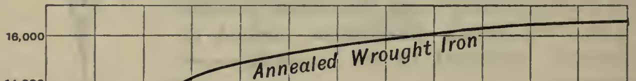

FIG. 277.—B/H Curves for various Metals.

form of the curve obtained with soft iron or steel. In the first portion of the curve, between O and A, the value of **B** *gradually* increases as **H** increases. Between A and C, **B** increases very rapidly during a small increase of **H**. At D the curve becomes almost flat, and an increase in **H** causes only a slight increase in **B**.

At any point, the ratio **B**/**H** gives the value of the permeability μ; and if the point is joined to the origin by a straight line, making an angle θ with the horizontal axis, then **B**/**H**$=\tan\theta=\mu$. In Fig. 276 it is evident that μ increases to a maximum value at the point C, and afterwards diminishes. At all points beyond C the iron is approximately *saturated*.

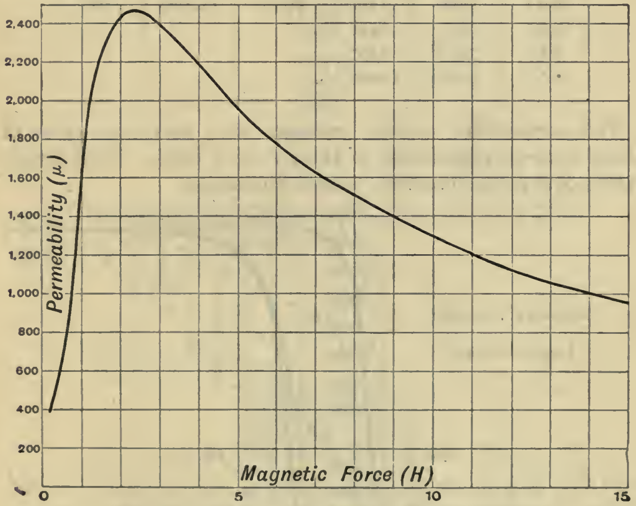

FIG. 278.—μ/H Curve for Annealed Wrought Iron.

Fig. 277 represents the **B**/**H** curves obtained with annealed wrought iron, cast iron, annealed and glass-hard steel, and nickel. It will be observed that annealed wrought iron gives a much higher value of **B** for a given value of **H**, and becomes saturated with a much lower value of **H**, than is the case with other kinds of iron or steel.

The relationship between μ and **H** for annealed wrought iron is shown in Fig. 278. The data, tabulated below, were obtained by Prof. Rowland[1] who used a ring of the specimen, and measured **B** by the ballistic method.

VALUES OF **H**, **B**, AND μ FOR ANNEALED WROUGHT IRON.

H.	**B.**	μ.	**H.**	**B.**	μ.
0.18	71	390	5.31	10,080	1899
0.69	600	869	8.48	12,270	1448
1.27	2460	1936	10.23	12,970	1269
1.41	2920	2078	11.99	13,630	1137
2.04	4960	2433	17.69	14,540	824
2.22	5480	2470	34.17	15,770	462
2.72	6650	2448			
3.16	7470	2367			
4.05	8940	2208			

The permeability rapidly increases to a maximum value of about 2400 corresponding to H = 2.7 C.G.S. units. With higher values of **H** the permeability slowly diminishes.

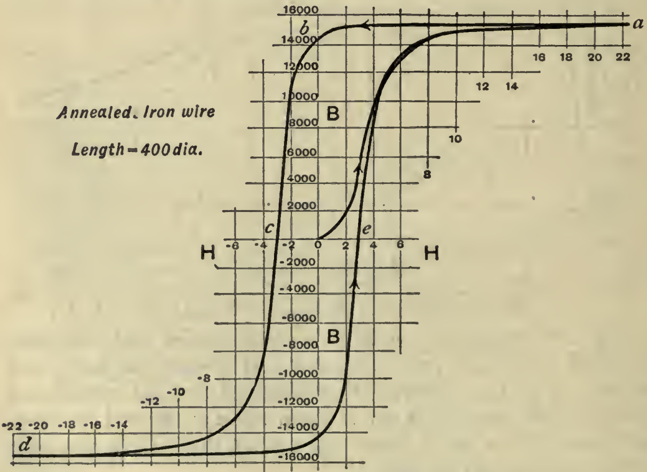

FIG. 279.—Hysteresis Curve for Annealed Iron Wire.

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[1] *Phil. Mag.*, xlvi., 1873.

Cycles of Magnetisation. Hysteresis.—The curves obtained by plotting the values of **B** produced by a cyclic change in **H** have the general form shown in Fig. 279, which has been obtained with a specimen of annealed soft iron. Commencing at *o*, the curve *oa* is obtained; on slowly diminishing **H** the magnetisation is largely retained, and even when **H** is reduced to zero permanent magnetisation represented by *ob* still remains. On applying a negative magnetising force, **B** is gradually reduced, and becomes zero when **H** $= -oc$. The curve *cd* is obtained if the negative force is gradually increased. If the negative force is now gradually reduced to zero and then made positive the curve *dea* is obtained. If the complete cycle is repeated a curve identical with *abcdea* is again obtained.

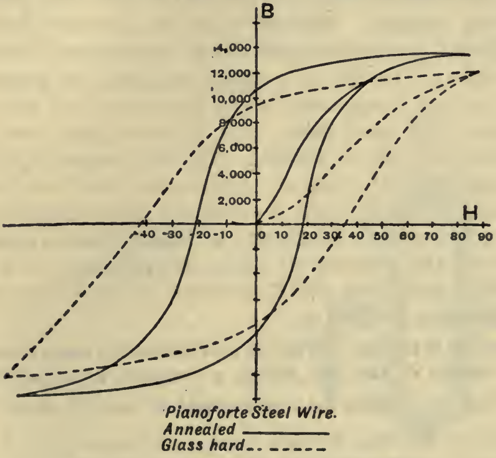

FIG. 280.—Hysteresis Curves for Steel Wire.

It will be noticed that a magnetic condition set up in the iron tends to persist, *e.g.* the value of **B** when **H** is diminishing is always greater than when **H** is increasing. Prof. Ewing has applied the term **hysteresis** (*ὑστερέω*, to lag behind) to this tendency.

According to Hopkinson, the quantity *ob* measures the *retentivity* of the iron, and *oc* measures the *coercivity*; or, in general, the retentivity is the value of $\mathbf{B}$ when $\mathbf{H}=0$, and the coercivity is the value of $\mathbf{H}$ when $\mathbf{B}=0$.

The form of the hysteresis curve obtained depends entirely upon *the absence of mechanical vibration*, which tends to destroy the retentivity, and results in the partial or complete coincidence of the two sides of the hysteresis curve. *The form of the curve is very characteristic of the magnetic material under examination.* Fig. 280 represents the hysteresis curves obtained with annealed and glass-hard piano wire. The presence of chromium or tungsten, particularly the latter, increases the coercivity to a remarkable degree; tungsten steel is therefore used largely for permanent magnets. Madame Curie has recently shown that molybdenum is even more effective than tungsten in increasing the coercivity. On the other hand, Hadfield's *manganese steel*, containing 12% manganese, has a permeability of only 1.5 in both strong and weak fields; it is therefore much used where magnetisation is to be avoided. Peculiar magnetic properties are possessed by *Hopkinson's alloy* (3 parts iron, 1 part nickel); at ordinary temperatures it is practically non-magnetisable, but if cooled below 0° C. it becomes as magnetisable as cast iron, and retains the property at higher temperatures; it loses its power when heated to 580° C. and does not recover until again cooled below 0° C.

Dissipation of Energy.—It can be proved that the energy dissipated in unit volume of the iron during a complete hysteresis cycle is represented by $\frac{1}{4\pi}$ times the area enclosed by the B/H curve.

Consider an iron ring, of length l cms., and cross-section s sq. cms., wound with a magnetising coil of n turns per unit length. If the current changes by an amount $d\mathrm{C}$ C.G.S. units, the induction will also increase by an amount represented by $d\mathbf{B}$. The *total* change in the induction will be $s\,.\,d\mathbf{B}$; this creates an opposing E.M.F. in the magnetising coil proportional to the *time-rate* of change in the induction, *i.e.* proportional to $s\,.\,\frac{d\mathbf{B}}{dt}$. The total opposing E.M.F. $=n\times l\times s\,.\,\frac{d\mathbf{B}}{dt}$. The work done by the current in overcoming this E.M.F. is equal to

$$\mathrm{C}\,.\,dt\times nls\frac{d\mathbf{B}}{dt}=n\mathrm{C}\,.\,ls\,.\,d\mathbf{B}.$$

But $4\pi n\mathrm{C}=\mathbf{H}$, and ls is the volume of the ring,

$$\therefore\ \text{work done per unit volume}=\frac{1}{4\pi}\,.\,\mathbf{H}\,.\,d\mathbf{B}.$$

The product $\mathbf{H}\,.\,d\mathbf{B}$ may be represented graphically as the area of a small strip such as *xy* (Fig. 281), and the sum of all these strips, $\int_0^{\mathbf{H}_1}\mathbf{H}d\mathbf{B}$, while $\mathbf{H}$ increases from 0 to $\mathbf{H}_1$ is equal to the area OAC. Or,

the work done per cm.^{3} *in passing from* O *to* A $= \frac{1}{4\pi} \times$ *area* OAC.

When $\mathbf{H}_1$ is reduced to zero the work recovered is represented by the area $\left(\frac{1}{4\pi} \times \text{area CAB}\right)$. Hence

total work done in the cycle OAB $= \left(\frac{1}{4\pi} \times \textit{area}\ \text{OAB}\right)$.

The energy so lost is derived from the magnetising current, and takes the form of heat. The number of heat units developed in each c.cm. of the metal is calculated by dividing the energy $\frac{1}{4\pi}\int \mathbf{H}\,.\,d\mathbf{B}$ by the *mechanical equivalent of heat* (p. 337).

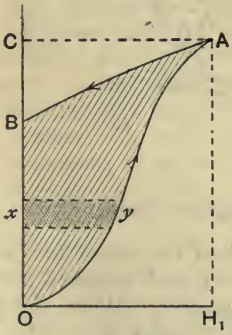

FIG. 281.

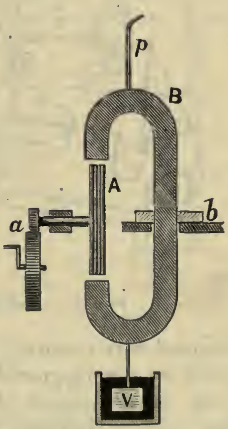

FIG. 282.—Ewing's Hysteresis Tester.

Ewing's Hysteresis Tester.—Fig. 282 represents the principle of Ewing's *Hysteresis Tester* which is largely used for the rapid comparison of the hysteresis in different specimens of iron. A bundle A of rectangular strips of the specimen is rapidly rotated by the gearing *a*. A C-shaped permanent magnet B, supported on knife-edges at *b* carries a pointer *p* and a vane V which dips into oil and serves to damp the vibrations. On rotating the specimen the magnet is deflected to an extent which depends directly upon the amount of hysteresis in the specimen and is practically independent of the speed of rotation. Numerical values of the hysteresis are obtained by making a similar observation with a specimen of a standard material, the hysteresis of which is known.

Ewing's Magnetic Curve Tracer.—Prof. Ewing has invented an instrument (Fig. 283) for rapidly tracing **B**/**H** curves by means of a spot of light reflected from a small mirror on to a screen. The mirror E is pivoted against a needle-point so as to have freedom of motion round both a vertical and a horizontal axis.

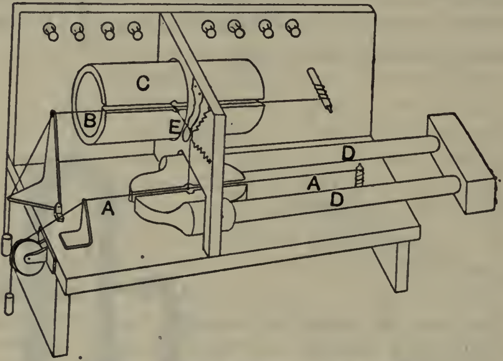

Fig. 283.—Ewing's Magnetic Curve Tracer.

A horizontal wire connects the back of the mirror to a wire B traversed by the magnetising current, and stretched between the poles of a constant electro-magnet C. When the magnetising current changes B tends to move across the lines of force between the pole-faces and causes a displacement of the mirror round a vertical axis. The specimen under examination is in the form of rods DD, fitted with a yoke and pole-pieces of soft iron; a wire AA is stretched between the pole-pieces, is also connected by a vertical wire to the back of the mirror, and carries a constant current. The wire AA is acted upon by vertical electro-magnetic forces proportional to the magnetic induction passing between the pole-pieces, and causes a movement of the mirror about a horizontal axis. Rapid cycles of magnetisation are obtained by means of an alternating current passing round the magnetising coils and through the wire B.

Weber's Molecular Theory.—According to this theory each molecule of magnetic material is a complete magnet. Before the mass is magnetised the molecules are arranged in a haphazard manner forming more or less complicated groups; the result of magnetisation is to re-arrange the individual molecules so that their similar poles point in one direction. It was suggested that the gradual increase in the intensity of magnetisation is due to a kind of *frictional resistance* which both impedes the magnetisation and also causes the acquired magnetisation to be retained more or less completely.

Several of the fundamental phenomena observed are not explained readily by the assumption of a frictional resistance, and Prof. Ewing has demonstrated completely that the constraint to re-arrangement may be attributed to the mutual magnetic forces which the molecules exert upon one another. To consider one point alone, it is necessary that any hypothesis should explain the three stages of magnetisation observed in Fig. 276. In the first stage there is no retentivity and the susceptibility is small; in the second stage the magnetisation is acquired rapidly and the retentivity is well marked; in the third stage the rate of acquiring magnetisation is again slow. These stages can be traced readily if we consider the behaviour of a group of four molecular magnets (Fig. 284, i) so arranged that they exhibit no external magnetic force. If a weak external magnetic field H is now applied the molecules are deflected but slightly (Fig. 284, ii); if H is increased gradually a stage is reached when the group breaks up and assumes the arrange-

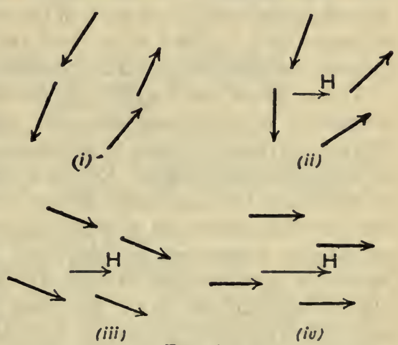

FIG. 284.

ment of Fig. 284, iii; any further increase in **H** will tend only to bring the molecules into more accurate parallelism. When **H** is reduced there will be but slight alteration in the group until the field is so weak that the molecules return suddenly to the arrangement in Fig. 284, ii, at which point **H** is less than that necessary to break up the group. In this manner, by considering the mutual magnetic forces in molecular groups, it evidently is possible to imitate not only the three stages of magnetisation (Fig. 276) but also the preliminary stage of hysteresis. Prof. Ewing has observed successfully all the main features of the magnetising forces by subjecting a group of small pivoted magnets, *e.g.* compass-needles, to a varying magnetic field produced by two coils of wire fixed on opposite sides of the group and traversed by an electric current. The magnetisation of the group may be observed by means of a magnetometer, providing that the direct effect of the coils on the magnetometer is neutralised by that of a third coil traversed by the same current and fixed in a suitable position.[1]

Theory alone can suggest why the molecules of iron have the properties of little magnets, and Ampère's theory, that an electric current circulates round each molecule, is regarded still as fundamentally correct. Recent investigation of the discharge of electricity through gases affords much evidence in favour of the view that each atom of matter includes, and perhaps consists of minute corpuscles, or electrons, charged negatively and revolving with extreme velocity like satellites in orbits within the boundary of the atom; and Prof. Rowland has shown that an electrostatic charge moving with high velocity has the same magnetic effect as an electric current.

Effect of Change of Temperature on Permeability.—Experiments conducted by Dr. Hopkinson have proved that the influence of temperature on the permeability of iron depends upon the strength of the magnetising force; thus, if the magnetising force is small, the permeability is increased largely when the temperature is raised; but with strong magnetic forces the permeability is diminished when the temperature is raised considerably. Fig. 285 represents the relation between permeability and temperature for three different magnetising forces: it is important to notice, when $\mathbf{H}=0.3$, the sudden increase and equally sudden loss of permeability at a temperature just below the critical temperature, 785° C., at which the iron ceases to be magnetic.

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[1] For further information on the subject of Magnetic Induction the student is referred to Ewing's *Magnetic Induction in Iron and other Metals* (Electrician Publishing Co.).

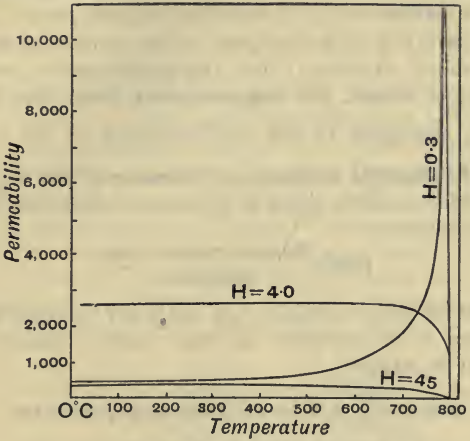


FIG. 285.—Permeability at different temperatures.

THE MAGNETIC CIRCUIT

Magnetic Reluctance and Reluctivity.—The total flux generated in a magnetic substance depends both on the magnetising force and also on the dimensions and permeability of the substance. Thus, consider the simplest case of an iron ring of circular cross-section, having a mean length l cms. and cross-section A sq. cms., round which is wrapped a magnetising coil of N turns. If the coil is traversed by a current of C C.G.S. units, then $\mathbf{H}=4\pi NC/l$.

$$\textbf{Magnetic flux } (\mathfrak{N}) = \mathbf{B} \times A = \mu \mathbf{H} A$$

$$= \frac{\mu \,.\, 4\pi NC \,.\, A}{l}$$

$$= \frac{4\pi NC}{\frac{l}{A} \times \frac{1}{\mu}} \quad \text{..................................(1)}$$

The following units were agreed upon by the Paris Congress of 1900.—

(i) *The* **Gauss** *is the* C.G.S. *unit of magnetic field*, and

(ii) *The* **Maxwell** *is the* C.G.S. *unit of magnetic flux.*

Equation (1) is quite analogous to the expression for Ohm's Law in voltaic electricity; thus, the quantity $4\pi NC$, analogous to E.M.F., is termed the **Magneto-motive Force** (M.M.F.); and $\left(\frac{l}{A} \times \frac{1}{\mu}\right)$, analogous to the total resistance of the circuit, is termed the **Magnetic Resistance** or **Reluctance** of the iron circuit. It is therefore usual to speak of a *magnetic circuit*, and we may generally say that

$$\textbf{Flux} = \frac{\textbf{Magneto-motive Force}}{\textbf{Reluctance}}.$$

It must be noted carefully that the M.M.F. is equal to $4\pi NC$ only when C is expressed in C.G.S. units. Hence if C is expressed in amperes,

$$\text{M.M.F.} = \frac{4\pi}{10} \times NC = 1.257 \times \textit{total ampere-turns.}$$

Also, since M.M.F. $= \mathbf{H} \times l$, or $\mathbf{H} = \text{M.M.F.}/l$

$$\therefore \ \mathbf{H} = 1.257 \times \textit{ampere-turns per cm. length.}$$

(From this we see that a field of unit strength, *i.e. one Gauss*, is obtained when the ampere-turns per cm. length $= \frac{1}{1.257} = 0.8$.)

Moreover, the expression $\left(\frac{l}{A} \times \frac{1}{\mu}\right)$ is comparable to $\left(\frac{l}{A} \times \rho\right)$, the expression for the electrical resistance of a wire having a specific resistance ρ. *The reciprocal* $\frac{1}{\mu}$ *is termed the* **reluctivity** *of the magnetic substance.* Hence

$$\textbf{Reluctance} = \textbf{Reluctivity} \times \frac{\textbf{Length}}{\textbf{Cross-Section}}.$$

Reluctivity may therefore be defined as **the magnetic resistance offered to magnetic flux between two opposite faces of a unit cube of the substance.**

The analogy of equation (1) to that of Ohm's Law is not complete, for the specific resistance of a conductor at a given temperature is independent of the current strength, while the value of $1/\mu$ varies considerably according to the strength of the magnetic field. Also, in an electric circuit, the expenditure of energy is maintained as long as the current continues, but in the case of the magnetic circuit energy is required to *create* the flux but not to maintain it.

The Magnetic Circuit.—In practical applications of the magnetic circuit, we seldom have the simple case in which the circuit is iron throughout; it generally includes one or more air-gaps, which offer great resistance to the passage of magnetic flux. The total reluctance is obtained by adding together the reluctances of the separate portions of the circuit. Thus, in the case of an anchor ring from which a small section has been cut and removed, if l_1 and l_2 are the lengths of the iron and air portions of the circuit, and if μ_1 and μ_2 are the permeabilities of iron and air, then (if C is expressed in amperes),

$$\text{Flux (N)} = \frac{4\pi NC}{10\left\{\frac{l_1}{A\mu_1}+\frac{l_2}{A\mu_2}\right\}} = \frac{1.257\times NC}{\frac{l_1}{A\mu_1}+\frac{l_2}{A\mu_2}} \quad \text{..............(2)}$$

Magnetic Leakage.—Allowance has not been made in equation (2) for the *leakage* of lines of force from the magnetic circuit. This leakage is due to the fact that there are no *insulators* for magnetic lines, and is a particularly important factor where the circuit includes air-gaps. If a given flux N_g is required across the air-gap, a greater flux N_m must be generated in the magnet, the difference being the quantity of flux lost by leakage. *The ratio* N_m/N_g *is termed the coefficient* (ν) of magnetic leakage; hence

$$\nu = \frac{\textit{Total Flux generated}}{\textit{Useful Flux}}.$$

Practical problems on the magnetic circuit consist largely in the determination of the ampere-turns necessary to produce a given flux in a certain portion of the magnetic circuit. In such cases it is best to calculate the ampere-turns required for each of the several portions of the circuit, and the *total* ampere-turns required is given by the sum of the ampere-turns necessary for each portion. Thus, when a flux **N** is required across an air-gap cut in an anchor ring, let l_1 = length of iron, l_2 = length of air-gap, A = cross-section of iron or air gap, ν = leakage coefficient, then $\nu\mathbf{N}$ lines must be generated in the iron, and

$$N_1C_1 \text{ required to generate } \nu\mathbf{N} \text{ lines in the iron} = \nu\mathbf{N} \times \frac{l_1}{A\mu_1} \div 1.257.$$

$$N_2C_2 \quad \text{,,} \qquad \text{,,} \qquad \mathbf{N} \qquad \text{,,} \qquad \text{air-gap} = \mathbf{N} \times \frac{l_2}{A\mu_2} \div 1.257.$$

$$\text{Total ampere-turns } (NC) = N_1C_2 + N_2C_2$$

$$= \frac{\mathbf{N}}{1.257}\left\{\frac{\nu l_1}{A\mu_1} + \frac{l_2}{A\mu_2}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In the example under consideration $\mu_2 = 1$, and the value of μ_1, which depends upon the flux νN, would be obtained from a **B**/**H** curve of the same material.

Example.—An iron ring, 10 cms. mean radius and 5 sq. cms. cross section, has an air-gap 1 cm. wide. The ring is wound uniformly with a coil of 900 turns. A field of 10,000 C.G.S. units is required in the air gap. If the permeability of the iron is 2000, and if there is no appreciable leakage, find the strength of current required.

$$l_1 = (2\pi r - 1) = 61.82 \text{ cms.},$$

$$l_2 = 1 \text{ cm.},$$

$$\mathbf{N} = 10,000,$$

$$\mu_1 = 2000,$$

N_1C_1 required to generate 10,000 lines in the iron

$$= \frac{10,000}{1.257} \times \frac{61.82}{5 \times 2000} = \frac{61.82}{1.257},$$

N_2C_2 required to generate 10,000 lines in the air

$$= \frac{10,000}{1.257} \times \frac{1}{5} = \frac{2000}{1.257}.$$

$$\text{Total ampere-turns } (NC) = \frac{2061.82}{1.257}.$$

But $\qquad N = 900;$

$$\therefore \; C = \frac{2061.82}{900 \times 1.257} = 1.82 \text{ amperes.}$$

Electro-magnets.—Fig. 3 represents the principle upon which a horse-shoe electro-magnet is constructed. The direction of the current round the limbs of the magnet must be in *opposite* directions, so as to give opposite polarity. The actual polarity is determined by the rule stated on p. 253.

The material and form of an electro-magnet which may be desirable depends upon the purpose for which the magnet is intended. *If only required to support a heavy weight*, a minimum reluctance is necessary, and the core should therefore be as short as possible. Electro-magnets required *to exert force on the armature, even at considerable distances*, should have their poles as far apart as possible, which will tend to prevent the lines of force from proceeding between the poles without also traversing the armature; also, since the reluctance of the air-gaps is so great compared with that of the core, it is not necessary to restrict the length of the core. Electro-magnets employed in *telegraph instruments or signalling appliances* must be capable of rapid action; the coercivity must therefore be very small, and the core should be of the softest wrought iron. Moreover, since the self-demagnetising action of short magnets is very marked, the cores should be very short; but, as the demagnetising action would be absent if the circuit were all iron, it is usual to prevent actual contact of the armature by means of stops, or by facing the poles with paper. If an electro-magnet is required *to exert maximum traction over a short distance*, it is important to have the reluctance low, and the flux as concentrated as possible. This is obtained in the **iron-clad** electro-magnet (Fig. 286), in which a bar electro-magnet is surrounded by an iron shell connected at one end to the iron core, thus giving a central pole surrounded by an annular opposite pole. A disc of iron forms a suitable armature.

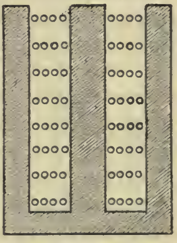


FIG. 286.—An Iron-clad Electromagnet.

The variation of magnetic flux density from the flat end of a bar electro-magnet is of considerable interest. The flux-density is much greater from the edge than from more central portions, which is probably due to the tendency of lines of force to take the shortest path through the air towards the opposite pole. The effect may be observed by suspending an iron ball from a spring balance, and determining the force necessary to separate the ball from various points of the surface. Also, since iron always tends to move from weaker to stronger portions of a field, an iron ball placed on the flat face of an electro-magnet will roll towards the outer edge.

It has been observed that if the surfaces of contact of an electro-magnet and its armature are rounded off, the lifting power is not diminished, and may even be increased. This apparent paradox is due to the fact that the flux, which naturally tends to restrict itself to the iron, is concentrated into a much smaller area, thus increasing the value of $\mathbf{B}$ at the surfaces of contact; since the lifting power (p. 390) is equal to $\mathbf{B}^2A/8\pi$, a diminution of A is more than balanced by the increased value of $\mathbf{B}^2$. For the same reason a bar magnet will exert a greater pull on its keeper when the pole faces are slightly rounded than when perfectly flat.

Dimensions of Electro-magnet Coils.—In designing electro-magnets it is necessary to calculate the number of ampere-turns necessary to impart to the iron a given lifting-power per unit area of pole surface. Suppose, for example, we require an electro-magnet of uniform cross-section to have a lifting power of 5840 gms. per sq. cm. The lifting power per unit area is given by the formula $Wg = \mathbf{B}^2/8\pi$, or $\mathbf{B} = \sqrt{8\pi Wg}$; hence, if $W = 5840$ gms., then $\mathbf{B} = 12000$ C.G.S. units. The value of μ, corresponding to this value of $\mathbf{B}$, must be obtained from a $\mathbf{B}/\mathbf{H}$ curve for the same material. Finally, from equation (3) (p. 402),

$$\mathrm{NC} = \frac{\mathbf{B}A}{1.257}\left\{\frac{\nu l_1}{A\mu_1} + \frac{l_2}{A\mu_2}\right\} = \frac{\mathbf{B}}{1.257}\left\{\frac{\nu l_1}{\mu_1} + \frac{l_2}{\mu_2}\right\} \text{............} (4)$$

EXAMPLE.—An electromagnet is required to exert a pull equal to the weight of 20 kilograms upon its armature when distant 0.5 cm. from each pole. The length of magnet core (l_m) is 40 cms., length of armature (l_a) is 10 cms., and cross-section of both core and armature is 5 sq. cms. If the coefficient of leakage is 1.4, determine the required number of ampere-turns.

(i) If $\mathbf{B}_g$ is the flux-density through the air gap, then

$$20{,}000 \times 981 = 2 \times \frac{\mathbf{B}_g{}^2 A}{8\pi} = \frac{10\mathbf{B}_g{}^2}{8\pi},$$

$$\text{or,} \qquad \mathbf{B}_g = 7020.$$

(ii) If $\mathbf{B}_a$ and $\mathbf{B}_m$ are the flux-densities throughout the armature and the magnet respectively; since $\mathbf{B}_a = \mathbf{B}_g = 7020$, then

$$\mathbf{B}_m = 7020 \times 1.4 = 9828.$$

From the **B/H** curve for wrought iron, p. 390, $\mu_a = 2510$, and $\mu_m = 2000$. The equation for the magnetic circuit is

$$\mathrm{NC} = \frac{\mathrm{B}}{1.257}\left\{\frac{\nu l_m}{\mu_m} + 2l_g + \frac{l_a}{\mu_a}\right\},$$

or,

$$\mathrm{NC} = \frac{7020}{1.257}\left\{\frac{1.4 \times 40}{2000} + 1 + \frac{10}{2510}\right\} = 5763 \text{ ampere-turns.}$$

Summary

Magnetic Induction.—The *magnetic induction* **B** in a piece of magnetic material is related to the *intensity of magnetisation* I and to the *intensity of the field* **H** by the equation $\mathbf{B} = 4\pi \mathrm{I} + \mathbf{H}$.

The **permeability** μ of the material is defined as the ratio of **B** to **H**, or

$$\mu = \mathbf{B}/\mathbf{H}.$$

The **susceptibility** κ of the material is defined as the ratio of I to **H**, or

$$\kappa = \mathrm{I}/\mathbf{H}.$$

Hence $$\mu = 1 + 4\pi\kappa.$$

The permeability of air is regarded usually as equal to unity.

Measurement of Permeability.—(i) In the *magnetometer method* the pole-strength acquired by the iron is observed by means of a magnetometer. (ii) In the *ballistic method* the current generated in a secondary coil surrounding the magnetising coil is measured by the throw of a ballistic galvanometer. (iii) In the *traction method* the induction **B** is calculated from the force required to separate two plane surfaces of the same material when magnetised to the same degree.

Curves of Magnetisation.—As the intensity of the magnetising force increases, the magnitude of the magnetic induction at first increases gradually; it then increases very rapidly even for small increments in **H**, and finally increases very slowly and tends to assume a maximum value. In this last stage the iron is approximately *saturated*. If now **H** is diminished gradually, the iron still indicates the effects of the previously stronger field; in fact the magnetic induction 'lags behind' the changing magnetising force, and gives rise to the phenomenon of **hysteresis.**

The **retentivity** of the material is measured by the residual induction when **H** is reduced to zero, and the **coercivity** is measured by the strength of the reversed force necessary to reduce the induction to zero.

If the field **H** is altered gradually through a complete cycle of changes, and if corresponding values of **B** and of **H** are plotted on squared paper, a *hysteresis curve* is obtained, which is characteristic of the material. The area of the curve is a measure of the energy dissipated in unit volume of the material during a complete cycle of changes.

Theory of Magnetisation.—Ewing's modification of Weber's theory explains all the phenomena of a magnetisation cycle and of hysteresis by a consideration of the mutual magnetic forces which the molecules exert upon one another.

The Magnetic Circuit

The **magnetic flux** (**N**) through a ring-shaped material of permeability μ, cross-section A, and length l, when magnetised by a coil of N turns and traversed by a current of C c.g.s. units, is expressed by the equation

$$\mathbf{N}=4\pi NC\Big/\left(\frac{l}{A}\cdot\frac{1}{\mu}\right).$$

The quantity $4\pi NC$ is termed the **magneto-motive force**, and $l/A\,.\,\mu$ is the **magnetic resistance** or **reluctance**. The reciprocal $1/\mu$ is termed the **reluctivity.**

In a compound circuit, consisting partly of iron and partly of other material, the above equation becomes

$$N=4\pi NC\Big/10\left(\frac{l_1}{A\mu_1}+\frac{l_2}{A\mu_2}\right),$$

where C is expressed in amperes.

Electro-magnets.—The dimensions of an electro-magnet may be calculated from the properties of the material by means of the principle of the magnetic circuit.

Questions on Chapter XXI

1. A ring of iron is uniformly wound with insulated copper wire, so that the annular core can be magnetised by means of an electric current. State how the resulting magnetisation can be calculated. Describe how it can be experimentally measured.

2. What is meant by the 'magnetic permeability' of a substance, and how can it be measured? Describe how the permeability of a piece of soft iron varies with the intensity of the magnetising force.

3. Describe the general construction of an electromagnet. Why is the copper wire insulated, and how? If an electromagnet is required to produce an effect at some distance from its poles should its core (of given length) be straight or curved to a horse-shoe form?

4. A magnet, dimensions $(10 \times 1 \times 0.5)$ cms., has a magnetic moment of 1200 C.G.S. units. Determine the intensity of magnetisation and the pole strength.

5. A straight piece of soft iron wire, 50 cms. long and 0.1 cm. radius, is placed horizontally in the magnetic meridian $(H=0.18)$. If the susceptibility is 30, find the intensity of magnetisation of the iron, and also its magnetic moment.

6. A bar of iron is magnetised to an intensity 500 when placed in a magnetic field of intensity 50 C.G.S. units. Determine the susceptibility, and also the flux density at the pole face.

7. It is found that a force equal to the weight of 5 kilograms is required to separate two bars, the faced ends of which are touching and surrounded by a magnetising coil. If the cross-section of each bar is 5 sq. cms., determine the magnetic induction per sq. cm. through the joint.

8. You have to make a long coil of wire such that when 10 amperes flow through it the magnetic field at the centre shall be 1000 times as strong as the earth's horizontal field $(H=0.18)$. How many turns per cm. length must be put on the coil?

9. Determine the intensity of the magnetic field at the centre of a solenoid 40 cms. long, containing 1600 turns, and traversed by a current of 0.3 amperes. How many lines of force per sq. cm. will there be at the centre of the solenoid?

10. An iron rod 1 metre long and 1 sq. cm. cross-section is surrounded by a solenoid of 850 turns. If the permeability is 700, find the magnetic flux through the iron when a current of 1.5 amperes traverses the solenoid.

11. A rod of annealed wrought iron, 0.4 cm. in diameter and 1 metre long, is wound uniformly with 500 turns of insulated wire. Find (from the data on p. 392) what strength of poles the rod will acquire when the coil is traversed by a current of 1.63 amperes.

12. A closed soft iron ring, 100 cms. mean circumference and 5 sq. cms. cross-section, is wound uniformly with 200 turns of insulated wire. Supposing that you have found the following relations to exist in iron of this quality,

B, -	10,200	12,000	13,700
μ, -	2,000	1,500	1,000,

calculate the current, in amperes, at which the total flux of magnetic lines is 65,000.

13. With the same specimen of iron as in Question 12, determine the magnetic reluctance of the core of a transformer, 80 cms. long and 20 sq. cms. cross-section, when the flux is (i) 10,200, (ii) 12,000, and

(iii) 13,700. Determine the number of ampere-turns required to produce each of these flux-densities.

14. An iron ring, 20 cms. mean diameter and 10 sq. cms. cross-section, is magnetised by a coil of 500 turns traversed by a current of 8 amperes. If the permeability is 600, determine the magneto-motive force of the coil, the reluctance of the ring, the total magnetic flux, and the magnetic induction.

Determine the reluctance and the total magnetic flux if an air-gap 10 mms. wide is made in the above ring.

15. A watch has had its rate spoiled by being brought near a dynamo. It is hung by a piece of string between the poles of a large steel magnet and made to spin round. While spinning it is slowly removed from the magnet and then is found to be nearly cured. How is this explained?

16. Show how it is possible to have a rod of soft iron magnetised to different intensities at different times, though under the influence of the same magnetising force. Explain this result.

17. In what respects do the magnetic properties of iron and steel differ? Define the terms intensity of magnetisation (I), induction (B), and magnetic force (H).

How do you obtain the relation $B=H+4\pi I$, either theoretically or experimentally?

18. Explain what is meant by residual magnetism, coercive force, permeability.

Draw a curve showing the manner in which the magnetism induced in a soft iron rod varies as the magnetising field is taken through a cycle, and state in a general way how from this diagram you would obtain the residual magnetism, coercive force and permeability of the iron.

19. What is meant by magnetic permeability? Describe and discuss any method for determining it.

CHAPTER XXII

ELECTROMAGNETIC INDUCTION

Faraday's Experiments.—We have seen already that a magnetic field is set up in the space surrounding a wire conveying a current. If the current and magnetic field are connected indissolubly we might anticipate that the creation of a magnetic field round a closed circuit will cause a current to traverse the circuit. Faraday, in 1831, showed that whenever a circuit is moved in a magnetic field in such a manner as to vary the number of lines of force passing through the circuit, then an E.M.F. is set up proportional to the rate of change of the lines of force traversing it and continuing so long as the change continues. The following statement is known as **Faraday's Law:—When the number of lines of force through a circuit is changing, an induced E.M.F. is set up, and the magnitude of the E.M.F. is proportional to the rate at which the number of lines of force changes.**

Fig. 287 represents Faraday's initial experiment. AB and CD are two parallel wires, the former joined to a battery and key, the latter to a galvanometer. AB is termed the *primary*, and CD the *secondary* circuit. On closing the primary

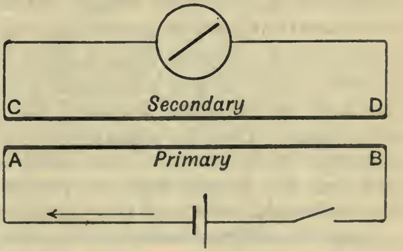


FIG. 287.—Faraday's Initial Experiment.

circuit a momentary current is observed in CD, and its direction is *opposite* to that in AB. On breaking the primary circuit a momentary current, in the *same* direction as that in AB, is observed in CD.

Faraday applied the term **Induced Currents** to currents generated in this manner. He also observed that (i) **an inverse induced current is obtained when the primary current begins, or when it increases, or when it is brought nearer to the secondary circuit**; and (ii) **a direct induced current is obtained when the primary current ceases, or diminishes, or is removed to a greater distance.** All of these effects can be observed more readily if coils of wire (Fig. 288) are used instead of straight wires. The effects are also more marked if a soft iron core is inserted in the primary coil. Since the magnetic field due to a current traversing a coil is similar to that of a bar magnet it follows that identical results are obtained if a bar magnet is used instead of a primary coil.

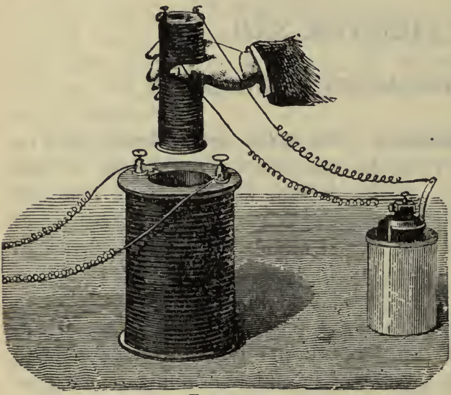


Fig. 288.

Expt. 77 (i).—Observe which terminal of the galvanometer must be positive in order to give a deflection to the right. Place the primary coil (Fig. 288) inside the secondary, and at some distance from the galvanometer. Close the primary circuit and observe the direction of deflection. Note how the needle comes back to zero, and is deflected in the opposite direction when the primary circuit is broken. Verify that the induced currents are *inverse* and *direct* respectively.

Repeat the observations with an iron core inserted in the primary.

(ii) Remove the primary coil to a distance, and close its circuit. Move it quickly towards the secondary, and (when the needle is again at rest) remove it to a distance. Verify the inverse and direct currents.

(iii) Insert the primary coil into the secondary, and include a resistance box in the primary circuit, so as to vary the primary current. Verify that an increase or diminution of the primary current induces an inverse or direct current in the secondary.

(iv) Repeat Expt. ii, using a bar-magnet instead of the primary coil. Verify that when a *north-seeking* pole is brought towards the secondary coil the induced current is such that the near end of the coil acquires *north-seeking* polarity; also that when the same pole is withdrawn the polarity of the same end of the coil is *south-seeking*.

Telegraphy by Electro-magnetic Induction.—The more important attempts to apply electro-magnetic induction to telegraphy were made by Sir William Preece in 1885 and following years. An experiment was conducted with parallel telegraph wires, about 10 miles apart, between Durham and Darlington; it was found that audible sounds could be produced in a telephone included in one circuit by the ordinary telegraphic currents transmitted in the other. In a later experiment signals were transmitted between the sea-coast near Cardiff and an island about 3 miles distant. A wire, over 1200 yards long, supported on insulators and earthed at both ends was connected in series with an alternator and a key; and a guttapercha covered wire, 600 yards long, was laid on the island, its ends also being earth-connected. A telephone included in the latter circuit was readily sensitive to the currents transmitted along the wire on the mainland.

Lenz's Law.—Energy is required in order to generate the induced currents observed in the preceding experiments. Since there is no source of energy in the secondary coil, the energy represented by the induced currents must be due to some external agency; in fact, in Expt. 77, (ii) and (iv), it originates from the *mechanical work* done in overcoming the mutual forces of attraction or repulsion which exist between the currents in the two circuits. Thus, if the circuits are approaching, the direction of the induced current is such that it exerts a repulsion and impedes the movement: also, if the circuits are receding, the induced current is such as to cause attraction. A general statement of this result is known as *Lenz's Law*, which may be stated thus: **The induced current is in such a direction that its reaction tends to stop the motion or change to which the induced current is due.**

General Principles.—Let AB (Fig. 289) represent a metal rod sliding on horizontal parallel rails, connected to a battery and placed in a uniform vertical magnetic field of intensity H. By applying Fleming's Rule (p. 262) we see that AB will tend to move in the direction of the force F; and since the work done by the force is equal to the product of the current and the number dN of lines of force cut by the rod (p. 265), then

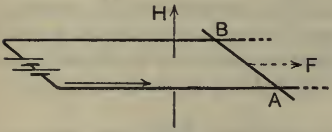


FIG. 289.

$$\text{Work done} = C \times dN.$$

But the energy required to do this work must be derived from the battery circuit; and we may apply to this case the principle explained on p. 351, according to which the total energy taken from the battery during a period of time dt is equal to the energy spent in generating heat and in doing any other work in the circuit. Hence

$$ECdt = C^2Rdt + CdN, \quad \ldots\ldots\ldots\ldots\ldots(1)$$

or

$$E - CR = \frac{dN}{dt}.$$

But, by Ohm's Law, CR is equal to the sum of the E.M.F. due to the battery and that which necessarily arises from the bar cutting lines of force. If e represents the latter, then $CR = E + e$.

Hence

$$E - (E + e) = \frac{dN}{dt},$$

or

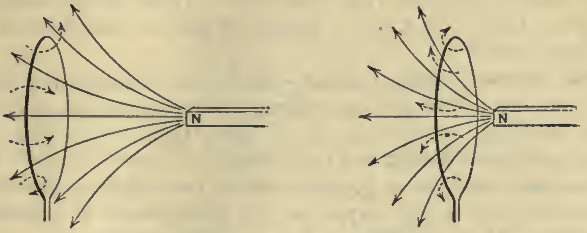
$$e = -\frac{dN}{dt} \text{ C.G.S. units.}$$

The induced E.M.F. is therefore equal to the rate at which lines of force are cut by the bar. The negative sign indicates that the direction of the induced E.M.F. is such that it *opposes* the change in flux to which the E.M.F. is due.

If the length of the bar is l cms., and its velocity is v cms. per sec., then

$$e = -\frac{dN}{dt} = -lvH \text{ C.G.S. units.}$$

By referring to Fig. 289 it is evident that an increase in the magnetic flux through the circuit induces a current which tends to create a flux in the *opposite* direction to that which causes the induced current. Also Fig. 290 (i) represents a north-seeking pole approaching a single turn of wire, the number of lines of force through the coil is being thereby increased ; the induced current will generate lines of force in the *opposite* direction thus



I. Coil approaching. *II. Coil receding.*

FIG. 290.

tending to neutralise the increase due to the magnet. The same reasoning applies to Fig. 290, (ii), which represents a magnet-pole receding from the coil. By considering these and any similar cases we obtain the following rule : **When the number of lines of force passing through a circuit is increased or diminished the induced current will be in that direction which tends to keep the number of lines of force constant.**

The direction of the induced current may also be deduced by means of Fleming's **Right-hand Rule** (Fig. 291) : **Hold the thumb and first finger of the right hand as fully extended as possible and bend the second finger at right angles to the palm. If the first finger represents the direction of the lines of force, and the thumb that of the motion, then the second finger will point in the direction of the induced current.**

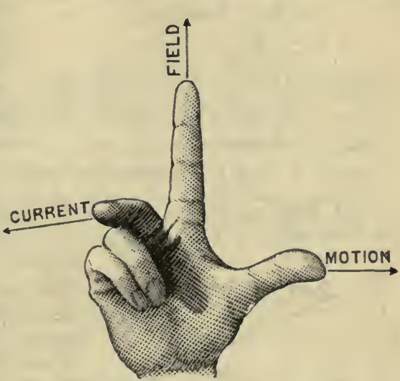


FIG. 291.

If the circuit through which the flux is changing consists of *two* complete turns, the same E.M.F. is induced in *each* turn, and the total E.M.F. between the extreme ends will be twice as great as that generated in a single turn. If the circuit consists of n turns, the total E.M.F. is n times as great as that induced in a single turn.

The absolute units of E.M.F.—If, in the equation $e = -Hlv$ (p. 412), the length and the velocity of the moving rod, and the field intensity, are each equal to unity, then $e = -1$, and the E.M.F. set up in the bar is 1 C.G.S. unit.

As mentioned on p. 293, this unit is too small for practical purposes, and a unit equal to 10^8 C.G.S. units is generally adopted. This practical unit is termed the **volt.**

EXAMPLES.—(i) A railway carriage is travelling at the rate of 40 miles an hour. If the axle is 4 ft. 8½ ins. long, and the vertical intensity of the earth's field is 0.44, find the E.M.F. between the wheels at opposite ends of an axle.

$$v = (\tfrac{2}{3} \times 88) \text{ feet. per sec.} = \tfrac{176}{3} \times 30.48 \text{ cms. per sec.}$$
$$l = 56.5 \times 2.54 \text{ cms.}$$
$$H = 0.44.$$

Number of lines cut per second $= Hlv$

$$= 0.44 \times (56.5 \times 2.54) \times (\tfrac{176}{3} \times 30.48)$$
$$= 112900;$$
$$\therefore \text{ E.M.F.} = 112900 \text{ C.G.S. units} = 0.001129 \text{ volt.}$$

(ii) A single north-seeking pole, of strength 250, is situated 20 cms. from the centre of a single ring of wire and on a line drawn perpendicularly to its plane (Fig. 292). Find the average current generated in the wire when the pole is removed in 1 second to a distance of 200 cms. from the ring (radius of wire ring $= 40$ cms., resistance $= 10^{-3}$ ohm).

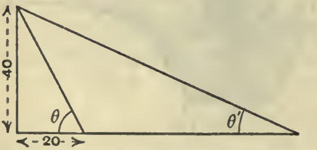


FIG. 292.

The solid angle (p. 563) subtended by the ring is $2\pi(1 - \cos\theta)$, and the number of lines of force passing through the ring is $2\pi m(1 - \cos\theta)$, where m is the pole-strength. In the final position, the number of lines enclosed is $2\pi m(1 - \cos\theta')$.

Since the change takes place in 1 second, the average *rate of change* of the number of enclosed lines is $2\pi m(\cos\theta' - \cos\theta)$.

$$\text{The average E.M.F.} = \frac{2\pi m(\cos\theta' - \cos\theta)}{10^8} \text{ volt.}$$

$$\text{,, ,, current} = \frac{2\pi m(\cos\theta' - \cos\theta)}{10^8 \times 10^{-3}} \text{ ampere.}$$

But $\cos\theta' = 0.9804$, $\cos\theta = 0.4474$, and $m = 250$.

Hence average current $= 0.00853$ ampere.

Faraday's Disc.—Faraday's disc was the first machine constructed to produce a steady current by the rotation of a conductor in a magnetic field. A copper disc (Fig. 293) is rotated between the poles of a horse-shoe magnet, and metal springs which touch the axle and circumference of the disc are connected to a galvanometer. When the disc is rotating the radius connecting the springs may be regarded as a conductor moving rapidly across a magnetic field, and an E.M.F. is set up along the radius. The direction of the induced E.M.F. may be deduced by applying Fleming's Rule. Assuming that the magnetic field is uniform, the magnitude of the E.M.F. is calculated in the following manner: If n = revolutions per sec., r = radius of disc, H = intensity of field, then

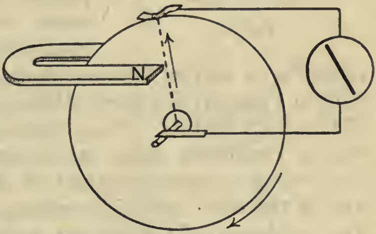


FIG. 293.—Faraday's Disc.

the area traced out by any radius $= n\pi r^2$ sq. cms. per sec.,

lines of force cut per sec. $= \mathrm{H}n\pi r^2$.

$\therefore$ E.M.F. $= \mathrm{H}n\pi r^2/10^8$ volt.

In this case we have an application of Lenz's Law. By Fleming's Left-hand Rule (p. 262) a current traversing the disc from axle to rim, as in Fig. 293, will tend to make the disc rotate in the opposite direction and thus to oppose the actual motion of the disc (cf. *Barlow's Wheel*, p. 265).

Foucault Currents.—Arago observed that if a copper disc

(Fig. 294), mounted horizontally and immediately below a pivoted magnet, is rapidly rotated, the magnet is deflected in the direction of the disc's motion, and that the deflection increases with the speed of the disc. This effect is due to the induced currents set up in the disc; the portion immediately underneath the N.-seeking pole will, by Fleming's Rule, carry an induced current proceeding from rim to axle; the current induced in the portion under the S.-seeking pole will be from axle to rim. These are equivalent to a current passing along a diameter (from left to right in Fig. 294) which will deflect the magnet in the direction of the disc's motion. The circuit of these induced currents will be completed along the circumference of the disc, thus constituting a magnetic shell of N.-seeking polarity in one half of the disc, and of S.-seeking polarity in the other half. The movement of the magnet may then be explained by the mutual forces of attraction and repulsion between these shells and the poles of the magnet. Moreover, the polarity of the shells may be determined by applying Lenz's law. Currents thus set up in solid conductors are sometimes called **Foucault currents.**

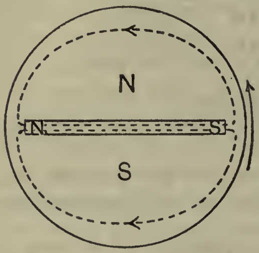


FIG. 294.

The energy represented by these induced currents is dissipated in the form of heat. The temperature of a metal plate is raised considerably when rotated rapidly between the poles of a strong electro-magnet. Also, the mechanical energy required to rotate the disc is far greater when the electro-magnet is excited than when the magnet is inactive; the difference in the energy required is represented by the heat developed in the disc.

This principle is applied in some dead-beat types of galvanometer by suspending the magnet in a solid metal chamber. The swinging magnet creates Foucault currents in the surrounding metal; such currents impede the movement of the magnet, and soon bring it to rest.

Self-Induction.—Faraday showed that when a current is established in a circuit the change in the magnetic field sets up in the same circuit induced currents which are superposed upon the principal current. By Lenz's law, the induced currents thus set up will tend to weaken an increasing current and to strengthen one which is diminishing. This phenomenon is termed **self-induction.** Owing to this effect, a current does not immediately assume its maximum value as soon as the circuit is closed; also, the momentary tendency of the current to continue, when the circuit is broken, is represented by the spark which appears at the point where the circuit is opened.

Faraday observed the effect in the following manner: The coil AB (Fig. 295) is connected to the galvanometer G and battery as shown in the illustration. If the needle is deflected into the position *ab* when the current is established, and is retained in that position by means of a stop, then on breaking and again closing the circuit the needle is deflected momentarily beyond *ab*: this is due to self-induction delaying the establishment of the current in AB, thus momentarily increasing the fraction of the total current which traverses G. Again, if the needle is held in its normal position *cd* by means of a stop, a deflection in the opposite direction is observed when the battery circuit is broken: in this case, the extra current in AB is in the *same* direction as the principal current, and proceeds through G from right to left.

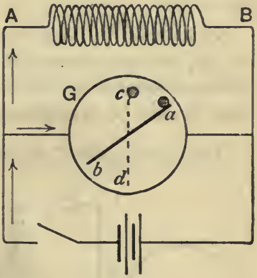


FIG. 295.

Coefficient of Self-Induction (or Self-Inductance).—The total number of lines of force passing through any circuit and due entirely to one C.G.S. unit current traversing the circuit is termed the Coefficient of Self-Induction (or the Self-Inductance) of the circuit.

Thus, if N is the number of lines of force linked with any circuit, of inductance L. and conveying a current C C.G.S. units, then

$$N = LC.$$

The induced E.M.F., due to a change in the flux through the circuit, is equal to $-d\mathrm{N}/dt$. Hence

$$e = -d\mathrm{N}/dt = -\mathrm{L}\,.\,d\mathrm{C}/dt.$$

The direction of the induced E.M.F. depends upon whether the current is increasing or decreasing. If $d\mathrm{C}/dt$ is positive, as when the circuit is first closed, e will then be negative. This explains the delay in the current assuming its maximum or *Ohm law* value. If $d\mathrm{C}/dt$ is negative, as when the circuit is broken, then e is positive, and gives rise to the *extra current* described on p. 417.

In general, it may be stated that the effect of self-induction is to oppose any *change* in the current traversing the circuit; and its resemblance to the inertia properties of matter is so marked that it may be termed *electro-magnetic inertia.*

This inertia effect is demonstrated clearly in the discharge of a Leyden jar through a single turn of wire (Fig. 296), two portions of which are about 1 mm. apart; the discharge may traverse either the wire circuit or the short air-gap, which constitutes an alternative path. Although the resistance of the wire is small, the sudden application of a high P.D. generates an induced E.M.F. in the wire in the opposite direction, and a portion of the discharge finds it easier to traverse the air gap, although it has a far higher ohmic resistance than the wire.

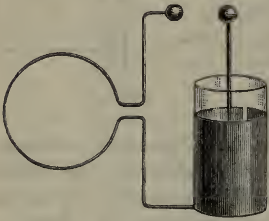


FIG. 296.

The practical unit of inductance is termed the **henry**; and a circuit has an inductance of one henry if a current of one ampere generates a flux of 10^9 lines of force through the circuit. Or, as is evident from the equation $e = -\mathrm{L}\,.\,d\mathrm{C}/dt$, we may say that **the inductance of a circuit is equal to one henry if an opposing E.M.F. of one volt is set up when the current in the circuit varies at the rate of one ampere per second.**[1] Profs. Ayrton and Perry have suggested the term **secohm** for the same unit.

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[1] If R is the resistance of the circuit in ohms, the ratio L/R is termed the *time-constant* (p. 431).

The flux through a solenoid of length l and with n turns per unit length, when traversed by a current of C C.G.S. units, is $4\pi n$C per sq. cm. (p. 259); if the cross-section is A sq. cms., the total flux is $4\pi n$CA. But each turn of the solenoid will embrace this number of lines of force, hence the inductance is

$$4\pi n \mathrm{CA} \times nl = 4\pi n^2 l \mathrm{CA} \text{ C.G.S. units.}$$

If a bar of iron, of cross-section A′, length l, and permeability μ, is inserted in the coil, the total flux is

$$4\pi n \mathrm{C}(\mathrm{A}-\mathrm{A}') + 4\pi n \mathrm{C} \, . \, \mu \mathrm{A}' = 4\pi n \mathrm{C}(\mathrm{A}-\mathrm{A}'+\mu \mathrm{A}') \, ;$$

hence the inductance is

$$4\pi n^2 l \mathrm{C}(\mathrm{A}-\mathrm{A}'+\mu \mathrm{A}').$$

EXAMPLE.—Calculate the inductance of a coil of 100 turns of wire wound on an iron ring of 20 cms. mean diameter and 10 sq. cms. cross-section, the permeability of the iron being 700.

If N = total number of turns, and r = mean radius of the ring, then, since $\mathrm{C}=1$,

$$\mathrm{L} = 4\pi n^2 l \mathrm{A}\mu = 4\pi \frac{\mathrm{N}^2}{l^2} \, . \, l \mathrm{A}\mu = 4\pi \frac{\mathrm{N}^2}{2\pi r} \, . \, \mathrm{A}\mu$$

$$= \frac{2 \times 10^4}{10} \times 10 \times 700 = 14 \times 10^6 = 0.014 \text{ henry.}$$

Mutual Induction.—The **Coefficient of Mutual Induction** of two coils may be defined as **the total magnetic flux which passes through one of the coils when the other is traversed by the C.G.S. unit of current.** As in self-induction, the practical unit is the **henry**, and is equivalent to a flux of 10^9 lines through the circuit.

The mutual induction is constant for any pair of coils in any given position, whatever may be the strength of the current, providing that no iron is within or near the coils. But, in the presence of iron, the flux is by no means proportional to the current, owing to variation of the permeability of iron.

Suppose two co-axial solenoids of wire. If the inner one, which we may call the primary, has length l, cross-section A, n turns per cm., and is traversed by a current C C.G.S. units, the flux through the primary is $4\pi n$CA. If the outer or secondary coil consists of n_1 turns in all, each turn is traversed by the above flux, and the total flux through the secondary is

$4\pi nn_1$CA. But, by definition, the total flux is MC, where M is the coefficient of mutual induction. Hence,

$$M = 4\pi nn_1 A = 4\pi nn_1 A/10^9 \textit{ henries.}$$

Since the E.M.F. induced in the secondary is proportional to the *rate of change* of the flux through the primary,

$$e = -d(MC)/dt = -M \, . \, dC/dt.$$

(If M and C are expressed in C.G.S. units, then e will also be in the same units. But if M and C are expressed in henries and amperes then e will be in volts.)

As shown on p. 419, if a bar of iron, cross-section A′ and permeability μ, is inserted in the primary coil, the total flux is $4\pi n C(A - A' + \mu A')$, and

$$M = 4\pi nn_1(A + \mu A' - A').$$

The two coils of an induction coil giving a 10-inch spark have a coefficient of mutual inductance of about 5 henries.

EXAMPLE.—If in an induction coil the value of M is 5 henries, and a current of 10 amperes is entirely cut off in a period of $\frac{1}{5000}$ sec., what is the E.M.F. at the terminals of the secondary coil?

$$e = -M \, . \, dC/dt = -5 \times 10/\tfrac{1}{5000} = -250{,}000 \text{ volts.}$$

Measurement of Self-Inductance and Mutual-Inductance.—(i) SELF-INDUCTANCE.—If the resistances of the branches of a Wheatstone bridge are adjusted so that there is no deflection when the system is traversed by a steady current, a momentary deflection will still be observed when the circuit is closed or broken, if one of the resistances has appreciable self-inductance. Thus, if L is the self-inductance of the arm X (Fig. 225) of the bridge, and if C is the current traversing it, the reverse E.M.F. set up during the growth of the current is equal to $-L \, . \, dC/dt$ (p. 418). But any electromotive force E which may be set up in X will cause a current kE to traverse the galvanometer, where k is a constant depending on the resistances of the galvanometer and of the arms of the bridge. Hence the E.M.F. due to self-induction will cause a current $kL \, . \, dC/dt$ to traverse the galvanometer. If the current suddenly grows from zero to C, the total quantity of electricity sent through the galvanometer is kLC, and this quantity is measured by the throw of the galvanometer (if the instrument is ballistic); hence (p. 283)

$$k\mathrm{LC}=\frac{\mathrm{H}}{\mathrm{G}}\cdot\frac{\mathrm{T}}{\pi}\cdot\sin\frac{\theta}{2}.$$

The quantities k and C are eliminated from this equation by the following method: If a small resistance r is inserted in the arm X, it will have the same effect as introducing a reverse E.M.F. equal to rC, since the addition of a small resistance will not alter the current to an appreciable degree; and the steady current which will now traverse the galvanometer is krC. If the steady deflection of the galvanometer thus caused is equal to α, then

$$kr\mathrm{C}=\frac{\mathrm{H}}{\mathrm{G}}\cdot\tan\alpha$$

$$\therefore\ \mathrm{L}=\frac{1}{k\mathrm{C}}\cdot\frac{\mathrm{HT}}{\mathrm{G}\pi}\cdot\sin\frac{\theta}{2}=r\cdot\frac{\mathrm{T}}{\pi}\cdot\frac{\sin\frac{\theta}{2}}{\tan\alpha}.$$

(ii) MUTUAL INDUCTANCE—Let A and B (Fig. 297) represent the two coils of which the mutual inductance is to be measured. The coil A is connected to a battery, and B to a ballistic galvanometer. If C is the steady current established through A, the total magnetic flux through B will be MC; and, if R is the resistance of the circuit through B, the quantity of electricity generated in the circuit is given by the equation

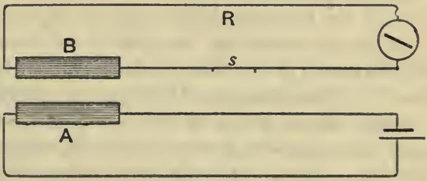

FIG. 297.

$$\frac{\mathrm{MC}}{\mathrm{R}}=\frac{\mathrm{H}}{\mathrm{G}}\cdot\frac{\mathrm{T}}{\pi}\cdot\sin\frac{\theta}{2}.$$

The following method[1] of eliminating the ratio H/G and C may be used: Break the A circuit and connect the ends to two points on the B circuit separated by a resistance s, which is small in comparison with the resistance R. The current through A is not appreciably altered, and the steady current passing through the galvanometer will be

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[1] J. J. Thomson's *Elements of Magnetism and Electricity* (Camb. Univ. Press), p. 438.

$$C \times \frac{s}{R+s}.$$

If α is the steady deflection produced, then

$$C \times \frac{s}{R+s} = \frac{H}{G} \cdot \tan \alpha.$$

$$\therefore\ M = \frac{T}{\pi} \cdot \frac{Rs}{R+s} \cdot \frac{\sin\frac{\theta}{2}}{\tan \alpha}.$$

The Ruhmkorff Induction Coil.—The principle of mutual induction between two circuits is applied in the Ruhmkorff coil to transform a *low* P.D. between the terminals of a primary coil into a *high* P.D. between those of a secondary coil. In any induction coil it is essential that

(i) the flux through the primary coil shall be as great as possible;

(ii) the secondary coil shall consist of as many turns of wire as possible; and

(iii) the primary circuit shall be rapidly made and broken.

Fig. 298 represents the details of a modern Ruhmkorff coil. The current enters the primary coil PP′ through the terminals at T. The primary coil consists of two or three layers of closely wound thick copper wire, and contains a core DD′ of soft iron wire—iron wire is used in preference to solid iron, so as to minimise the induced Foucault currents in the core. The secondary coil SS′, of very thin wire, is wound on an ebonite tube, with ebonite flanges at the ends; the length of wire in SS′ depends upon the length of spark required—usually about 1 mile is required for each inch length of spark. Owing to the high P.D. generated in the secondary, the proper insulation of each layer of wire is perhaps the most important point in the construction of the coil. It is necessary to keep well apart those portions of the coil between which there is considerable P.D., otherwise the insulation may break down. The diagram shows how the coil may be wound in separate sections, separated from each other by ebonite discs, thus avoiding any high P.D. between contiguous layers. The Apps *contact-breaker* consists of an iron hammer H carried by a brass spring S, and provided with a platinum contact piece p. In line with H is an adjustable screw B, with platinum contact p', carried by a stout brass pillar A. The screw U enables an adjustment of the free movement of the hammer, and is serviceable in modifying the character of the spark.

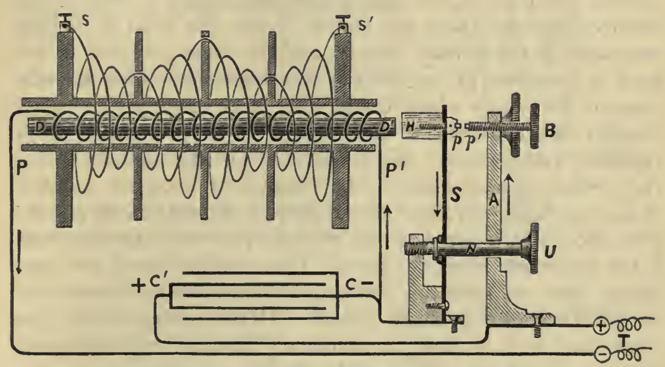

FIG. 298.—The Construction of a Ruhmkorff Coil.

The coil is started by screwing in B until p and p' are *nearly* in contact, then, by means of screw U, the hammer is drawn back until p and p' touch. The primary circuit is now closed, the iron core becomes magnetised, attracts the hammer, and breaks the circuit. The consequent removal of magnetic flux induces a high E.M.F. in the secondary in the same direction. The core having now lost its magnetisation, the hammer returns and again completes the primary circuit, thus creating an induced E.M.F. in the opposite direction in the secondary. This process is continued automatically by the movement of the hammer.

The self-inductance in the primary circuit creates a direct induced E.M.F. in PP′ when the circuit is broken; this is sufficiently great to cause an arc between the platinum contacts, and thus retards the cessation of the current. So also the induced E.M.F. on completing the circuit retards the growth of the current. In both cases the P.D. induced in the secondary is less than it would be if self-induction in the primary were absent. Moreover, the spark between the contacts rapidly burns away the platinum. This effect is minimised by connecting a *condenser* CC′, consisting of tinfoil sheets separated by paraffined paper, in parallel with the spark gap.

The action of the condenser is as follows: On breaking the circuit, the high E.M.F. due to self-inductance is distributed mainly along the spark gap (which is then the point of highest resistance in the primary circuit), and the same potential difference is imparted to the coats of the condenser, which absorbs most of the energy which would have been spent in the transmission of the spark. The charge in the condenser then *rebounds* and traverses the primary coil in the *opposite* direction, which is equivalent to the removal of a certain number of lines of force *and their re-insertion in the opposite direction*; thus, the E.M.F. induced in the secondary is much greater than if the condenser were not there. On the other hand, the condenser does not influence the magnitude of the induced E.M.F. on 'making' the primary circuit. Hence, except when the spark-discharge is short, a secondary spark is only obtained when the primary circuit is 'broken': in which case the secondary discharge is *an intermittent current in one direction only*, and not an alternating discharge.

The Wehnelt Electrolytic Interruptor.—The frequency of the interruptions in the primary of an induction coil is far greater when the Wehnelt interruptor is used instead of the ordinary hammer break. It was introduced by Dr. Wehnelt, of Charlottenberg, in 1899, and consists of a glass vessel which is filled with dilute sulphuric acid (one part of acid to five of water) and contains two electrodes, of which one is a large lead plate and the other is the extreme end of a thin platinum wire projecting from the end of a glass tube. The cell is included in the primary circuit, so that the lead plate forms the kathode (p. 454), and the condenser and hammer-break are removed from the circuit. If a voltage lower than 20 volts is applied to the primary circuit, ordinary electrolysis proceeds within the cell and no discharge is obtained from the secondary circuit; but if the voltage is increased to 25 volts or more the current becomes intermittent, and a torrent of dense sparks is obtained from the secondary. The frequency of the interruptions increases if the voltage is increased, but the interruptions cease if the voltage is increased beyond 100 or 120 volts. The cell does not act unless the circuit has self-induction, but as a rule the primary of an ordinary coil has sufficient for the purpose. Although the length of spark is not increased by using the interruptor, yet a torrent of sparks is obtained having a density in marked contrast to that observed with a hammer-break. The theory of the action ot the cell is not yet clearly understood.

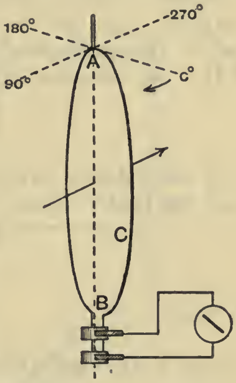

FIG. 299.—The Earth Inductor.

The Earth Inductor.—The earth-inductor (Fig. 299) is a circular or rectangular conductor, consisting of one or more turns of insulated wire, mounted on an axle so that it can be rotated through any given angle. Any induced currents set up in the circuit may be conveyed to a galvanometer by means of metal brushes in contact with ring terminals mounted on one of the axles and joined to the ends of the conductor.

If A = area of an inductor, of a single turn of wire,
F = intensity of a uniform magnetic field,
R = total resistance (C.G.S. units),

then, if the plane of the coil is initially perpendicular to the lines of force, *i.e.* in the position marked 0°, and is rotated rapidly through 90°, an induced current is set up in the direction ACB. The number of lines of force cut by the inductor is AF, and the *quantity* of electricity set in motion is AF/R C.G.S. units. If rotated from 90° to 180°, the number of lines through the inductor increases from zero to AF, and the induced current will be in the same direction as before. The total quantity of electricity set in motion while the inductor is rotated from 0° to 180° is

$$q = \frac{2AF}{R} \quad \dots\dots\dots\dots\dots\dots (1)$$

The same quantity will be set in motion when the inductor is rotated from 180° through 270° to 0°, but its direction will be opposite to that previously obtained. The quantity of electricity may be determined by means of a ballistic galvanometer (p. 281).

Weber used the earth inductor in order to determine the magnetic dip. If the inductor is rotated round a vertical axis in the earth's field through 180° as described above, then, if H is the horizontal intensity of the earth's field

$$q_1 = \frac{2AH}{R}.$$

If the inductor is rotated similarly round a horizontal axis, only the earth's vertical component V is now effective, and

$$q_2 = \frac{2AV}{R}.$$

Hence $$\frac{q_2}{q_1} = \frac{V}{H} = \tan \delta$$

(where δ is the angle of dip).

If the inductor consists of n turns, each of area A, equation (1) is written $q = 2nAF/R$.

Measurement of an Intense Magnetic Field.—The method described in the previous paragraph is convenient for the measurement of an intense field, *e.g.* that between the poles of a horse-shoe electromagnet. A small coil, of known dimensions and wound on an ebonite disc, is placed with its plane perpendicular to the lines of force and connected to a ballistic galvanometer of which the *constant* is known. When the coil is rotated suddenly through 180°, the throw of the galvanometer enables the *quantity* (q_1) of electricity to be calculated. The intensity F of the field is given by the equation $F = q_1 R/2nA$. The determination of the effective area nA of the coil is dispensed with if the experiment is repeated in a magnetic field of known intensity.

Another method of measuring a strong magnetic field is given on p. 263.

Instantaneous Value of the Induced E.M.F. in an Earth Inductor.—Let ABCD (Fig. 300, i) represent a rectangular inductor, initially perpendicular to a uniform field of intensity F. The sides AB and DC only are effective in inducing an E.M.F. when the inductor is rotated round its vertical axis.

If l and r are the lengths of AB and AD respectively, and ω the angular velocity; then, when the inductor has rotated

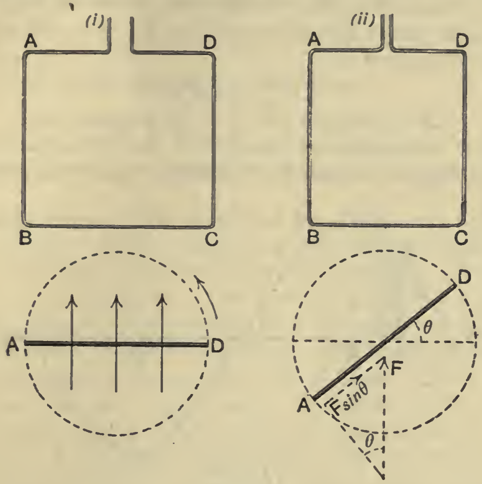

FIG. 300.

through an angle θ, only the component of F perpendicular to the direction of motion of AB, *i.e.* along AD (Fig. 300, ii), is effective in creating an induced E.M.F., and this component is $F \sin \theta$.

The linear velocity of AB is $\omega \times \frac{r}{2}$.

Hence, the E.M.F. set up in AB is $\left(\omega \times \frac{r}{2}\right) \times l \times F \sin \theta$;

$$\therefore \text{ total E.M.F.} = \omega r l \times F \sin \theta = \omega A \times F \sin \theta,$$

where A is the area of the inductor.

Or, if T is the period of one revolution, and t is the time occupied in rotating through the angle θ, then $t/T = \theta/2\pi$, or $\theta = 2\pi t/T$;

$$\therefore \text{ total E.M.F.} = \omega A \times F \sin \frac{2\pi t}{T}.$$

Hence, the instantaneous E.M.F. is proportional to the sine of the angle described from its initial position. It is consequently equal to zero when the plane of the inductor is perpendicular to the lines of force, and has a maximum value, equal to ωAF, when the plane coincides with the direction of the lines of force. If the maximum E.M.F. is denoted by E_0, then the current at any instant is given by

$$C=\frac{E_0}{R}\cdot\sin\frac{2\pi t}{T}.$$

Fig. 301 represents the variation of the momentary induced E.M.F. during one complete revolution. Successive positions of the inductor are measured along the horizontal axis XX′, and ordinates are drawn from these points proportional to the sine of the angle in each case. The ordinates are above or below the horizontal line according as the value of the sine is positive or negative. The curve obtained is termed a **Sine Curve**, and the E.M.F. in such a case is said to vary **harmonically**. *In the absence of self-induction*, the current would vary in the same manner.

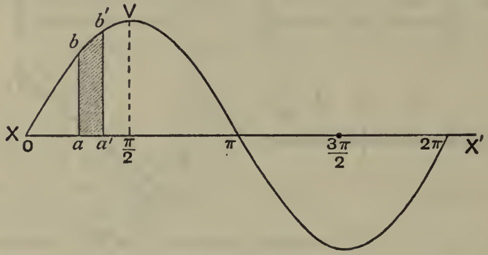

FIG. 301.

The average E.M.F. in an Earth Inductor.—We require to find the average height of the ordinates, such as *ab* (Fig. 301) drawn from the axis XX′ to touch the sine curve. Imagine that the axis from 0 to π is divided into a large number n of equal parts, each equal to $d\theta$, and that ordinates are drawn from each of these points. The average length is obtained by adding these together and dividing the sum by the number of ordinates taken. If y represents an ordinate such as *ab*, and if Σy represents the sum of these ordinates,

$$\text{the average height}=\frac{\Sigma y}{n}=\frac{\Sigma y\,.\,d\theta}{n\,.\,d\theta}=\frac{\Sigma y\,.\,d\theta}{\pi};$$

$\Sigma y \,.\, d\theta$ is the integration of $y \,.\, d\theta$ between the values $\theta = 0$ and $\theta = \pi$. But $y = \omega \mathrm{AF} \,.\, \sin\theta$,

$$\therefore \; \Sigma y \,.\, d\theta = \omega \mathrm{AF} \int_0^\pi \sin\theta \,.\, d\theta = \left[-\omega \mathrm{AF} \cos\theta \right]_\pi^0 = 2\omega \mathrm{AF}.$$

Hence, $\qquad$ the average height $= \dfrac{2\omega \mathrm{AF}}{\pi}$.

But $\omega \mathrm{AF}$ is the maximum value of the ordinate,

$$\therefore \; \textbf{the average E.M.F.} = \frac{2}{\pi} \times \textbf{the maximum E.M.F.}$$

Also, $\qquad$ the average current $= \dfrac{2}{\pi} \times$ the maximum current.

Determination of the value of the Ohm.—(i) ROTATING COIL METHOD. Weber suggested that the principle of the rotating coil might be used in order to determine the absolute resistance of a conductor. The method, which was elaborated by Clerk Maxwell and adopted by a committee of the British Association in a series of experiments conducted in 1863, consisted in spinning a circular coil with uniform velocity about a vertical axis in the earth's field. The ends of the coil are joined together, and a small magnet is suspended at the centre of the coil. A periodic current is set up in the coil, and in all positions of the coil the direction of the current is such as to deflect the needle in the same direction. If the dimensions and angular velocity of the coil, and the intensity of the magnetic field are known, the induced E.M.F. can be calculated ; also the deflection of the needle serves to measure the current. Hence, knowing the E.M.F. and current strength, the resistance of the coil can be calculated.[1]

Standard 1-ohm coils, of which the resistances were measured by comparison with the above coil, were termed **B.A. units.** The resistance of these standards was subsequently found to be too low, and a more accurate series of observations with the earth inductor, conducted by Lord Rayleigh,[2] showed that the *B.A. ohm is equal to* 0.9865 *true ohm.*

(ii) LORENZ'S METHOD.—It has been shown on p. 415 that, when a metal disc of radius r rotates at a uniform speed of n

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[1] The approximate formula is $R = \dfrac{2\pi^3 a n^2}{t \,.\, \tan\phi}$, where a is the radius of coil, n the number of turns, t the time of one revolution, and ϕ the deflection of the magnet.

[2] See Gray's *Absolute Measurements*, vol. ii., Part 2 (Macmillan).

revolutions per second in a plane perpendicular to the lines of force of a uniform field H, the potential difference set up between the axis and the circumference is given by the equation $e = \mathrm{H}n\pi r^2$ C.G.S. units. Lorenz has applied this principle to the measurement of a resistance in C.G.S. units.

The axle of the disc D (Fig. 302) coincides with that of a circular coil C mounted with its plane in the magnetic meridian and traversed by a current C C.G.S. units. Then, if M is the magnetic flux through the disc due to unit current, the flux due to a current C is equal to MC. The potential difference set up in the disc is then

$$e = \mathrm{MC}n.$$

If the circumference and the axle are connected through a galvanometer G with two points on the main circuit and separated by a resistance R, the speed of the disc may be adjusted so that the P.D. set up is just balanced by that which exists between the ends of R and is equal to RC. Hence,

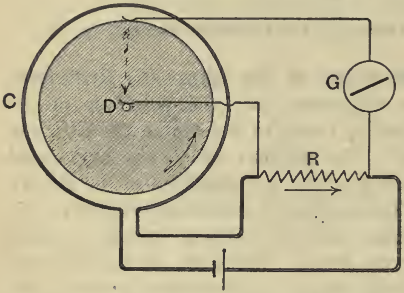

FIG. 302.—Lorenz's Method of determining the Ohm.

$$\mathrm{RC} = \mathrm{MC}n$$

$$\text{or,} \qquad \mathrm{R} = \mathrm{M}n.$$

The value of M can be calculated from the dimensions and number of turns of the coil, and n can be measured directly.

In a very careful determination, in 1883, of the value of the B.A. unit by this method, Lord Rayleigh and Mrs. Sidgwick found that the *B.A. ohm is equal to* 0.98677 *true ohm.* A still more careful re-determination was made, in 1890, by the late Prof. Viriamu Jones, who used a column of mercury as a resistance for balancing the E.M.F. set up in the disc, and thus obtained directly the specific resistance of mercury. This was found to be 94067 C.G.S. units at 0° C., and the true ohm is therefore equal to the resistance at 0° C. of a column of mercury 106.307 cms. long and 1 square millimetre in cross-section.

in accordance with the recommendation of an International Congress of Electricians, the Board of Trade in 1894 specified that *the standard ohm shall be the resistance of a column of mercury*, 106.3 *cms. long and having a mass of* 14.4521 *grams, at the temperature of melting ice.*

The Ohm expressed as a Velocity.—The ohm is defined as equal to 10^9 C.G.S. units of resistance, and may be expressed as *a velocity of* 10^9 *cms. per second.* This may be understood more clearly by considering the E.M.F. induced in a metal bar AB (Fig. 289) sliding along two horizontal metal bars, of negligible resistance and connected together at their distant ends. If the bar intersects the lines of force of a vertical magnetic field of unit intensity, an E.M.F. is induced in the bar tending to send a current from B to A.

If AB is 1 cm. long, and if v is its velocity, then

$$\text{E.M.F.} = v \text{ C.G.S. units, and}$$

$$\text{current} = \frac{v}{R} \text{ C.G.S. units.}$$

If the velocity is increased or diminished until the current generated is equal to 1 C.G.S. unit, then

$$R = v,$$

or, the resistance of the bar is expressed as a velocity.

Helmholtz's Equation. Time-Constant.—It has already been stated that the self-induction in a circuit impedes the growth and decay of a current set up in it. Helmholtz has shown how this may be represented by equations. Consider the case of a battery, E.M.F. $= E$, joined in series with a circuit of resistance R and inductance L. While the current is increasing the conditions may be represented by the fundamental equation (p. 412),

$$ECdt = C^2Rdt + LC \,.\, dC,$$

$$\text{or,} \qquad \frac{E}{R} - C = \frac{L}{R} \cdot \frac{dC}{dt}.$$

But, since E and R are constants, dC/dt may be written

$$-d\left(\frac{E}{R} - C\right)\Big/dt.$$

Hence,

$$\frac{d\left(\frac{E}{R} - C\right)}{\frac{E}{R} - C} = -\frac{R}{L} \cdot dt.$$

In order to find the current strength after any interval t, this expression on the left must be integrated between the limits 0 and C, and that on the right between limits 0 and t. Hence, if $e=2.7183$, the base of the natural system of logarithms,

$$\log_e\left(\frac{E}{R}-C\right)-\log_e\frac{E}{R}=-\frac{R}{L}\cdot t;$$

or

$$\log_e\left(\frac{\frac{E}{R}-C}{\frac{E}{R}}\right)=-\frac{R}{L}\cdot t;$$

or,

$$\frac{E}{R}-C=\frac{E}{R}\cdot e^{-\frac{Rt}{L}};$$

$$\therefore\ C=\frac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right)=\frac{E}{R}\left(1-\frac{1}{e^{Rt/L}}\right)$$

But $\frac{E}{R}$ is the maximum strength of the current when fully established, which may be denoted by $\mathbf{C}$. When L/R is equal to t, then

$$C=\mathbf{C}\left(1-\frac{1}{e}\right)=\mathbf{C}\times 0.6321.$$

Hence the ratio L/R, termed the **time-constant** of the circuit, may be defined as equal to *the time required for the current to attain a strength equal to* 0.6321 × *its maximum value.* If L is very small compared with R, then $C=\frac{E}{R}$ approximately, and the current assumes its maximum value almost instantaneously.

Fig. 303 represents the growth during 4 seconds of a current through a circuit of 1 ohm resistance and 1 henry inductance, the impressed E.M.F. being 1 volt. From the above equation, the current at any instant is $C=\left(1-\frac{1}{e^t}\right)$;

$$\text{when } t=0.1 \text{ sec., } C=1-\frac{1}{e^{0\cdot 1}}=0.0952,$$

$$\text{,, } t=0.2 \text{ sec., } C=1-\frac{1}{e^{0\cdot 2}}=0.1813, \text{ and so on}$$

On breaking the circuit, everything ceases except the induced current due to the self-induction of the circuit. But it is impossible to calculate the actual current strength in any ordinary case since the resistance of the circuit is increased indefinitely by an amount depending on the resistance of the spark gap; in any case this is very great, and the removal of magnetic flux is correspondingly sudden. Consequently the

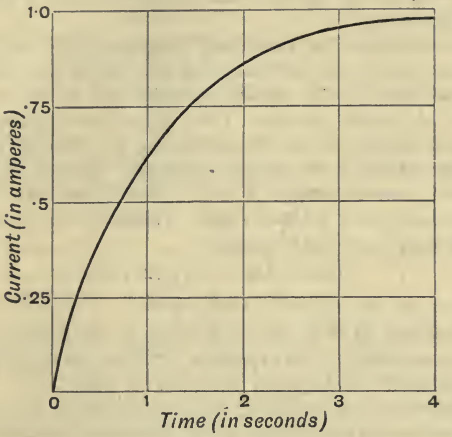

FIG. 303.

induced E.M.F. is much greater at 'break' than at 'make.' In a theoretical case, *in which the resistance remains constant*, the instantaneous current is given by the equation

$$C=\mathbf{C}e^{-\frac{Rt}{L}}.$$

Energy Stored up in the Medium.—The fundamental equation (1) on p. 412 may be written $EC=C^2R+LC\frac{dC}{dt}$. But EC is *the rate of supply of energy* from the battery, and the other factors express the rate of dissipation of energy as heat and the rate of absorption of energy in the medium respectively. Hence

$$\left.\begin{array}{c}\text{Rate of supply}\\ \text{of energy}\end{array}\right\}=\begin{array}{c}\text{Rate of dissipation}\\ \text{as heat}\end{array}+\begin{array}{c}\text{Rate of storage of energy}\\ \text{in the magnetic field.}\end{array}$$

The *total* energy stored up in the medium is determined by integrating the expression $LC\frac{dC}{dt}$ between the limits $C=0$ and $C=\mathbf{C}$, where $\mathbf{C}$ is the final value of C. Hence

$$\textbf{total energy}=\int_0^{\mathbf{C}} LC\,.\,dC=\tfrac{1}{2}L\mathbf{C}^2.$$

Since, by definition (p. 417), the product LC is equal to the number of lines of force passing through a circuit of self-inductance L and traversed by a current C, the total energy in the magnetic field is equal to *half the product of the current in the circuit and the number of lines of force through the circuit.*

A further consideration is necessary in the case where two independent circuits are present, for we have then to consider the energy in the medium due both to the *mutual induction* and to the separate self-inductions of the two circuits. Thus, suppose two circuits **A** and **B**, traversed by currents C and C′, and having self-inductances L and N respectively. If M is the mutual inductance, the number of lines of force passing through circuit A is $LC+MC'$; similarly, the number passing through circuit B is $NC'+MC$. Hence, by the above definition, the total energy in the field is equal to

$$\tfrac{1}{2}C(LC+MC')+\tfrac{1}{2}C'(NC'+MC)$$

$$\text{or,}\qquad \tfrac{1}{2}LC^2+MCC'+\tfrac{1}{2}NC'^2.$$

The expression $\frac{1}{2}LC^2$ for the total energy in the field due to a single circuit is comparable to the expression $\frac{1}{2}CV^2$ for the total energy in the electrostatic field of a simple air condenser (p. 190).

By means of the expression $\frac{1}{2}LC^2$, it can be shown that the quantity $4\pi\mu$ may be termed the *electromagnetic density* of the medium. Thus, consider the energy in the medium within a uniform coil, wound in the form of an anchor ring, of length l, cross-section A, having n_1 turns per unit length, and traversed by a current C. If the permeability of the medium is μ, then (p. 419) $L=4\pi n_1^2 lA\mu$;

$$\text{hence}\qquad \tfrac{1}{2}LC^2=\tfrac{1}{2}\,.\,4\pi\mu\,.\,Al\,.\,(n_1C)^2.$$

But Al is the volume of the field, therefore the energy per unit volume is

$$\tfrac{1}{2}\,.\,4\pi\mu(n_1C)^2.$$

This formula is analogous to the expression $\frac{1}{2}mv^2$ for the kinetic energy of a mass m moving with velocity v. In the electromagnetic case n_1C may be regarded as the 'electric velocity,' and $4\pi\mu$ may be termed the *electric mass per unit volume* (or the *electric density*) of the medium.

SUMMARY

Faraday's Law.—*When the number of lines of force through a circuit is changing, an induced E.M.F. is set up, and the magnitude of the E.M.F. is proportional to the rate at which the number of lines of force changes.*

If two circuits, which may be termed the *primary* and *secondary* circuits, are placed near together, the following phenomena can be observed: (i) *an inverse induced current traverses the secondary when the primary current begins, increases, or is brought nearer to the secondary;* (ii) *a direct induced current is obtained when the primary current ceases, diminishes, or is removed to a greater distance.*

Lenz's Law.—*The induced current is in such a direction that its reaction tends to stop the motion or change to which the induced current is due.*

General Principles.—(i) If the energy in a simple battery circuit is expended partly in generating heat and partly in increasing the lines of force included in the circuit, the equation of energy is

$$ECdt = C^2Rdt + Cd\mathbf{N}.$$

Also, the induced E.M.F. due to the cutting of the lines of force is equal to $-d\mathbf{N}/dt$ C.G.S. units.

(ii) When the number of lines of force passing through a circuit is increased or diminished the induced current will be in that direction which tends to keep the number of lines of force constant.

Fleming's Right-hand Rule.—*When the thumb and the first and second fingers of the right hand are extended at right angles to each other, then, if the first finger represents the direction of the field and the thumb the direction of motion, the second finger will point in the direction of the induced current.*

The C.G.S. Unit of E.M.F.—If a conductor of unit length moves with unit velocity in a direction at right angles to its own length and to the direction of a magnetic field of unit intensity, the E.M.F. set up in the conductor is equal to one C.G.S. unit.

The practical unit of E.M.F. is equal to 10^8 C.G.S. units, and is termed the *volt*.

Faraday's Disc.—If a disc of radius r rotates in a plane perpendicular to the lines of force of a field of intensity H at a speed of n revolutions per second, the E.M.F. generated between the axle and the edge of the disc is equal to $H \,.\, n \,.\, \pi r^2/10^8$ volt.

Foucault Currents are set up in any conducting sheet rotated near a source of magnetic lines of force, except when the plane of rotation coincides with the direction of a uniform magnetic field.

Self-Induction.—The change of magnetic field which takes place when a current is being established in any circuit gives rise to an induced E.M.F., and therefore to an induced current which is superimposed upon the original current. This property of a circuit is termed *self-induction.*

The **Coefficient of Self-Induction** is defined as the number of lines of force passing through any circuit when traversed by one C.G.S. unit of current. If L is this coefficient, then $\mathbf{N} = LC$; hence the induced E.M.F. is $e = -d\mathrm{N}/dt = -\mathrm{L} \,.\, d\mathrm{C}/dt$.

The practical unit of self-induction is the **henry**, and is equivalent to a flux of 10^9 lines of force through a circuit when traversed by one ampere.

The **Coefficient of Mutual Induction** of two coils is defined as the magnetic flux through one of the coils when the other is traversed by the C.G.S. unit of current.

The **Ruhmkorff Coil** is an appliance designed for the transformation of a low P.D. between the ends of one coil into a high P.D. between those of a neighbouring coil.

The **Earth Inductor.**—If a coil, consisting of one turn of wire of area A and resistance R, is supported with its plane perpendicular to the lines of force of a field of intensity F, and is rotated through 180°, the quantity q of electricity set in motion is equal to $2\mathrm{AF}/\mathrm{R}$.

If ω is the angular velocity and T is the period of one revolution, the E.M.F. at an instant t after passing its initial position is equal to $\omega \mathrm{A} \times \mathrm{F} \sin \frac{2\pi t}{\mathrm{T}}$. The maximum E.M.F. is therefore $\omega\mathrm{AF}$; also, if this is denoted by E_0, the current at any instant may be written

$$\mathrm{C} = \frac{\mathrm{E}_0}{\mathrm{R}} \,.\, \sin \frac{2\pi t}{\mathrm{T}}.$$

The *average* E.M.F. is equal to $\frac{2}{\pi} \times \omega\mathrm{AF}$.

Determination of the Ohm.—(i) *Weber's method* of the rotating coil.

(ii) *Lorenz's method* of a rotating disc within a magnetic field.

The ohm may be expressed as a *velocity*.

Helmholtz's Equation.—If an E.M.F. equal to E is impressed on a circuit of resistance R and self-inductance L, the current strength after an interval t is expressed by the equation

$$\mathrm{C} = \frac{\mathrm{E}}{\mathrm{R}}\left(1 - \frac{1}{e^{\mathrm{R}t/\mathrm{L}}}\right).$$

The **total energy** stored up in the medium surrounding a conductor of self-inductance L traversed by a current C is equal to $\frac{1}{2}\mathrm{LC}^2$.

QUESTIONS ON CHAPTER XXII

1. A flat coil of wire, the ends of which are connected to a sensitive galvanometer, is (1) moved up towards the north end of a long bar-

magnet, (2) threaded on the bar, (3) moved along it to the south end, (4) removed to a distance. What indications does the galvanometer give during these operations?

2. A few turns of fine insulated wire are wrapped round the centre of an iron nail, and the ends of the wire connected to a sensitive galvanometer. State and explain the effect on the galvanometer (1) when the nail is put across the poles of a horse-shoe magnet, slowly or quickly, (2) when it is removed slowly or quickly.

3. A bar-magnet is suspended on a stirrup by a string, and oscillates in a horizontal plane. How are the oscillations affected (if at all) when a thick non-magnetic metal plate is placed horizontally beneath the needle so as to be close to without touching it?

4. A vertical hoop of wire, at right angles to the magnetic meridian, is quickly but with uniform speed turned through 180° about a vertical axis, its originally eastern half moving northward at first. State the direction in which the induced current passes round the wire, and determine the position of the hoop in which the induced E.M.F. is the greatest.

5. A circuit consisting of a battery and an electro-magnet can be made and broken by a wire dipping into mercury. At every break a bright spark occurs. If the electromagnet be removed from the circuit, the spark is much less bright. Explain these phenomena.

6. Show how to determine approximately the direction of the magnetic meridian at a place by experimenting with a coil of wire movable about a vertical axis, and connected with a sensitive galvanometer.

7. Describe an arrangement for producing a continuous current by making a conductor rotate between the poles of a horse-shoe magnet.

8. Explain how it is possible by means of induction currents to transmit signals between two places which are not connected by a conductor.

9. Describe an induction coil, and explain why the core is made of wire.

10. Describe Barlow's wheel, and explain its action (i) when used as a dynamo, (ii) when used as a motor.

11. Calculate the electromotive force generated, by virtue of the vertical component of the earth's field (which may be taken as $= 0.41 \text{ cm.}^{-\frac{1}{2}} \text{ gm.}^{\frac{1}{2}} \text{ sec.}^{-1}$) in the axle of a railway carriage, of length 150 cm., travelling with a speed of 75 kilometres per hour. In what units is your answer given? How could you observe the existence of this electromotive force?

12. A circular coil of wire is rotated clockwise as seen from above, about the vertical diameter (i) in England, (ii) in Australia. Explain how you obtain the direction of the current for different positions of the coil, and discuss whether there is any difference in the two cases.

13. A circular horizontal coil of wire is connected to a distant galvanometer. What effect is shown by the galvanometer in the following two cases: (i) A vertical magnet, previously placed in the axis of the coil and half-way through it, is allowed to drop. (ii) The same magnet, previously placed in a horizontal position with its centre at the centre of the coil, is allowed to drop.

14. A single Daniell cell, an electromagnet and an electrolytic cell containing dilute sulphuric acid with small platinum electrodes are connected in series. State what happens. If the terminals of the electrolytic cell are connected by a wire and this connexion is rapidly made and broken, gas comes off at the electrodes. Why is this?

15. Supposing that a railway line is laid in England on insulating material, and that the two rails are connected at a certain station by a cross wire, show that a current will flow in the cross wire and the rails, when a train is moving on the line, and draw a figure showing its direction when the train is moving from the station. Give a reason for the direction you assign.

16. A circular coil of wire is rotating round a vertical axis in its own plane. Describe how the E.M.F. due to its cutting the lines of the earth's magnetic field varies as it rotates, and show how to calculate its value. Show also how to calculate the resulting current. Does the self-induction of the circuit affect the result?

17. A solenoid an inch in diameter and a yard long is connected with a distant galvanometer. A bar-magnet 6 inches long is quickly thrust half-way into one end of the solenoid. When the needle is at rest it is thrust in a foot further and again when all is quiet it is thrust in another foot. Finally it is pulled out at the other end very quickly. What response will the needle make in the four cases?

18. A copper ring is hung by a torsionless thread between the vertical parallel and flat opposed poles of an electro-magnet. The plane of the ring is vertical, and is inclined at 45° to that of either pole face. If a current is started round the magnet, the ring turns through a moderate angle, but quickly comes to rest. If it is replaced in its former position and the current is stopped, it starts twisting in the opposite direction and keeps on spinning. Account for these actions.

19. A metal hoop is held vertically with its plane in the magnetic meridian, and is allowed to fall freely towards the east. In what direction does the induced current traverse the part of the hoop touching

the ground? If the radius of the hoop is 30 cms., and its resistance is 0.2 ohm, find the total quantity of electricity developed (angle of dip = 67°, $H = 0.18$).

20. A coil of 100 turns and 10 cms. mean diameter is rotated at a speed of 90 turns per minute about a diameter lying magnetic east and west in the earth's field of 0.4 dynes per unit pole. Calculate the average E.M.F. during a quarter revolution beginning with zero E.M.F.

21. How would you show by experiment that the E.M.F. induced in a conductor is accurately proportional to the rate at which it cuts lines of magnetic induction?

A solid fly-wheel, 1 metre in diameter, spins on an axis which is directed towards the north, at a place where $H = 0.18$. It makes 250 revolutions per minute. Calculate the difference of potential between the centre and the circumference of the wheel.

22. A circular coil, of 200 turns of wire and mean diameter 20 cms., revolves round a vertical axis at the rate of 12 revolutions per second. If $H = 0.18$, what will be the instantaneous E.M.F. induced in the coil (i) when it is at right angles to, (ii) when it is at 30° to, (iii) when it coincides with, the magnetic meridian?

23. A square conducting frame cut through at one place rotates in the earth's magnetic field about a vertical axis, passing through the middle points of opposite sides. Describe the variation in E.M.F. between the two sides of the break which consequently occurs, and calculate its maximum amount when there are 120 revolutions per minute, if the edge of the square is 25 cms. and the intensity of the earth's horizontal force is 0.18.

24. A coil of wire of inductive area 15000 sq. cms. is connected with a galvanometer. The coil lies flat on a table, and when it is turned over the galvanometer is momentarily deflected. Discharging a condenser of 1 microfarad capacity charged to 1.5 volts through the same galvanometer produces the same kick as with the coil. If the vertical component of the earth's field is 0.4 C.G.S. units, what is the resistance of the coil and galvanometer together with the connecting leads?

25. What is the mutual inductance of two concentric coils, 40 cms. long, of 200 and 600 turns respectively, the inner coil being wound on a paper tube 10 cms. diameter?

26. What is the inductance of a coil of 100 turns wound on a paper tube 25 cms. long and 4 cms. radius?

27. Two coils, a primary of 400 turns and a secondary of 20 turns, are wound on an iron ring of mean diameter 20 cms. and cross-section 2 cms. radius. Find the inductance of each when $\mu = 800$.

28. If 500 volts are impressed on a circuit of 10 henrys inductance and 5 ohms resistance, what will be the current at the end of (i) $\frac{1}{100}$ sec., (ii) 1 sec., (iii) 5 secs.? What will be its final value?

29. A transformer has an iron core of 200 sq. cms. area and 40 cms. long. There are 220 turns in the primary and 22 turns in the secondary coils. If $\mu = 2000$, what are the inductances in the two circuits?

30. Describe the action of an induction coil, and explain the effect of having a condenser in the primary circuit in parallel with the interrupter.

31. Give Lorenz's method for determining the ohm in absolute measure, and describe in detail the apparatus required for the purpose.

32. A metal cylinder is kept spinning about its axis with uniform angular velocity ω between the poles of an electro-magnet, the axis of rotation being perpendicular to the direction of the magnetic force H. Show that the work required per second to maintain the rotation is proportional to $H^2\omega^2$.

33. The wire supporting a heavy pendulum bob hangs in the space between the poles of an electro-magnet, which produces a field perpendicular to the plane of oscillation of the pendulum. Calculate the total quantity of electricity sent through a circuit of resistance r, in series with the wire, at each swing.

34. Find the nature and magnitude of the electromotive force which is induced in a plane coil of wire when it rotates (*a*) about its own axis, (*b*) about an axis in its own plane, in a uniform magnetic field the lines of which are in each case perpendicular to the axis of rotation of the coil.

35. Define self-induction, and show that when an E.M.F. E is applied to a circuit of self-induction L and resistance R the current is equal to $\frac{E}{R}(1 - e^{-Rt/L})$, where t denotes the time after the E.M.F. is applied.

CHAPTER XXIII

ALTERNATING CURRENTS

Graphical Representation of Alternating Currents.—The principle of an alternating current may be understood by considering the rotation of an earth-inductor (p. 425) in a uniform magnetic field. In this case the direction of the E.M.F., and therefore of the current, is reversed once during each half-revolution of the inductor; the **periodic time** of the current is equal to the time occupied by one complete revolution of the inductor, and the **frequency** of the current is the number of complete periods described in one second.

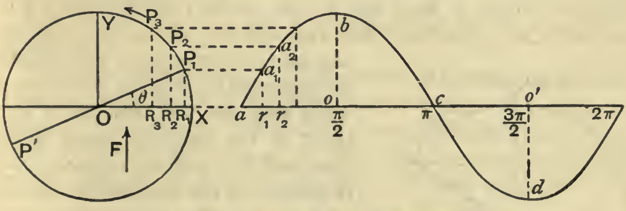

FIG. 304.—Wave Diagram of an Alternating Current.

It is necessary to have some method of representing, in the form of a diagram, the manner in which the current, or the E.M.F., varies with the time. One method is shown in Fig. 304, where $P'P_1$ is an earth-inductor, rotating at a uniform rate round an axis O. When passing the position OX, and when its plane is perpendicular to the lines of force of the magnetic field F, the instantaneous E.M.F. is zero; and, when passing the position OP_1, the E.M.F. is $\omega AF \sin \theta$ or $\omega AF \sin \frac{2\pi}{T} . t$ (p. 427), where t is the time interval occupied in passing between the positions OX and OP_1. The E.M.F. is therefore proportional to P_1R_1. Similarly, in the positions OP_2 and OP_3 the E.M.F. will be proportional to P_2R_2 and P_3R_3 respectively. The successive values of the E.M.F. for various values of the **phase-angle** θ are shown by the curve on the right of Fig. 304. Starting from the point a, the length ar_1 represents the value of the angle θ (expressed in circular measure), and the height of the vertical line a_1r_1 is proportional to the instantaneous E.M.F. when the inductor is in the position OP_1. Similarly the E.M.F. set up in the position OP_2 is represented by a_2r_2. The E.M.F. is a maximum when $\theta = \pi/2$, and becomes zero when $\theta = \pi$. During the remainder of the period the E.M.F. is reversed, and the ordinates are drawn below the horizontal line. By joining the ends of the ordinates, a curved line is obtained which represents the variation in the E.M.F. during one revolution of the inductor. The curve, which may be termed a **wave diagram**, is evidently a **sine curve**, since the height of each ordinate is proportional to the *sine* of the corresponding phase-angle. The form of the wave diagram obtained in practice is not necessarily so simple as this, and it may differ considerably. The resultant E.M.F. obtained when two separate alternating E.M.F.'s are applied to the same circuit may be represented also by means of a wave diagram, the ordinate at any point of the resultant curve being obtained by adding together the ordinates of the component curves at the same point.

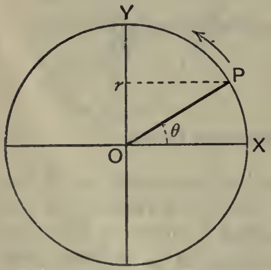

FIG. 305.—Clock-face Diagram.

Fig. 305 is an example of another method of representing an alternating E.M.F., and is known as a **clock-face diagram**; thus, if the line OP, of which the length (drawn to a given scale) represents the maximum E.M.F., rotates round O at a uniform speed such that the time of one revolution is equal to the period of the alternating E.M.F., and if the position OX corresponds to the instant when the E.M.F. is zero, then the length of the projection Or will represent the instantaneous E.M.F. when the phase is θ. This method is especially applicable to the representation of the resultant E.M.F. due to two or more separate E.M.F.'s which differ in phase. Thus, let OP and OQ (Fig. 306) represent the maxima E.M.F.'s applied to a circuit, the difference in phase being equal to the angle POQ; then, since alternating E.M.F.'s may be regarded as *vector quantities* —having both direction and magnitude—the resultant may be found by a graphical method resembling the principle of the *parallelogram of forces*. The resultant is represented therefore by OR, and differs in phase from OP and OQ by the angles POR and QOR respectively. From the geometry of the figure it is evident that the instantaneous value Or of the resultant E.M.F. is equal to the sum of the instantaneous values Op and Oq of the components.

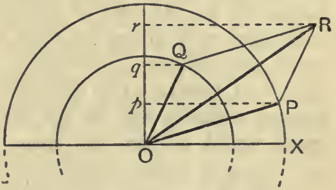

FIG. 306.

Alternating currents may be represented by the same forms of diagram.

The Root Mean Square Value of an Alternating Current. —Since the strength of an alternating current is changing continually, a numerical value can be given to the current strength only by taking an average of the instantaneous strengths at equal intervals of time during one complete period. It has been shown (p. 429) that the *true mean current* is equal to $\frac{2}{\pi}$ times the maximum current. Simple reasoning will show, however, that this mean value does not express the strength of a steady current which would have the same effect as the alternating current under consideration—and this is what we mean by the *strength* of an alternating current. Thus, when a straight wire is traversed by a simple harmonic current, its temperature rises to a constant value which it attains when the heat is radiated out at the same rate as it is generated in the wire. The heat generated at any instant is, by Joule's law, proportional to the *square* of the current strength at that instant. If we construct, from the curve of a simple harmonic current, a curve of which the ordinates are proportional to the **squares of the instantaneous current strengths**, then such a curve will represent *the variation in the rate of generation of heat.* Also, since the horizontal axis measures *time*, the area enclosed between the curve and the axis, multiplied by the resistance, will give the total heat generated in the same period. **The square root of the average ordinate will be the magnitude of the constant current which would develop the same heat in the same time.** This equivalent constant current is sometimes termed the **virtual** strength of the alternating current.

The relationship between the *virtual* strength and the *maximum* strength **C** of a simple harmonic current may be determined by the method adopted on p. 428. If y is any ordinate in the curve, then at any instant

$$y = \mathbf{C} \sin \theta.$$

If, in the curve representing the squares of the current strength, ordinates are drawn at n equidistant points, the average ordinate is

$$\frac{\Sigma y^2}{n} = \frac{\Sigma y^2 \,.\, d\theta}{n \,.\, d\theta}.$$

But, if we consider only those ordinates between the points o and π, then $n \,.\, d\theta = \pi$, and

$$\textit{mean ordinate} = \int_0^\pi \frac{y^2 \,.\, d\theta}{n \,.\, d\theta} = \frac{1}{\pi}\int_0^\pi \mathbf{C}^2 \sin^2\theta \,.\, d\theta$$

$$= \frac{\mathbf{C}^2}{2\pi}\int_0^\pi (1 - \cos 2\theta)\, d\theta$$

$$= \frac{\mathbf{C}^2}{2\pi} \times \pi = \frac{\mathbf{C}^2}{2}.$$

Hence, the **virtual current** $= \frac{\mathbf{C}}{\sqrt{2}} = 0.707 \times \mathbf{C}$;

and (p. 429), the **true mean current** $= \frac{2\mathbf{C}}{\pi} = 0.637 \times \mathbf{C}$.

Similarly, the *virtual* E.M.F. is equal to $\frac{1}{\sqrt{2}} \times$ maximum E.M.F., and is numerically equal to the product of the virtual current and the *impedance* (p. 447).

An alternating E.M.F. is measured usually by means of an *electrostatic* voltmeter (p. 209) or a *Cardew* voltmeter, both of which indicate the *virtual* value of the E.M.F.; in the case of the latter instrument, since it depends for its action upon the expansion of a wire due to the heat generated in it by a small current, it will indicate the virtual E.M.F. as defined above. Alternating currents may be measured by some type of *electro-dynamometer* (p. 288). In this case also the virtual current is measured since the electromagnetic force acting on the suspended coil is proportional to the *square* of the current strength; and, if the period of vibration of the coil is great compared with the period of the current, the position of the coil is not affected by the rapid alternations of the current.

Relation between Current and E.M.F. (alternating).—When a simple harmonic E.M.F. is applied to a circuit, then

$$E_i = \mathbf{E} \sin \omega t$$

where E_i and $\mathbf{E}$ are the instantaneous and maximum E.M.F.'s; and, *if the circuit is free from self-induction*, the instantaneous current C_i is given by the equation

$$C_i = \frac{\mathbf{E}}{R} . \sin \omega t, \quad \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$= \mathbf{C} \sin \omega t;$$

also,

$$C_v = \frac{E_v}{R}$$

where C_v and E_v are the virtual values of the current and E.M.F. But, *if the circuit has appreciable self-induction*, the impressed E.M.F. is modified by an E.M.F. due to self-induction; and it has been shown (p. 431) that, in such cases, the instantaneous current C_i is given by the equation

$$E - L\frac{dC_i}{dt} = C_i R$$

where E is the instantaneous *impressed* E.M.F. (*i.e.*, the E.M.F. applied externally to the circuit). It is evident that the effective E.M.F., and therefore the resultant current, in the circuit is *retarded* always behind the impressed E.M.F. Consequently the current will be increasing at the instant when the applied E.M.F. is a maximum, and there will be at the same instant an opposing E.M.F. due to self-induction, and the maximum effective E.M.F. will be less than the impressed E.M.F. Hence an E.M.F. greater than C_vR is required in order to transmit a current C_v through a circuit of which the ohmic resistance is R and which has appreciable self-induction; and the additional E.M.F. required at any instant is $L\frac{dC_i}{dt}$.

But $$C_i = \mathbf{C} \sin \omega t, \quad \text{.................................(2)}$$

and $$\frac{dC_i}{dt} = \omega \mathbf{C} \cos \omega t. \quad \text{...............................(3)}$$

Hence $$L\frac{dC_i}{dt} = \omega L \mathbf{C} \cos \omega t,$$

and the maximum value of this is $\omega L\mathbf{C}$. The virtual value of this additional E.M.F. is equal to

$$\sqrt{\frac{(\omega L\mathbf{C})^2}{2}} = \omega L \sqrt{\frac{\mathbf{C}^2}{2}} = \omega L C_v.$$

If this E.M.F. coincided in phase with the externally impressed E.M.F., the total virtual E.M.F. required to transmit the current C_v would be $C_v(R + \omega L)$. But it is evident that the phase of the opposing E.M.F. due to self-induction lags behind the resultant E.M.F., for it is evident from equations (2) and (3) that dC_i/dt is a maximum when C_i is zero, and vice versa. The opposing E.M.F. therefore lags behind the resultant E.M.F. by ¼ period: and this is also evident if a *cosine* curve is superimposed on the *sine* curve of Fig. 304.

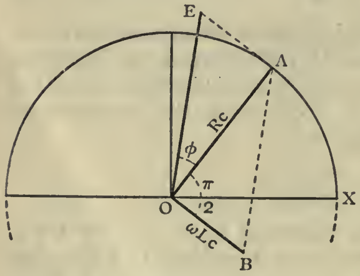

FIG. 307.

In Fig. 307 OA represents the maximum E.M.F. required to transmit a current C_v through the resistance R in the absence of self-induction, and OB is the maximum self-induced E.M.F. The line OE therefore represents, in magnitude and phase, the E.M.F. which must be impressed in order that the resultant E.M.F. may coincide with OA. Since the angle AOB is a right angle,

$$OE = \sqrt{(OA)^2 + (OB)^2},$$

or,

$$\mathbf{E} = \sqrt{(R\mathbf{C})^2 + (\omega L\mathbf{C})^2}$$

$$= \mathbf{C}\sqrt{R^2 + (\omega L)^2}.$$

Hence

$$C_v = \frac{E_v}{\sqrt{R^2 + (\omega L)^2}}.$$

Since the phase of the current is coincident with that of the resultant E.M.F., the current *lags* behind the impressed E.M.F. by an angle ϕ, and it acquires a maximum value at an instant $T \times \phi / 2\pi$ *after* the maximum value of the impressed E.M.F. has occurred. Under certain conditions (p. 449) the phase of the current may be in advance of that of the impressed E.M.F., in which case a **leading** current is obtained. In the case of a *lagging* current, traversing a circuit of which the capacity is negligible, the *angle of lag* ϕ is given by the equation

$$\tan\phi = \frac{\omega L}{R},$$

$$\text{or } \cos\phi = \frac{R}{\sqrt{R^2 + (\omega L)^2}}.$$

Hence

$$C_v = \frac{E_v}{R} \cdot \cos\phi.$$

The quantity $\sqrt{R^2 + (\omega L)^2}$ is termed the **impedance**, and ωL is termed the **reactance**. Fig. 308 represents more clearly the relationship between impedance, reactance, and ohmic resistance.

Since the current lags behind the impressed E.M.F. by an angle ϕ, and since the *impedance* takes the place of ohmic resistance, equation 1 (p. 445) for the instantaneous current must be written

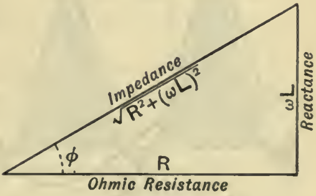

FIG. 308.

$$C_i = \frac{\mathbf{E}}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t - \phi) = \mathbf{C} \sin(\omega t - \phi).$$

EXAMPLE I.—A virtual current of 80 amperes is required through a resistance of 2 ohms. If the periodicity is 50, and if the self-inductance is 0.004 henry, find the impressed voltage required.

$$\begin{aligned} E_v &= C_v\sqrt{R^2+(2\pi nL)^2} \\ &= 80\sqrt{4+1.577} \\ &= 188.8 \text{ volts.} \end{aligned}$$

Power, in an Alternating Circuit. In a circuit traversed by a steady current, the *power* is expressed in *watts* by the product EC, where E is the E.M.F. in volts between the ends of the circuit and C is the current in amperes. In the case of a circuit traversed by an alternating current, the current and E.M.F. vary harmonically and differ in phase, and the power at any instant is measured by the product of their simultaneous values; this may be written

$$P_i = \mathbf{C}\sin(\omega t-\phi)\times\mathbf{E}\sin\omega t,$$

or, since $\cos(A-B)-\cos(A+B)=2\sin A\sin B$,

$$P_i = \tfrac{1}{2}\mathbf{CE}\{\cos\phi-\cos(2\omega t-\phi)\}$$

The average power during one-half of a complete period is equal to the average value of P_i while t changes from zero to T/2, or while $(2\omega t-\phi)$ changes from $-\phi$ to $(2\pi-\phi)$. But, within these limits, $\cos(2\omega t-\phi)$ has all possible values between $+1$ and -1, and its average value is zero; hence, the mean power, or work generated in unit time, is

$$\tfrac{1}{2}\mathbf{CE}\cos\phi.$$

But $C_v=\mathbf{C}/\sqrt{2}$, and $E_v=\mathbf{E}/\sqrt{2}$;

hence, $\text{power}=C_vE_v\cos\phi$, or

the average power is equal to the product of the virtual current, the virtual E.M.F. and the cosine of the angle of lag. From this it is evident that *the power is zero when* $\phi=90°$.

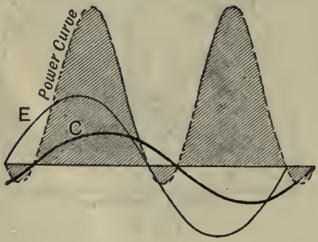

FIG. 309.

The effect of an alteration in the angle of lag on the power developed is shown in Figs. 309 and 310, where the thin and thick continuous lines represent the E.M.F. and current, and the dotted line represents the variation in the power developed. The dotted curve is drawn below the horizontal axis at points where the product of E and C has a negative value. The total power developed during any time interval is equal to the *algebraic* sum of the shaded areas included in that interval, and enclosed between the power curve and the horizontal axis. In Fig. 309 the angle of lag is $\pi/5$, and the lag is increased to $\pi/2$ in Fig. 310. In the latter case it is evident that the shaded area is distributed equally above and below the horizontal axis, indicating that the power developed in any number of complete periods is equal to zero.

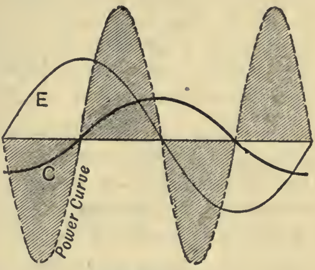

FIG. 310.

The virtual E.M.F., acting between two points of a circuit, may be measured by means of an electrostatic voltmeter, and the virtual current by means of a Siemens' or a Weber electrodynamometer. But the product of the readings will indicate the true power developed only when the reactance of the circuit is zero, since the readings would not be influenced by the angle of lag. For this reason, a *wattmeter* is used for the purpose generally. The construction of this instrument is the same as that of the Siemens' dynamometer, except that the two coils are connected to separate pairs of terminals. The suspended coil consists of extremely fine wire wound on a frame of non-metallic material, and the fixed coil consists of a few turns of thick wire. The former is joined to the ends of the circuit in which the power is to be measured, and is therefore traversed by a current proportional to the voltage; the fixed coil is joined in series with the main circuit. In order to prevent any lag in phase of the current in the suspended coil, its self-induction is reduced by winding only a few turns on the coil, the remainder of the fine wire being wound *non-inductively* on a separate frame.

Effect of Capacity in an Alternating Circuit.—Every electric circuit possesses more or less capacity, owing to the proximity of the conductors to the earth and to each other; the conditions are then equivalent to a condenser being connected *in parallel* with the circuit. If the circuit is traversed by an alternating current, a condenser may be connected even *in series* with the circuit (providing that its capacity is sufficiently great) without necessarily interfering with the transmission of power along the circuit. In either case, the effect of capacity in a circuit is to assist the rise and fall of the current; it therefore tends to cause the phase of the current to be in advance of that of the impressed E.M.F., and it tends to neutralise the lag caused by self-induction.

Fig. 311 may assist in making this point more clear: Suppose the conductors A and B to be joined to the alternator D, which, in the first instance, charges A positively and B negatively. When the E.M.F. of D is afterwards diminishing, the weakening of the current and its subsequent reversal will be *aided* by the discharge through D of the charges on A and B. Then the conductors will acquire reversed charges, and their discharge will assist the next reversal of the current.

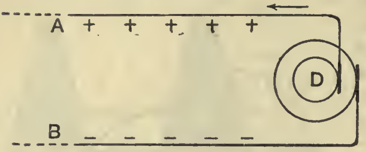

FIG. 311.

It can be shown that, if the capacity and inductance of the circuit are S and L, the reactance is equal to $\left(\omega L - \frac{1}{\omega S}\right)$. Hence, the reactance is zero when $LS = \frac{1}{\omega^2}$; and the alternating current will then obey Ohm's law.

Choking Coils.—It is evident, from the preceding paragraphs, that there is a wide difference between the manner in which ohmic resistance and in which reactance affect an alternating current. The former dissipates the energy in the form of heat, and the latter diminishes the current by setting up an opposing E.M.F.; **hence reactance diminishes the current without any waste of energy.** Therefore, the most efficient method of regulating an alternating current is to insert in the circuit a conductor having *small resistance and variable reactance.* Such a device is known as a **choking coil** or **reactance coil.** From p. 447 it is evident that if R is negligible compared with ωL, then $C_v = E_v/\omega L$, and the current may be diminished to any required extent by increasing ωL. The simplest form of choking coil consists of a coil of thick wire *w* (Fig. 312), the reactance of which can be varied by means of an adjustable core, *c*, of soft-iron wires. The reactance is a maximum when the core is inserted completely within the coil.

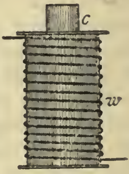

FIG. 312.

In practice, there is always a certain amount of energy lost owing to hysteresis and to Foucault currents set up in the iron core; but the former is minimised by a careful selection of the iron used, and the latter by coating each wire with a layer of varnish and by arranging the wires so that their lengths are parallel to the axis of the coil.

In determining the inductance which must be introduced in order to reduce the voltage to a required extent, it is necessary to remember that the square of the impressed E.M.F. is equal to the sum of the squares of the E.M.F. required to overcome the ohmic resistance and the reactance respectively: this is evident from Fig. 308. Thus, if several lamps required 7 amperes at 200 volts, supplied from mains in which the voltage is 220 and the frequency 100, then, assuming that the lamps are non-inductive and that the choking-coil has negligible resistance,

$$\overline{220}|^2=\overline{200}|^2+v^2$$

where v is the E.M.F. which must be balanced by the reactance of the choking coil; hence,

$$v=91 \text{ volts.}$$

But this E.M.F. must be equal to ωLC_v; therefore

$$L=\frac{91}{2\pi\times 100\times 7}=0.0207 \text{ henry.}$$

The *angle of lag* within the choking coil is given by the equation $\tan\phi=\omega L/R$ (p. 447); hence, if L is made sufficiently great, $\phi=\pi/2$. Also, since $C_v=\frac{E_v}{R}\cdot\cos\phi$ (p. 447) the current traversing the coil is zero when $\phi=\pi/2$; and, under the same conditions, the power generated in the coil is zero.

Transformers.—The cost of transmission of electric power to a distance is prohibitive if low voltage is used, since the size of the conductors must then be considerable; but, by using alternating currents generated at high voltage, it is possible to use comparatively thin conductors, providing that some device is available at the end of the circuit for lowering the voltage to a suitable extent. Any device for changing the voltage of a current-supply is termed a **transformer**; if used for lowering the voltage, it is known as a *step-down* transformer, and a *step-up* transformer if used to raise the voltage. The Ruhmkorff-coil (p. 422) is a typical example of a step-up transformer.

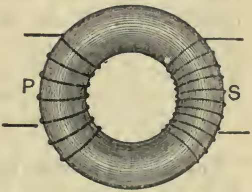

Fig. 313.

The earliest form of transformer was used by Faraday, and consisted of an iron ring, on which two coils of wire, P and S (Fig. 313), termed the primary and secondary coils, were wound. When a current traversing coil P is altered, a change is produced in the magnetic induction through S, in which an induced E.M.F. is thereby set up. If the number of turns of wire in P and S are the same, any change in voltage at the ends of P will produce the same change in voltage at the ends of S, providing that there is no magnetic leakage from the ring and no loss of energy due to hysteresis and Foucault currents in the iron. If the number of turns of wire in S is greater than the number in P, the voltage at the ends of S will be greater than the change in voltage at the ends of P in practically the same ratio as the number of turns. Thus, in order to transform up from 220 volts to 2200 the ratio of the windings would be 1 : 10. In practice, the type of transformer represented in Fig. 313 is not economical, unless the ring consists of varnished wires or thin stampings, and unless the coils are wound either near to each other or one over the other in order to prevent loss by leakage of magnetic lines of force.

If the losses are negligible, the power spent in the primary is equal to that generated in the secondary, and therefore the current is diminished (or increased) in the same ratio as the voltage is increased (or diminished). A transformer serves as an automatic regulator of the power transmitted; for, if the secondary circuit is open, the transformer acts simply as a choking-coil, which practically prevents the passage of current through the primary. On taking a small current from the secondary, the choking action is reduced and a commensurate current traverses the primary, and the choking action is reduced again by any further increase in the secondary current.

SUMMARY.

Alternating Currents may be represented graphically by means of either a *wave-diagram* or a *clock-face* diagram.

The **virtual strength** of an alternating current is the numerical value of that steady continuous current which will have the same heating effect. It may be defined also as the *root mean square value* of the alternating current: this is *the square root of the average ordinate of a curve of which the ordinates are proportional to the squares of the instantaneous current strengths.*

The *virtual strength* = 0.707 × *maximum strength*, and
the *true mean* ,, = 0.637 × ,, ,,

The relation between current and E.M.F., in an alternating circuit, is given by the equation $C_v = E_v/\sqrt{R^2 + (\omega L)^2}$. The quantity in the denominator is termed the *impedance*, and ωL is termed the *reactance*. If the circuit has self-induction, the current *lags* in phase behind that of the E.M.F.; the angle of lag ϕ is given by the equation $\tan\phi = \omega L/R$.

The **average power** developed in an alternating circuit is equal to the product $C_vE_v \cos\phi$, where C_v and E_v are the virtual values of current and E.M.F., and ϕ is the angle of lag. The power is zero when $\phi = 90°$.

Capacity, in an alternating circuit, tends to diminish the lag of the current due to self-induction. The lag is reduced to zero when $LS = \frac{1}{\omega^2}$, where S is the capacity of the circuit.

Alternating currents are regulated by means of **choking coils**, which may be described as conductors possessing small resistance and variable reactance.

A **transformer** is a device for raising or lowering the voltage in an alternating circuit.

QUESTIONS ON CHAPTER XXIII

1. With a frequency of 100, what is the impedance of a coil of 10 ohms resistance and 0.02 henry inductance? By what angle does the current lag behind the E.M.F.?

2. An alternating E.M.F. of 200 (virtual) volts, with frequency of 60 cycles per second, is applied to a circuit having a resistance of 10 ohms and inductance of 0.02 henry. Find the current and the angle of lag.

3. A circuit has a resistance of 12 ohms and an inductance of 0.02 henry. Find graphically the impedance at a frequency of 50.

4. What is the impedance of a circuit of 12 ohms resistance, 0.12 henry inductance, and 25 microfarads capacity, when the frequency is 100? Will the current lag or lead, and by what angle?

5. A circuit has an inductance of 0.25 henry and a capacity of 10 microfarads. With what frequency will the impedance be equal to the ohmic resistance?

6. If the inductance of a circuit is 0.1 henry, and its resistance 10 ohms, at what frequency will an increase of 1 per cent. in the inductance be more important than a diminution of 50 per cent. in the resistance? Prove that, when very high frequency currents are employed, the self-induction of even a straight wire is more important than its resistance.

7. Describe the construction of an electrostatic voltmeter. An electrostatic voltmeter gives deflections of 15, 18 and 21 scale divisions for constant potentials of 50, 60 and 70 volts respectively: what deflections will be produced by an alternating electromotive force $E \sin pt$, (*a*) when the amplitude E is 70 volts and (*b*) when E is 90 volts?

CHAPTER XXIV

CHEMICAL EFFECTS OF THE ELECTRIC CURRENT

Electrolysis.—All conductors of electricity may be divided into two groups,

(i) *pure metals, whether solid or molten, mercury and liquids which are not decomposed when a current passes through them, and*

(ii) *those compounds, whether fused or in solution, which undergo decomposition by the current.*

The latter are termed **electrolytes.** Perfectly pure liquids, *e.g.* water, sulphuric acid, alcohol, and ether, are not capable of electrolytic decomposition.

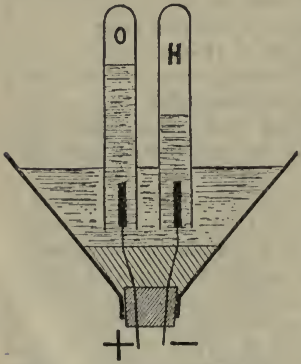

FIG. 314.—Electrolysis of Water.

A current is conveyed to and from an electrolyte by immersing in it rods or plates of a conductor of the first group; these are termed the **electrodes**—that by which the current enters is termed the **anode** (*ἀνοδος*, a way up), and that by which it leaves is the **kathode** (*κάθοδος*, a way down). The electrolyte undergoes decomposition, and, under the influence of a potential difference, the constituent portions tend to move in opposite directions, and are set free at the electrodes. The elements, or groups of elements, liberated are termed **ions** (or wanderers); the ion liberated at the anode is termed the **anion**, and that liberated at the kathode is termed the **kation**. Also, kations and anions are sometimes referred to as **electro-positive** and **electro-negative** ions respectively, since the former travel through the electrolyte *with* the current.

Fig. 314 represents a simple form of apparatus for the electrolysis of water. The lower end of the vessel, made from a glass funnel, is closed with a cork and a layer of paraffin wax. The wires through the cork terminate in strips of platinum foil. The vessel and test-tubes are filled with dilute sulphuric acid. Hydrogen is liberated at the kathode and oxygen at the anode. The chemical change due to the current may be presented thus:

$$\begin{array}{ccccc} H_2O & = & H_2 & + & O \\ \textit{Water} & \left\{\begin{array}{c}\text{is decomposed}\\ \text{into}\end{array}\right\} & \begin{array}{c}\text{2 vols.}\\ \textit{Hydrogen}\end{array} & \text{and} & \begin{array}{c}\text{1 vol.}\\ \textit{Oxygen.}\end{array}\end{array}$$

Faraday's Laws.—Faraday, in 1833, fully investigated the phenomena of electrolysis, and deduced the following laws:

(i) **The mass of an ion set free by a current is proportional to the quantity of electricity which has passed.**

(ii) **If several different electrolytes are included in the same circuit, the relative masses of the liberated ions are proportional to their chemical equivalents.**

The **chemical equivalent** of an element is the weight of it which will combine with, or replace, 1 part by weight of hydrogen, and is numerically equal to the ratio of the atomic weight of the element to that of hydrogen divided by the valency; the **valency** of an element being the number of hydrogen atoms which will combine with, or are replaced by, one atom of the element.

Thus, in comparing the formulae of sulphuric acid H_2SO_4, and of sodium sulphate Na_2SO_4, one atom of sodium replaces one atom of hydrogen; the sodium atom is therefore *univalent*. Hence, since the atomic weights of sodium and hydrogen are 23.05 and 1.008 respectively, the chemical equivalent of sodium is $23.05/1.008=22.87$. Similarly, in copper sulphate, $CuSO_4$, copper is *divalent*, and its atomic weight is 63.6; hence the equivalent of copper is $63.6/(1.008\times 2)=31.54$. In *cuprous* compounds the copper is univalent, and its equivalent is then 63.09. Also in ferrous and ferric compounds iron is divalent and trivalent respectively, and the equivalent is either 27.73 or 18.485.

Electro-chemical Equivalents of Elements.—**The electro-chemical equivalent** (E.C.E.) **of an element is the weight in grams**

which is deposited by the passage of the unit quantity of electricity (1 coulomb). The accurate determination of the E.C.E. of at least one element is a most important experiment, since by the second law of Faraday this determination may be used for the purpose of calculating the E.C.E. of all other elements. Lord Rayleigh and Mrs. Sidgwick[1] found that 1 coulomb deposits 0.001118 gram of silver.[2] The chemical equivalents of silver and hydrogen are 107.073 and 1 respectively, hence the E.C.E. of hydrogen is $0.001118 \times 1/107.073 = 0.00001044$. In a similar manner the E.C.E. of any other element may be calculated. In the following table the system of atomic weights based upon **O = 16** is used.

ELECTRO-CHEMICAL EQUIVALENTS.

Element.	Atomic Weight.	Chemical Equivalent.	Electro-chemical Equivalent. (Grammes per coulomb.)
Electro-positive.			
Aluminium, - -	27.1	8.96	0.00009355
Copper (Cuprous),	63.6	63.09	0.0006587
,, (Cupric), -	,,	31.54	0.0003293
Gold, - - -	197.2	65.21	0.0006808
Hydrogen, - -	1.008	(1)	0.00001044
Iron (Ferrous), -	55.9	27.73	0.0002895
,, (Ferric), -	,,	18.485	0.0001930
Nickel,- - -	58.7	29.12	0.0003040
Potassium, - -	39.15	38.84	0.0004055
Silver, - - -	107.93	107.073	0.001118
Sodium, - -	23.05	22.87	0.0002387
Zinc, - - -	65.4	32.44	0.0003387
Electro-negative.			
Chlorine, - -	35.45	35.17	0.0003672
Oxygen, - -	16.00	7.935	0.00008285

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[1] *Phil. Trans.*, Part ii., 1884.

[2] More recent determinations indicate that the correctness of the last figure in this quantity is not yet established.

The following definitions will be found useful:

A **Gram Atom** of an element is a quantity of the element equal in grams to the number denoting its atomic weight; thus the gram atom of silver is 107.93 grams.

A **Gram Ion** is a quantity of an ion equal in grams to the number denoting its combining weight; thus a gram ion of sulphion SO_4 is 96.06 grams.

A **Gram Molecule** of a substance is a quantity equal in grams to the number denoting its molecular weight; thus a gram molecule of HCl is 36.458 grams.

A **Gram Equivalent** is a quantity equal to a gram molecule divided by the valency of the compound.

Since 0.001118 gram of silver is deposited by 1 coulomb, the quantity of electricity required to deposit 107.93 grams is 107.93/0.001118 = 96550 coulombs.[1] The same quantity will be required to deposit a *gram-atom* of any univalent element. In the case of a divalent element the gram-atom will require (2 × 96550) coulombs; thus this quantity will be required to deposit 63.6 grams of copper from copper sulphate.

EXAMPLE.—If the E.C.E. of nickel is 0.000304, calculate how much electricity is required to give a coating of nickel 0.1 mm. thick to a surface of 1000 sq. cms.? Density of nickel = 8.8.

Volume of nickel = 1000 × .01 = 10 c.c.
Mass of ,, = 10 × 8.8 = 88 grams.
1 coulomb of electricity deposits 0.000304 gm.

$$\therefore \text{ quantity of electricity required} = \frac{88}{0.000304} = 289{,}600 \text{ coulombs.}$$

Voltameters.—The accuracy with which the E.C.E.s of several elements are known enables the process of electrolysis to be used for the measurement of current: the method is especially useful in the case of very weak currents. As a general rule, the deposition of silver or copper, or the decomposition of water is adopted, and the apparatus used is termed a **Voltameter.**

(i) THE SILVER VOLTAMETER.—The kathode consists of a platinum dish, resting on a metal ring, which contains a 10% aqueous solution of pure silver nitrate. The anode is a circular disc of pure silver suspended horizontally in the solution by means of platinum wires ; the disc is wrapped in filter paper to prevent any disintegrated silver from falling into the dish. Currents up to 1.5 amperes may be measured readily by this method.

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[1] Some writers apply the term *faraday* to this quantity of electricity.

(ii) THE COPPER VOLTAMETER.—Fig. 315 represents a convenient form of this voltameter ; the two outer plates of copper form the anode, and the central plate is the kathode, which should be much smaller than the anode plate. The plates hang from copper wires, *C*, which are supported by two vulcanite rods, *V* and *V′*. The solution used is a 15% solution of copper sulphate, to each litre of which 5 c.c. of concentrated sulphuric acid have been added. The cathode should be sufficiently large to allow 50 sq. cms. of surface for each ampere of current.

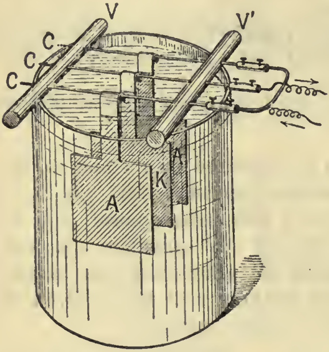

FIG. 315.—A Copper Voltameter.

(iii) THE WATER VOLTAMETER.—The mass of hydrogen liberated by 1 ampere in 1 second is 1.044×10^{-5} gm.; since the density of hydrogen at 0° C. and 76 cms. pressure is 0.0000896, this mass of gas will occupy 0.1165 c.c. The volume of oxygen liberated under similar conditions is 0.0582 c.c. Hence the total volume of mixed gases set free by 1 ampere in 1 second is 0.1747 c.c.

A simple form of water voltameter is represented in Fig. 316. Two platinum wires are fixed through a rubber stopper and terminate in platinum strips immersed in dilute

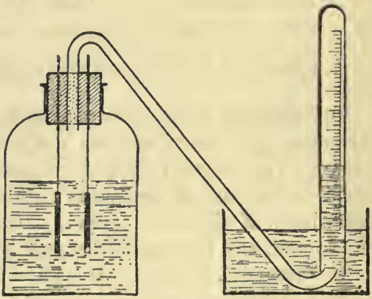

FIG. 316.—A Water Voltameter.

sulphuric acid (1 to 4).[1] The observed volume of gases must be corrected for temperature and pressure, and also for the vapour tension of water. It should be borne in mind that the gases are appreciably soluble in water; the current should therefore be passed through the voltameter for some time before the evolved gases are collected.

Secondary Reactions in Electrolysis.—Based upon the fact that pure water is non-electrolytic but that the addition of sulphuric acid renders the water electrolytic, it is now generally assumed that the primary action of the current is to liberate hydrogen ions at the kathode and sulphion (SO_4) ions at the anode; a **secondary** action simultaneously proceeds between the sulphion and the water, oxygen being liberated and sulphuric acid re-formed; thus,

$$SO_4 + H_2O = H_2SO_4 + O.$$

In the electrolysis of copper sulphate, using *copper* electrodes, copper is deposited on the kathode, and the sulphion ions attack the anode reforming copper sulphate, which is dissolved in the surrounding water—thus maintaining the total quantity of the salt originally present. This effect can be observed by using a narrow glass vessel with a copper anode placed horizontally at the bottom, and the kathode parallel to and above it. Changes in the intensity of the colour indicate that the solution near the anode becomes more concentrated, while that near to the kathode becomes less so.

With a solution of a salt of an alkali metal, such as sodium sulphate, Na_2SO_4, hydrogen and oxygen are set free at the kathode and anode respectively. We may assume that, initially, sodium and sulphion ions are liberated, and that each of these re-acts upon the surrounding water, forming caustic soda and hydrogen in the former case and sulphuric acid and oxygen in the latter case, thus

$$Na_2 + 2H_2O = 2NaHO + H_2.$$
$$SO_4 + H_2O = H_2SO_4 + O.$$

This result can be observed readily by conducting the electrolysis in a narrow glass cell divided into two compartments by a porous partition—platinum electrodes being used and the solution coloured with neutral litmus.

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[1] A solution (15°/₀) of caustic soda is preferable, since, when dilute acid is used, there is a tendency for the oxygen to be converted into ozone.

The gases liberated in the electrolysis of hydrochloric acid are somewhat variable. The gas liberated at the anode consists of chlorine mixed with a small quantity of oxygen, and smaller quantities of other chlorine compounds may also be formed. The oxygen is due to a secondary action of chlorine on water, thus $Cl_2+H_2O=2HCl+O$.

Commercial Applications of Electrolysis. **Sodium and Potassium.**—Previous to 1808 the caustic alkalis were supposed to be chemical elements, but in that year Davy succeeded in decomposing caustic soda and potash by electrolysis. In his initial experiments he placed a piece of slightly moistened caustic soda on a platinum plate connected to the positive pole of a battery and touched the upper surface of the caustic with a platinum wire joined to the negative pole. Oxygen gas was liberated from the plate, and on the wire small globules of metal appeared which rapidly tarnished in the air and detonated when the wire was immersed in water. Fig. 317 represents a simpler method of conducting the experiment. Mercury is poured into a glass vessel and covered with a concentrated aqueous solution of caustic soda. A platinum wire, joined to the negative pole of a battery of 10 or 12 cells, dips into the mercury, the exposed portion being protected by insulating material. A platinum plate connected to the positive pole of the battery is immersed in the soda solution. The sodium forms an amalgam with the mercury and may be detected by its action upon pure water; or the mercury may be separated by distillation in a vacuum.

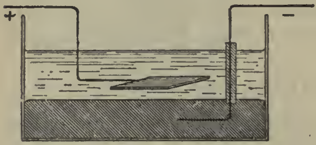

FIG. 317.—Electrolysis of Caustic Soda.

Until recently sodium and potassium were manufactured by the electrolysis of the fused chlorides. But this method is now almost completely replaced by Castner's process, in which fused caustic soda is electrolysed between iron electrodes; the metal being lighter than the electrolyte rises to the top and is removed from time to time.

Caustic Soda.—The Castner process for making caustic soda and chlorine from common salt is an important industry in this country. The cell (Fig. 318) is constructed of slate and is divided into three compartments by vertical partitions which do not reach quite to the floor. The floor is covered with mercury; the end compartments contain strong brine and the middle compartment contains water. Carbon anodes (A) are inserted through the outer walls. In the earliest form of the cell the mercury was used as a cathode. On passing a current through the liquid, chlorine is liberated in the outer compartments and is conducted away, while the sodium forms an amalgam with the mercury. One end of the cell is pivoted on a fixed support, and the other end rests on a revolving eccentric; in this manner a slow rocking movement is imparted to the cell, thus ensuring a constant circulation of the mercury whereby it is brought into contact with the water in the middle compartment; here the amalgam is decomposed forming a solution of caustic soda, which is drawn off slowly and replaced by fresh water. In a more recent improvement, patented by Kellner, the E.M.F. generated in the middle compartment is utilised by introducing an iron cathode C into this compartment, the mercury thus becoming an intermediate electrode. The improved cell is known as the Castner-Kellner.

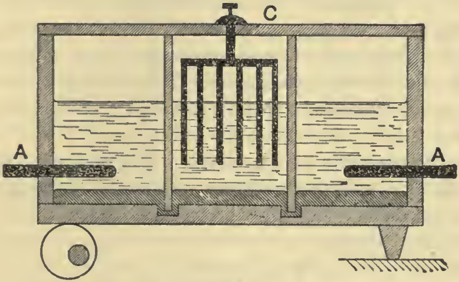

FIG. 318.—The Castner-Kellner Electrolytic Cell.

Aluminium.—Aluminium is now obtained by the electrolysis of fused cryolite (a double fluoride of aluminium and sodium) and alumina, contained in a carbon-lined iron tank (Fig. 319) which serves as the cathode. The anode consists of one or more carbon rods. The cryolite itself is not decomposed, but it is the *dissolved* alumina which is electrolysed. The aluminium is deposited on the bottom and walls of the tank; and since the

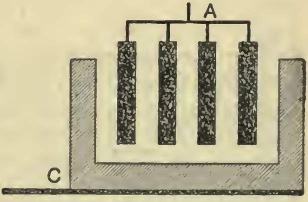

FIG. 319.

temperature is about 800° C., the metal is fluid and may be tapped out. The carbon anodes are oxidised slowly to carbon monoxide by the oxygen liberated on their surface, and this contributes towards the temperature of the furnace.

The Work of Electrolysis.—According to Faraday's laws, one coulomb of electricity decomposes *equivalent* quantities of all electrolytes ; but, since these, in recombining, would develop very different quantities of heat, the amount of work done by one coulomb must vary over wide limits according to the electrolyte through which it is passing. This amount of work is equal to the product Q × E, where E is the potential difference between the electrodes. Hence E must vary according to the electrolyte.

If Q is the quantity of electricity required to decompose one *gram-equivalent* of the electrolyte, and if q is the heat developed by the recombination of one gram-equivalent, then the work Q × E *joules* is equal to

$$\frac{Q \times E}{4.2} \text{ therms} = q,$$

$$\text{or } E = (4.2 \times q)/96550 \text{ volts.}$$

In the case of hydrogen, q is equal to 34000 therms ; hence, the minimum potential difference required to electrolyse water is

$$\frac{4.2 \times 34000}{96550} = 1.48 \text{ volt.}$$

Ionisation.—More than one theory has been offered as an explanation of the phenomena of electrolysis. Fig. 320 represents the theory of Grotthus (1806), according to which the molecules of an electrolyte are, under normal conditions, arranged in a haphazard manner as shown in the top row of molecules in the diagram—the shaded and clear portions represent the kation and anion respectively. The opposite electric charges on the ions will cause the molecules to assume the linear

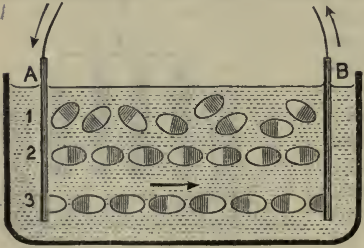

FIG. 320.—The Theory of Grotthus.

arrangement of the 2nd row when a potential difference is set up between the electrodes. The 3rd row shows how, if the potential difference is sufficiently great, the molecule next to the anode is decomposed, the anion portion being set free, while the kation portion attacks the next molecule, seizing the anion and liberating the kation which in turn decomposes the third molecule. This process extends through the liquid to the kathode, where the the kation portion of the last molecule is set free.

Subsequent experiments, combined with observations on the phenomena of vapour pressure, osmotic pressure, and freezing point of solutions, have provided strong evidence in favour of the opinion that the molecules of a dissolved substance are in a physical condition different from that when the solid is anhydrous. For example, scarcely any transference of electricity takes place between platinum electrodes joined to the poles of a battery and immersed either in *absolutely pure* water or in dry hydrochloric acid gas; yet a current readily traverses a solution of the gas in water. We may assume therefore that the molecular condition of the gas when dissolved in water is *not* the same as when the gas is anhydrous. On the other hand, a solution of hydrochloric acid gas in chloroform is non-conducting; hence the nature of the solvent plays an important part in determining whether the solution will or will not conduct electricity. It is supposed that in an electrolyte some or all of the molecules of the dissolved substance are broken up into their constituent atoms *which are oppositely charged: such charged atoms are termed* **ions.** According to the modern electron theory (pp. 121 and 499) the ion of a univalent element is an atom associated with one too many or one too few electrons; in the former case it is negatively charged, and in the latter case it is positively charged. Similarly the ion of a divalent element is the atom associated with two too many or two too few electrons. The phenomenon of the separation of a molecule into oppositely charged ions is termed **ionisation.**

The degree to which ionisation proceeds depends largely upon the concentration of the solution; it is more marked in dilute than in strong solutions, and it is only complete in solutions of extreme dilution. There is reason to believe that, in the case of pure water, about 10^{-9} gram in each c.c. is dissociated into ions of hydrogen and of hydroxyl. At first sight it may be difficult to understand how the strong chemical affinity between the elements is inoperative when the compound is ionised; but, if we attribute the affinity to the attraction of opposite electric charges, it must be borne in mind that the solvent has usually a high specific inductive capacity and that the force of attraction in such a medium is inversely proportional to such capacity (p. 182). Moreover, the molecules of a dissolved solid have somewhat similar properties to those of a dry gas; they possess a considerable *free-path*, and undergo frequent collisions, which would tend to break up the molecules into their constituent ions. Again, there would be frequent collisions between free and oppositely charged ions, and in a certain fraction of such cases complete molecules would be re-established. In this manner we are able to imagine a constant interchange of partners going on and the simultaneous existence of both complete molecules and free ions. This phenomenon must be clearly distinguished from that of chemical *dissociation*: for example, in the case of ammonium chloride the ions in an aqueous solution would be NH_4 and Cl whereas dissociation breaks up the molcules into NH_3 and HCl.

Assuming the truth of the ionisation theory, the introduction of an electric field of force within the solution will create a tendency for the positively charged kations to move towards the kathode and the negatively charged anions towards the anode. In this sense the effect of introducing electrodes at different potentials is only *directive*, and no expenditure of energy is required in order to do chemical work within the solution. This statement is supported by the fact that if suitable terminals are introduced into the electrolyte *at other points than at the electrodes* Ohm's law is obeyed, *i.e.* the current between any two points is proportional to the potential difference; consequently, Joule's law will also be obeyed, and all the electrical energy will be converted into heat and not partly into heat and partly into chemical change.

The passage of a current through an electrolyte may therefore be regarded as a process of *convection* of the oppositely charged ions. Also, the quantity of positive electricity conveyed by the kations is equal to the quantity of negative electricity conveyed by the anions, and *each* of these is equal to the product of the current strength and the time. If the total current is i, and the charge on each ion is e, the number of ions conveyed both to the kathode and the anode in unit time is i/e. Since both anions and kations may be regarded as carriers of electricity it might appear that *one half* of the current would be carried by the anions and the other half by the kations; but the following argument will show that the physical processes taking place near to the electrodes are much more complicated than those taking place at the centre of the cell.

Assuming that all the ions have the same velocity and equal charges, then across any section AB (Fig. 321) there must be a transference of i/e ions in unit time, and these consist of $i/2e$ kations and $i/2e$ anions. At a section CD, very near to the kathode, $i/2e$ kations are brought up by the potential gradient and strike against the kathode; but there are also $i/2e$ anions which cross CD out of the kathode space. Assuming that the liquid never has free electrification, so that in a given space there must be the same number of positive and negative ions, the liberation of these $i/2e$ anions necessitates the removal of the same number of kations from the kathode space. Hence, in this space there is a total loss of i/e kations, one half of these being brought up by the potential gradient, the remainder simply dropping out of the kathode space and thus setting free $i/2e$ anions which travel across to the anode. Similarly, of the i/e anions striking the anode, one-half are brought up by the potential gradient and the remainder are taken from the liquid quite near to the anode.

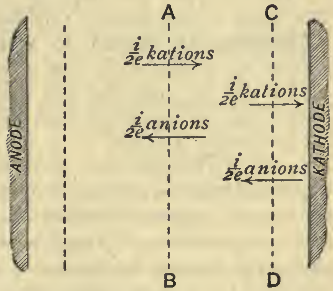

FIG. 321.

Ionic Charge—It has been stated already that the passage of 96550 coulombs will liberate 1 gram equivalent of any element or compound radicle. If n is the number of H atoms in 1 gram of the gas, and if e is the charge on each atom, then

$$ne = 96550 \text{ coulombs.}$$

But Lord Kelvin and Dr. Johnstone Stoney have proved that the hydrogen atom weighs approximately 10^{-25} gm., hence $n = 1/10^{-25} = 10^{25}$.

$$\therefore\ e = 96550/10^{25} = 10^{-20} \text{ coulomb, approximately.}$$

Since the same quantity of electricity is conveyed by 107 grams of silver, and since the silver atom weighs 107 times as much as the hydrogen atom, the same atomic charge of 10^{-20} coulomb is present on each atom of silver. In fact, the same charge is conveyed by each atom of any univalent element. This atomic charge of a univalent element may be regarded as a natural unit of quantity of electricity, and Dr. Stoney has suggested the term **electron** for this atomic charge. The terms *ion* and *electron* may be distinguished by remembering that the ion is the material atom with its associated electric charge, while the electron is the charge without the atom.

With divalent elements each atom is *equivalent* to two atoms of a univalent element, and therefore has a charge $2e$ associated with it. The charge on each atom of a trivalent element will be $3e$.

Ionisation, as applied to voltameters.—The ionisation theory enables us, in any given case of electrolysis, to anticipate how the current may be conducted into and from the electrolyte. Thus, the current may be transferred from the anode into the electrolyte by (i) *the formation of a kation*, or by (ii) *the deposition and discharge of an anion*, or (iii) by both these processes. Similarly, the current may be transferred from the electrolyte to the cathode by (i) *the deposition and discharge of a kation*, or (ii) *by the formation of an anion*, or (iii) by both these processes. Thus, in the **copper voltameter,** since the electrolyte contains the divalent ions of copper and sulphion and the monovalent ions of hydrogen and hydroxyl, the current may be transferred at the anode by one or more of the following processes:

(i) *liberation of copper ions from the anode*;
(ii) *deposition of sulphion ions*;
(iii) *deposition of hydroxyl ions.*

In ordinary cases the first process alone is evident, but the third process can be detected in the blackening of the copper plate, especially when the current density is abnormally high. The current may be transferred from the electrolyte to the kathode by one or more of the following processes:

(i) *deposition of positively charged copper ions*;
(ii) ,, ,, ,, ,, *hydrogen ions*;
(iii) *transfer of only one-half the ionic charge from divalent copper ions, leaving in solution monovalent cuprous ions.*

The second process is evident only when an excessive current density is used, and it may be minimised by keeping the electrolyte in motion. The third process may be present to a slight extent, and it tends to make the deposition of copper somewhat less than normal.

When platinum plates are used in the electrolysis of copper sulphate, the possible reactions at the anode are (i) *the deposition of hydroxyl ions, or of sulphion ions, or of both*, and (ii) *the liberation of platinum ions.* The latter effect is practically absent, and the reactions are limited to the deposition of H and SO_4 ions, both resulting in the liberation of oxygen. The reactions at the kathode are identical with those obtained when a copper kathode is used.

When dilute sulphuric acid is electrolysed between platinum plates the deposition of hydrogen ions is the only possible reaction at the kathode, and *it may be regarded, therefore, as a trustworthy means of current measurement.* At the anode both hydroxyl and sulphion ions may be liberated ; but this assumes that hydrogen and sulphion are the only possible ions to be derived from sulphuric acid, and recent research has shown that it is also possible for the acid to dissociate into ions of H and HSO_4, and the reaction at the anode *may* include the discharge of the latter ions, with subsequent combination to form persulphuric acid $H_2S_2O_8$. If a strong solution of sulphuric acid is electrolysed the persulphuric acid is detected readily. For this reason the reaction at the anode is not trustworthy as a means of current measurement.

Migration of the Ions.—When a solution of copper sulphate is electrolysed in a cell, arranged so that the copper electrodes are horizontal and the anode underneath, a change in the intensity of colour is noticeable ; the solution near to the anode becomes more deeply coloured, while that near the kathode becomes lighter. Hittorf has attributed this to a difference in the velocity of the ions, and the change in concentration of the solution can be readily explained if we assume that the SO_4 ions travel more rapidly than the Cu ions. It might, at first, be thought that the anion and kation must move at the same speed since they are liberated in chemically equivalent proportions at the opposite electrodes : but this equivalence is maintained for any relative rate of motion. In Fig. 322 the black dots and circles represent kations and anions respectively. If the anion travels twice as rapidly as the kation, the arrangement of the ions after a short interval may be represented by the 2nd column of

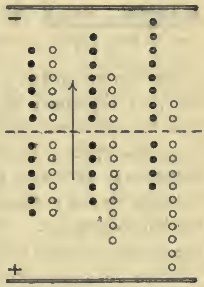

FIG. 322.

the diagram; three ions of each kind have been liberated, also five molecules remain undecomposed below the central line and four molecules remain above. After a further equal time interval, there are four undecomposed molecules below, and two above, the central line. The solution near to the anode therefore becomes more concentrated than that near to the kathode. Moreover, the ratio of the molecules lost in the separate halves of the cell is equal to the ratio of the relative velocities, or

$$\frac{\text{Velocity of anion}}{\text{Velocity of kation}}=\frac{\text{Molecules lost round kathode}}{\text{Molecules lost round anode}}.$$

Hence, by analysing the solution on opposite sides of the cell after the passage of a current, it is possible to calculate the *ratio* of the velocities of the ions.

In order to obtain the *absolute* values of the ionic velocities, it is necessary to obtain another relation between them, and Kohlrausch has shown how this may be obtained from a knowledge of the conductivity[1] of the solution. If u and v are the velocities of the kation and anion respectively, and if there are N gram molecules of copper sulphate per unit volume; then, assuming that the solution is so dilute that ionisation is complete, Nu gram-ions of the kation and Nv gram-ions of the anion will in unit time cross unit area of any section perpendicular to the direction of the current and at a distance from the electrodes (Fig. 323). Moreover, as explained on p. 465, the liberation of Nv anions from the kathode space necessitates the removal of the same number of kations from this space; hence the quantity of the kation deposited on the kathode is $N(u+v)$ gram-ions, of which Nu are moved up to the kathode by the potential gradient, the remainder Nv being taken from the kathode space. Similarly, the quantity of the anion deposited on the anode will also be $N(u+v)$ gram-ions.

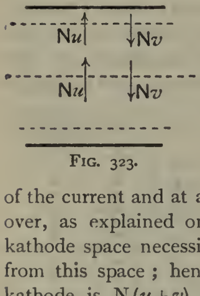

FIG. 323.

Since $N(u+v)$ gram-ions, including anion and kation, cross unit area in unit time, and each gram-ion conveys $n \times 9655$ E.M. units of electricity, where n is the valency of the ions, the total quantity of electricity conveyed across unit area in unit time is

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[1] The conductivity may be determined by the method described on p. 329: or, the solution may be placed in a long trough, and the current observed by means of an ammeter; the P.D. between any two points, at known distance apart, is observed by connecting two thin wires, which dip into the electrolyte at the points, to the terminals of an electrostatic voltmeter.

$$N(u+v)n \times 9655 \text{ C.G.S. units.}$$

The value of $(u+v)$ depends upon the electric force acting upon the ions. This force is equal to the rate of fall of potential between the electrodes, and may be expressed as dV/dx. Hence, if k is the specific conductivity, *i.e.* the reciprocal of the specific resistance, then, by Ohm's law,

$$N(u+v)n \times 9655 = k \cdot \frac{dV}{dx}.$$

If the fall of potential is 1 volt per cm.; then $dV/dx = 10^8$ C.G.S. units. Hence

$$(u+v) = \frac{k}{N} \times \frac{1}{n} \times \frac{10^8}{9655}.$$

But, if N_1 is the number of *gram-equivalents* of the electrolyte *per litre*, then $N_1 = Nn \times 10^3$;

$$\therefore \ (u+v) = \frac{k}{N_1} \times (1.036 \times 10^7).$$

The ratio k/N_1 is termed the *molecular conductivity*: its value, for infinite dilution, is obtained by plotting a curve of *conductivity* and *strength of solution*; from such a curve the conductivity for infinite dilution may be estimated.

By measuring the specific resistance of any electrolyte Kohlrausch obtained values for $(u+v)$ which, when combined with the known value of u/v, enabled the absolute values of u and v to be determined. The following table gives the velocities of ions in infinitely dilute solution, the potential-gradient being 1 volt per cm.:

Ion.	Velocity.	Ion.	Velocity.
H	3.2×10^{-3} cm. per sec.	OH	1.82×10^{-3} cm. per sec.
K	0.66 ,, ,,	Cl	0.59 ,, ,,
Na	0.45 ,, ,,	I	0.61 ,, ,,
Ag	0.57 ,, ,,		

The student is referred, for further information on the subject matter of Chaps. XXIV and XXV, to the following works: Lehfeldt's *Electro-Chemistry* (Longmans); Whetham's *Treatise on the Theory of Solution* (Camb. Univ. Press); and to Walker's *Introduction to Physical Chemistry* (Macmillan).

SUMMARY

Definitions.—An *electrolyte* is a conductor which undergoes chemical decomposition when traversed by a current of electricity.

The terminals, or *electrodes*, by which the current enters and leaves the electrolyte are termed the *anode* and *kathode* respectively.

The elements, or groups of elements, liberated at the electrodes are termed *ions*. The ion liberated at the anode is termed the *anion*, and that liberated at the kathode is termed the *kation*.

Faraday's Laws.—(i) *The mass of an ion set free by a current is proportional to the quantity of electricity which has passed.*

(ii) *If several different electrolytes are included in the same circuit, the relative masses of the liberated ions are proportional to their chemical equivalents.*

The **Electro-Chemical Equivalent** (E.C.E.) *of an element is the weight in grams of it which is liberated by the passage of 1 coulomb of electricity.*

The deposition of 1 *gram-equivalent* of any element requires the passage of 96550 coulombs.

Voltameters are instruments in which the strength of a current is determined by the quantity of an element liberated by the current in a given time.

In many examples of electrolysis the ions initially set free by the current react either upon the liquid solvent or upon the electrodes and cause the liberation of other ions. This is termed a *secondary* action.

Electrolysis is commercially applied in the manufacture of (i) Sodium and Potassium, (ii) Caustic Soda, (iii) Aluminium, etc.

Ionisation.—It is now supposed that in an electrolytic solution some or all of the molecules of the dissolved substance are broken up into their constituent atoms, which are *oppositely charged*; such charged atoms are called *ions*, and the phenomenon is termed *ionisation.*

In an electrolytic cell, the quantity of positive electricity conveyed by the kations is equal to the quantity of negative electricity conveyed by the anions, and *each* of these is equal to the product of the current-strength and the time.

Ionic Charge.—The charge e on each monovalent ion is 10^{-20} coulomb approximately. The charge on any divalent ion is $2e$, and $3e$ on any trivalent ion.

Migration of the Ions.—When an electrolyte is placed in an electric field of force the ions do not always travel with the same velocity; thus, in the case of copper sulphate, the SO_4 ions travel more rapidly than the Cu ions.

QUESTIONS ON CHAPTER XXIV

1. Describe a method of investigating the relation between the strength of an electric current and the rate of chemical change produced by it.

2. How may the strength of an electric current be measured by means of a copper voltameter?

3. Is the strength of the current that passes through a simple circuit the same at all points of the circuit however its parts differ in resistance? How would you justify your answer by experiment?

4. Explain the term *electro-chemical equivalent*. If 3 amperes deposit 4 grams of silver in 20 minutes, what is the electro-chemical equivalent of silver?

5. If the electro-chemical equivalent of silver is 0.01118, what is the E.C.E. of oxygen (Ag = 108)?

6. How much silver will be deposited by a current of 5 amperes in 1 minute?

How long will it take a current of 5 amperes to deposit 5 grams of copper?

7. A piece of metal weighing 200 grams is to be plated with 2½% of its weight in gold. If the current strength is 1 ampere, how long will it take to deposit the required weight of gold?

8. A tangent galvanometer and a copper voltameter were connected in series and included in the same circuit. A constant current was sent through the circuit for 30 minutes, and the weight of copper deposited was found to be 0.272 gram. If the deflection of the galvanometer needle was 30°, calculate the *reduction factor* of the galvanometer.

9. A metal plate, having a surface of 200 sq. cms. is to be silver-plated. If a current of 0.5 ampere is used for a period of 1 hour, what thickness of silver will be deposited on the plate (density of silver = 10.6 grams per c.c.)?

10. A copper voltameter, consisting of 3 cells connected in parallel, is joined in series with an ammeter, an accumulator, and a variable resistance. After a period of 30 minutes, the increase in weight of the kathodes was found to be 0.763, 0.742, and 0.785 gram respectively. What should have been the ammeter reading?

11. A current is passed through a voltameter and through a coil of wire in series with it. If the current is altered in such a way that the heat produced in the coil is doubled, show what change will be produced in the rate at which chemical action takes place in the voltameter.

12. Distinguish between the chemical and electro-chemical equivalents of an element. What weight of hydrogen is separated from water by the

passage of 1000 coulombs of electricity, given that the chemical equivalent of copper is 31.5, and its electro-chemical equivalent = 0.000328 grams per coulomb?

13. How long must a constant current of 500 amperes pass through a bath for the electrolytic deposition of copper in order to deposit sufficient copper to make 1 kilometre of No. 16 S.W.G. wire (diam. = 0.163 cm.)? The density of copper is 8.95 gms. per c.c.

14. A current is measured by means of a water voltameter. The density of the dilute acid was 1.2 gms. per c.c. In 5 minutes 25 c.c. of mixed gases were collected. Assuming that the gas was saturated with moisture, calculate the current strength, having given that

height of column of dilute acid = 10 cms.,
density of mercury = 13.56,
reading (corrected) of barometer = 75.62 cms.,
temperature of room = 20° C.,
vapour tension of water at 20° C. = 17.4 mm.

15. A Daniell's cell has an E.M.F. of 1.07 volts, and its internal resistance is 2 ohms. Its terminals are connected by two wires in parallel of 3 and 4 ohms respectively. If the electro-chemical equivalent of copper is 0.000328 grams per coulomb, calculate the quantity of copper deposited in the cell; and also calculate the heat developed (*a*) in the cell, (*b*) in each of the wires, during one hour of working.

16. A tangent galvanometer has a current passed through it which deflects it 45°. The same current passes through a copper voltameter, where it deposits 0.3 gram of copper in 30 minutes. If the electro-chemical equivalent of copper is 0.00033 gram/ampere-second, find the value of the current, and show how to determine the current for any other reading of the galvanometer.

17. A current of electricity driven by an electromotive force of 10 volts traverses a water voltameter in which there is a resistance of 2 ohms and a back electromotive force of 1.5 volts. Calculate the weight of hydrogen, and the number of calories developed in the voltameter per hour, assuming the resistance of other parts to be negligible. [The electro-chemical equivalent of hydrogen is 0.000010384 gram per coulomb, and the mechanical equivalent of one calorie is 42,000,000 $\text{cm.}^2\,\text{gm.}^1\,\text{sec.}^{-2}$.]

18. An E.M.F. of 3 volts is required to force a current of 1 ampere through a voltameter containing acidulated water. If the work required to separate one gram of hydrogen is 142,000 watt-seconds, and the electro-chemical equivalent of hydrogen is 0.00001035, find the resistance of the voltameter.

CHAPTER XXV

THEORY OF THE VOLTAIC CELL. POLARISATION. ACCUMULATORS

Osmotic Pressure.—If a layer of pure water is poured carefully on to the top of an aqueous solution of a soluble solid, *e.g.* copper sulphate $CuSO_4$, contained in a glass cylinder, the molecules of the sulphate will *diffuse* gradually into the pure water; this process continues until the concentration of the solution is uniform. The same transfer occurs even when the water and the solution are separated by a porous membrane which allows the transference through its substance of molecules both of water and of the sulphate. *If a membrane is used which is permeable to pure water alone*, and if it completely surrounds the solution of sulphate, then pure water will diffuse inwards more rapidly than water permeates outwards; if the partition is mechanically weak, this process may continue until the internal pressure is sufficiently great to break it. The phenomenon is termed *osmosis.* If a manometer is connected to the vessel the pressure gradually increases to a maximum value when the pressure due to the manometer just balances the excess of pressure from the outside of the vessel. The pressure measured by the manometer would be termed the *osmotic pressure* of copper sulphate.

This may be explained more clearly in the following manner: The rate of diffusion of the molecules of any given substance depends directly upon the pressure exerted by that substance upon the permeable membrane. The total pressures on each side of the partition are initially equal; that on the outside is due *entirely* to water molecules, but that on the inside is due partly to water molecules and partly to molecules of the sulphate; the difference in pressure exerted by the *water* on opposite sides of the membrane will be equal therefore to the pressure of the sulphate molecules. Hence, water will diffuse inwards more rapidly than it will diffuse outwards; and the process will continue until the hydrostatic pressure of the manometer balances the difference of pressure of the water molecules on opposite sides of the partition. The manometer thus indicates the magnitude of the partial pressure due to the copper sulphate molecules.

Observations on osmotic pressure suggest that the molecules of a dissolved solid have properties resembling those of a gas.

Electrolytic Solution Pressure. Electrode Potentials.—When a solid is in contact with a *saturated* aqueous solution of the same solid, its mass remains unaltered. We may regard this condition of equilibrium as the balancing of two opposite effects, viz. the osmotic pressure of the *dissolved* solid and the tendency of the molecules of the solid to enter the solution; the latter may be termed the **solution pressure**, or the tendency to pass into solution, of the solid.

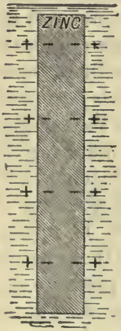

FIG. 324.

If a zinc plate is immersed in a solution of zinc sulphate *no apparent loss or gain in the weight of the zinc is detected, whatever the concentration of the solution*, but a potential-difference is set up between the metal and the solution.[1] We may suppose that, in the first instance, the solution-pressure of the zinc is greater than the osmotic pressure of the zinc ions in the solution, and that some zinc ions, which are of course positively charged, proceed from the plate into the solution, so that the plate and the solution become charged negatively and positively respectively. There is thus formed an **electrical double layer** (Fig. 324) at the surface of separation between the metal and the electrolyte; and the intensity of this electrical field will increase until it is sufficient to prevent the transference of any more positively charged zinc ions from the metal. The solution-pressure of the metal is balanced therefore by the combined effects of the osmotic pressure of the metal-ions in solution and of the electrical double layer. The whole process is restricted to a layer of liquid of molecular thickness, and, as stated above, it is quite impossible to detect the small amount of chemical change involved. Nernst has applied the term *electrolytic solution pressure* to the special cases of metals immersed in an electrolyte.

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[1] It has been calculated that the P.D. observed in the case of zinc and zinc sulphate would be obtained by the oxidation of 5.5×10^{-8} gram of zinc per sq. cm.

In the previous paragraph we have only considered a case in which the solution-pressure is greater than the osmotic pressure of the ions of the metal in the solution. If the reverse is the case, and the osmotic pressure is greater than the solution-pressure of the solid metal, ions of the metal will be deposited on the solid metal, and the plate and solution will be charged positively and negatively respectively. Thus, an electric double layer, *opposite in sign* to the previous case, is set up and will prevent the further deposition of metal. This condition is obtained when a copper plate is immersed in a solution of copper sulphate. In special cases where the solution-pressure and the osmotic pressure are equal, no electrical double layer is formed.

The formation of an electrical double layer constitutes a *difference of potential* between the metal and the electrolyte in which it is immersed, and a P.D. of this nature is termed an **electrode potential.** Moreover, **the electrode potentials between metals and solutions form the basis of the electro-motive-force of all voltaic cells.**

Measurement of Electrode Potentials.—In order to measure an electrode potential it is necessary to add another metal contact, and sometimes another electrolyte also, in order to connect the combination to the electrometer or other equipment for measuring the P.D. The P.D. measured is therefore the resultant of *three* contacts ; but that between the two electrolytes, especially if they have approximately equal ionic velocities or have equi-molecular concentrations, may be disregarded. Also, the P.D. between various metals may, for reasons explained on p. 485, be disregarded. Hence it is only necessary to use a *standard* electrode, of constant and known

P.D., or an electrode of negligible P.D. The latter is obtained by means of mercury which is allowed to flow from a narrow jet in a stream of rapid drops into an electrolyte: at the surface of contact the mercury will have positive potential as compared with the electrolyte, and as each drop forms it will take positive electricity from the mercury in order to form the electrical double layer; this, if the stream of drops is sufficiently rapid, will maintain the mercury at the same potential as that of the electrolyte.

As an alternative, the combination *metal | electrolyte | mercury* could be used, since the P.D. due to the *electrolyte | mercury* contact could be measured by means of Lippmann's Capillary Electrometer (described in the next section). The following table gives the P.D. obtained at the surfaces of contact of different metals in *normal* solutions, *the potential of the liquid being zero.*

Metal.	Sulphuric Acid.	Hydrochloric Acid.
Zinc . .	−0.62	−0.54
Copper . .	+0.46	+0.35
Silver . .	+0.73	+0.57
Mercury .	+0.86	+0.57

Lippmann's Capillary Electrometer.—Fig. 325 represents a convenient and simple form of Capillary Electrometer. It consists of two glass vessels *d* and *b* connected by a capillary tube *c*. Mercury is poured into *d* and *b*, and *b* is then nearly filled with dilute sulphuric acid. Contact is made with the masses of mercury by means of platinum wires; the one leading into *b* is protected from contact with the acid by being fused into a glass tube. The angle of slope of *c* is adjusted by attaching it to a wooden strip, as shown. A millimetre scale is fixed below the capillary tube. The position of the mercury meniscus in *c* depends upon the surface tension of mercury in contact with sulphuric acid; and the surface tension is balanced by the hydrostatic pressure of the mercury in *d*. If the surface tension is increased the meniscus moves to the left, and if diminished it moves to the right.

According to Helmholtz, the P.D. at the surface of contact of mercury and acid must cause an electrical double-layer distributed over the contact surfaces, the mercury being positive and the acid negative. Each surface, owing to the self-repulsion of its charge, tends to expand and therefore to neutralise the true surface tension. If, by external means, the P.D. between the mercury and acid is *increased*, the surface tension will be reduced still further and *the meniscus will move to the right.*

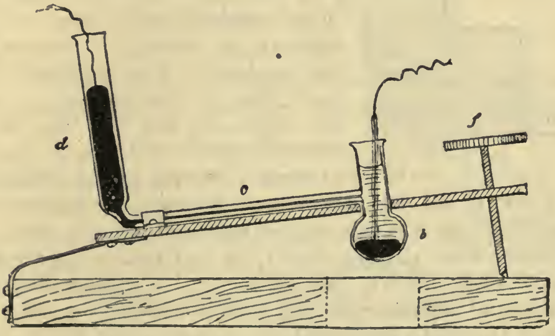

FIG. 325.—Lippmann's Capillary Electrometer.

On the other hand, if the P.D. is *diminished* the surface tension will increase and *the meniscus will move to the left*; a point is reached finally when the double layer is neutralised exactly, and the surface tension will have its maximum value: with a further diminution of P.D. a *reversed* double layer is formed and the surface tension again diminishes. The P.D. between mercury and any electrolyte may be measured therefore by connecting the mercury to the negative terminal and the acid to the positive terminal of a source of known E.M.F., and observing the magnitude of the E.M.F. required to give a *maximum surface tension* to the mercury.

This type of electrometer has the special merit of being insensible to electric and magnetic disturbances. It is usually used as a *zero* instrument, *i.e.* for verifying the equality of opposing E.M.F.'s, as in the potentiometer method of comparing the E.M.F. of cells. On the other hand, by previously calibrating the instrument it may be used for determining the actual magnitude of an E.M.F.

The Electro-motive Force of Cells.—The *Electro-motive force* of a cell is equal to the algebraic sum of the differences of potential occurring at all the surfaces of contact of the materials constituting the cell. Two examples of this will suffice to explain the principle. Thus, in the simple voltaic cell (Fig. 326), consisting of zinc and copper plates immersed in dilute sulphuric acid, there are two surfaces of contact, viz., copper/acid and acid/zinc. The potential difference due to the former is equal to +0.46 volt, and that due to the latter is equal to +0.62 volt; the sum of these, 1.08, approximates closely to the E.M.F. of the cell as observed in practice. Again, in the Daniell cell the E.M.F. is due to four surfaces of contact, viz., $Cu/CuSO_4$, $CuSO_4/ZnSO_4$, $ZnSO_4/Zn$, and Zn/Cu. Of these the contacts $CuSO_4/ZnSO_4$ and Zn/Cu may be disregarded, since the potential difference in each case is extremely small. The contacts $Cu/CuSO_4$ and $ZnSO_4/Zn$ are both equal to +0.52 volt approximately, and the resultant E.M.F., 1.04 volt, agrees fairly well with the observed value.

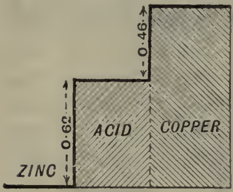

FIG. 326.

The Source of the Energy of the Current from a Cell.—It is now necessary to consider the source of the energy of the current derived from voltaic cells. It may be stated generally that the energy is derived from the chemical changes which occur inside the cell, although there are certain exceptional cases in which this general statement requires modification. The question is whether, in any typical case, the energy represented by the heat generated by a chemical change in which no electricity is developed is equal to the energy of the current obtained when the whole of the chemical action is used in generating electricity. In the Daniell cell the chemical changes are (i) the action of zinc on sulphuric acid, with liberation of hydrogen, (ii) the action of hydrogen on copper sulphate, with liberation of copper. In the following equations the symbol Aq attached to a formula indicates that the compound is in the form of an aqueous solution; this is important since the solution of substances is nearly always accompanied by thermal changes. The following thermal data[1], which have been obtained experimentally, assume that 65 grams of zinc are dissolved in the sulphuric acid.

(i) $Zn + H_2SO_4Aq = ZnSO_4Aq + H_2$.

Formation of $ZnSO_4Aq = 248{,}500$ calories generated.
Decomposition of $H_2SO_4Aq = \underline{210{,}900}$ calories absorbed.
Heat generated = 37,600 calories.

(ii) $H_2 + CuSO_4Aq = H_2SO_4Aq + Cu$.

Formation of $H_2SO_4Aq = 210{,}900$ calories generated.
Decomposition of $CuSO_4Aq = \underline{198{,}400}$ calories absorbed.
Heat generated = 12,500 calories.

The total heat generated by the solution of 65 grams of zinc is therefore $37{,}600 + 12{,}500 = 50{,}100$ calories.

∴ heat generated by the solution of one electro-chemical equivalent of zinc ($32.44 \times 1.044 \times 10^{-5}$ gram) is

$$\frac{50{,}100 \times 32.44 \times 1.044 \times 10^{-5}}{65} = 0.261$$

or, since 1 calorie is equivalent to (4.2×10^7) ergs,

$$= (.261 \times 4.2 \times 10^7) = 1.09 \times 10^7 \text{ ergs.}$$

But the solution of this quantity of zinc is accompanied by the transference of 1 coulomb, or 0.1 C.G.S. unit, of electricity; and the electrical work done $= (E \times q)$ C.G.S. units. Hence,

$$E \times 10^{-1} = 1.09 \times 10^7$$

$$\therefore \; E = 1.09 \times 10^8 \text{ C.G.S. units} = 1.09 \text{ volt.}$$

Polarisation.—When an electric current is passed between two platinum plates immersed in dilute sulphuric acid the current rapidly diminishes, and if the E.M.F. of the battery is below a certain value the current may cease completely. If the circuit is now broken and the plates quickly short-circuited through a galvanometer, a current is observed to pass through the cell in an *opposite* direction to that due to the battery. The E.M.F. between the platinum plates, immediately after breaking the circuit, is equal to 1.47 volts; consequently it is impossible to decompose water with a voltaic cell of smaller E.M.F. than 1.47 volts. The platinum plates are said to be **polarised**, and the effect is due to the oxygen and hydrogen accumulated on and absorbed by the plates. We may assume that, after disconnecting the battery, positively charged hydrogen ions proceed from the kathode, leaving it negatively charged; also that negatively charged oxygen ions proceed from the anode, leaving it positively charged. Hence the current will pass through the outer circuit from the anode to the kathode. Fig. 327 represents a method of observing the reverse current due to polarisation; in the first instance the movable arm S of the switch is in the lower position; the battery is then disconnected and the cell C joined up to the galvanometer by moving S into the upper position.

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[1] Ostwald's *Principles of General Chemistry* (Macmillan).

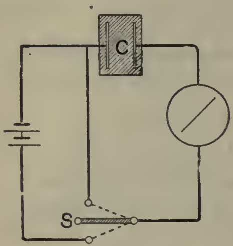

FIG. 327.

The measurement of the E.M.F. of polarisation is difficult, since it rapidly diminishes after the circuit is broken; in fact, the form of the discharge curve closely resembles that obtained by the discharge of a Leyden jar through a high resistance (Fig. 358, p. 541). For this reason the measurement can be made only with the aid of a rapidly moving commutator which alternately connects the cell to the battery and to a quadrant electrometer; the cycle of operations should be performed at the rate of at least 100 per second to ensure trustworthy results.

It has been stated previously (p. 467) that a difference of concentration is set up in a solution of copper sulphate when electrolysed between copper terminals, and that this is due to a difference in the velocities of the Cu and the SO_4 ions. This difference in concentration gives rise to a small reverse E.M.F. due to two solutions of different concentration. Such a polarisation effect would be absent only if the ionic velocities were the same. The same remarks would apply to the electrolysis of a solution of zinc sulphate between zinc terminals.

Secondary Cells, or Accumulators.—When dilute sulphuric acid is electrolysed between lead plates the anode becomes coated with lead peroxide, PbO_2, while the kathode remains unaltered. On breaking the circuit, and connecting the terminals by a wire, a *polarisation current* is obtained which passes through the cell in the reverse direction to that of the charging current. An arrangement of this kind is termed a **Secondary Cell.** The original type of this cell, devised by M. Planté, consisted of two sheets of lead rolled up together and separated by felt or similar material. Frequent charging and discharging results in the formation of porous or spongy lead on the surface of the plates, thus increasing the amount of available surface. With a view to accelerate this process of *forming* the plate, M. Faure coated the lead plates with a paste of red lead, Pb_3O_4, and sulphuric acid, which react to form lead sulphate; on charging the cell the anode is oxidised to PbO_2 and the kathode is reduced to spongy lead. It was found difficult to make the paste adhere and the chief modern improvements have been introduced to ensure a more permanent attachment of the paste. Fig. 328 represents the form of lead *grid*, now used in a leading type of modern accumulator, the paste being firmly pressed into the spaces. The paste now generally used for the negative plates is a mixture of litharge PbO and sulphuric acid, and a mixture of red lead and acid is used for the positive plates; in both cases lead sulphate is formed.

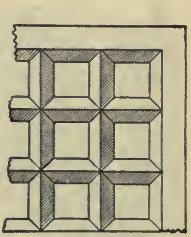

FIG. 328.

The reactions which take place during the *forming* of the plates is as follows:

At anode: $\mathbf{PbSO_4} + SO_4 + 2H_2O = \mathbf{PbO_2} + 2H_2SO_4$.
At kathode: $\mathbf{PbSO_4} + H_2 = \mathbf{Pb} + H_2SO_4$.

During the discharge of the cell, the surface of the anode is reduced to PbO and finally to $PbSO_4$; the kathode is at the same time converted into $PbSO_4$. These changes may be represented in a symbolic manner thus :

$$\mathbf{PbO_2} \mid 2H_2SO_4 \mid \mathbf{Pb} \text{ changes to } \mathbf{PbO}, \mid H_2O,\ H_2SO_4, \mid \mathbf{PbSO_4},$$

and finally to $$\mathbf{PbSO_4} \mid 2H_2O \mid \mathbf{PbSO_4}.$$

The voltage of an accumulator is about 2.2 volts when fully charged, and it is detrimental to the life of the cell to allow the voltage to fall below 1.8 volt. When the cell is charged fully the acid should have a density of about 1.20, and this gradually falls during discharge to about 1.17.

The **capacity** of a cell or battery of cells may be expressed in *ampere-hours*, which is equal to the product of the total discharge current which may be taken safely from a fully charged cell and its duration in hours. Or, we may refer to the *energy*-capacity of the battery in terms of *kilowatt-hours*, which involves both the voltage and the current. The capacity of a cell depends, of course, upon the number and size of the plates. Cells are made usually with several positive and negative plates arranged alternately and close together, the two outer plates always being negative. Fig. 329 represents a typical modern storage cell.

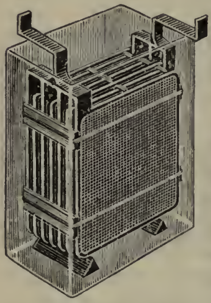

Fig. 329.

The **efficiency** of a cell is the ratio of the discharging capacity to the charging capacity. The maximum available energy is about 80% of that spent in charging, and this is obtained only providing that the discharge takes place slowly and soon after the charging.

Edison's Accumulator.—A type of accumulator has been introduced recently by Mr. Edison in which the positive and negative plates consist of finely divided iron and of nickel peroxide respectively. The grids or frames supporting these consist of shallow pockets of thin perforated steel. The plates are immersed in a 20% solution of caustic potash contained in a nickel-plated steel box. The *weight-efficiency* of the cell, or the ratio between the discharging capacity and the weight, is stated to be about 13 or 14 watt-hours per pound-weight of cell, which is somewhat greater than the weight-efficiency of the lead-plate accumulator.

The Contact Theory of E.M.F.—Fig. 330 represents an experiment carried out by Lord Kelvin many years ago. Two semicircular plates of copper and zinc are placed horizontally with their straight edges parallel and nearly touching, and a light metal vane is suspended at the point A and symmetrically over the plates. If the vane is charged positively, and the plates connected together by a wire the vane moves towards the copper plate. This effect was attributed to a *true contact potential-difference* between the copper and the zinc, and subsequent experiments proved that the P.D. was equal to about 0.8 volt. The same effect may be observed by means of a condensing electroscope arranged as in Fig. 331. A and B are two discs of brass (about 20 cms. diameter) insulated by rods of vulcanite and separated by a

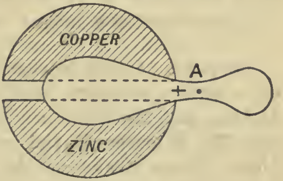

FIG. 330.

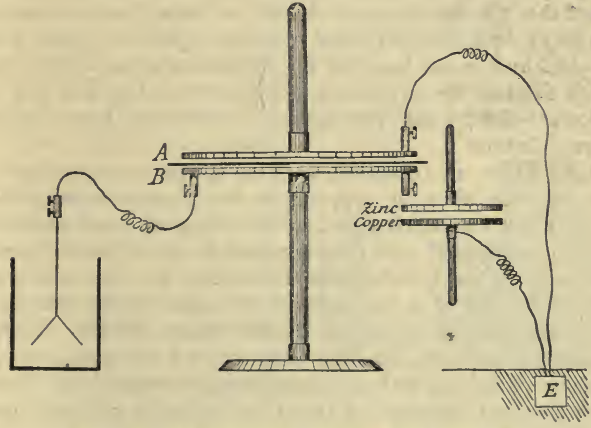

FIG. 331.—Demonstration of Contact Potential-difference.

sheet of paraffined paper. The plate B is connected to a gold-leaf electroscope, the disc of which has been removed and replaced by a binding-screw ; the plate A is earth connected. If a slight charge is given to B no divergence of the leaves is observed ; but if A is raised, the potential of B will be increased and the leaves will diverge.

EXPT. 77. Obtain two flat discs of zinc and copper (5 cms. diameter) each with a vulcanite handle. Connect the copper disc to earth. Holding each disc by its handle, bring them into contact, raise the zinc disc until it touches B, and bring it back into contact with the copper plate. Repeat this several times. Raise A, and observe the divergence of the leaves. Remove A completely and verify that B has a positive charge. Discharge the condenser and invert the copper and zinc discs. Repeat the experiment, and verify that B now has a negative charge.

By means of an experiment of this nature Volta, in 1801, found that a list of elements—known as *Volta's Series*—could be drawn up such that any metal acquires a positive charge when touched by a metal lower in the series, and a negative charge when touched by any metal preceding it. The series is as follows :—Zinc, Lead, Tin, Bismuth, Antimony, Iron, Copper, Silver, Carbon.

Lord Kelvin also observed, with the apparatus represented in Fig. 330, that when the copper and zinc plates are not quite in contact but connected together by a drop of dilute sulphuric acid the suspended vane does not tend to move from its normal position. It was concluded from this that the true seat of the E.M.F. of a cell is at the surfaces of contact of the metals—a principle which is termed the **Contact Theory of E.M.F.** Such a conclusion does not deny the presence of a P.D. at the contact surfaces of the acid with the copper and zinc respectively ; but if such potential differences do exist the numerical values of these must be the same in order to cause no movement of the vane. Evidence of a P.D. between zinc or copper and dilute sulphuric acid may be obtained with the aid of the condensing electroscope (Fig. 331) by substituting for the lower metal plate a piece of filter paper moistened with the acid and spread on a sheet of glass ; the zero potential of the acid may be ensured by touching it with the finger.

On the other hand, the **Chemical Theory** of E.M.F. suggests that in Kelvin's experiment allowance has to be made for the fact that the plates are in contact with air, the oxygen of which behaves as an electrolyte and establishes a P.D. between the plate and the air near to it. According to this theory, if we assume that the oxygen molecule consists of two atoms equally and oppositely charged and that some of the molecules are dissociated, some of the negatively charged atoms combine with the metal, and give up their charge to it, thus setting up an electric field at the surface of contact, the air being positively charged and the metal negatively charged. The effects would be the same both with zinc and with copper, but more pronounced in the former case. Moreover, if the two metals are in contact, their potentials would be practically the same, while the P.D. at the air/zinc contact would be greater than that at the air/copper contact ; hence the air near to the zinc is at a higher potential than that near to the copper, consequently an electric field is set up in the air, and a positively charged conductor will tend to move towards the copper.

According to the *contact theory*, as previously stated, there is no P.D. between a zinc and a copper plate immersed in acid because there is no metallic contact. But, according to the *chemical theory*, there is a P.D. of about 1 volt between the plates, but no apparent P.D. between the air in contact with the copper and that in contact with the zinc, because the air behaves towards the metals in the same way as the acid (Fig. 332).

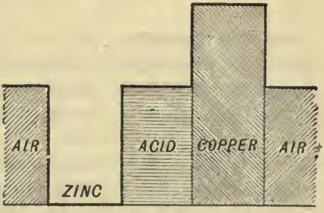

FIG. 332.

Supporters of the contact theory maintain that the energy represented by the P.D. set up at the surface of contact is derived from the heat in the metals, but it is difficult to reconcile the idea of a P.D. of about 1 volt with the experimental results deduced from observations on the Peltier Effect, which is a true contact P.D. The data on p. 366 indicate that the true contact P.D. between zinc and copper only amounts to about 0.58×10^{-4}

volt. On the other hand, if the energy is derived from the oxidation of the zinc, the quantity of zinc required to produce the observed effects is excessively small. Thus, the quantity Q of electricity required to charge the condenser used in Expt. 77 to a P.D. of 1 volt may be calculated, and the weight of zinc which must be oxidised to produce this quantity can be determined by means of the electro-chemical equivalent of zinc. If the radius of the condenser is 10 cms., and the distance apart of the plates is 1/100 cm., then the capacity C of the condenser is

$$\frac{A}{4\pi d}=2500.$$

$$\text{But } V=1 \text{ volt}=1/300 \text{ electrostatic unit};$$

$$\therefore\ Q=C\times V=2500/300=8.3 \text{ electrostatic units.}$$

The weight of zinc required to produce 9655 C.G.S. units, or $9655\times3\times10^{10}$ electrostatic units, is 32.7 grams. Hence the weight required to produce 8.3 electrostatic units is

$$\frac{32.7\times8.3}{9655\times3\times10^{10}}=9.37\times10^{-13} \text{ gram.}$$

SUMMARY

Osmotic Pressure.—If two different liquids are separated by a permeable diaphragm the quantities of the liquids passing in opposite directions through the membrane are unequal. This phenomenon is termed osmosis, and is attributed to the different *osmotic pressures* characteristic of the liquids. If an aqueous solution of a crystalline solid is contained in a semi-permeable vessel (*i.e.* one which is permeable to pure water only) which is immersed in pure water, more water diffuses inwards than outwards, and the increase of the internal pressure may be measured by a manometer connected to the interior. The hydrostatic pressure observed represents the *osmotic pressure* of the dissolved solid.

Electrode Potentials.—The ions of a zinc plate, immersed in a solution of zinc sulphate, exert a certain solution-pressure (or, tendency to pass into solution); on the other hand, the zinc ions in the solution exert osmotic pressure and tend to become deposited on the plate. Since the ions which enter the solution are charged positively, the plate and the solution become charged negatively and positively respectively;

and an *electrical double layer* is set up which tends to stop the further transference of ions. The solution pressure is then balanced by the osmotic pressure and the electrostatic field of the double layer. The potential difference thus established between a metal and an electrolyte is termed an *electrode potential.*

An electrode potential can only be measured by introducing another surface of contact between a metal and the same or another electrolyte; but this surface of contact may be chosen so as to have a P.D. which is either negligible or known with considerable accuracy. Either of these conditions is obtained by the use of mercury.

Lippmann's Capillary Electrometer consists of mercury in contact with dilute sulphuric acid, both liquids being contained in a capillary glass tube. If a potential difference is set up at the surface of contact by connecting the two liquids to an external source of E.M.F. an alteration in the surface tension results and the surface of contact moves along the tube to an extent depending upon the magnitude of the applied E.M.F.

The **Electromotive force** of a voltaic cell is equal to the sum of the electrode potentials at the several surfaces of contact which constitute the cell.

E.M.F. of a cell calculated from the chemical changes taking place within it.—In certain cases, the E.M.F. of a cell may be calculated by applying the principle that the heat generated by a chemical change in which no electricity is generated is equivalent to the energy of the current obtained when the whole of the chemical action is used in generating electricity.

Polarisation is the term applied to the E.M.F. observed between two plates of platinum, or other metal immersed in acid, immediately after the plates are disconnected from an external source of E.M.F. The maximum polarisation observed with platinum plates immersed in dilute sulphuric acid is 1.47 volt.

A **secondary cell** (or **accumulator**) usually consists of two lead plates (or a series of which alternate plates are connected together) immersed in dilute sulphuric acid.

The Contact Theory of E.M.F.—According to the *contact theory*, the energy represented by the P.D. set up at the surface of contact of two metals, *e.g.* zinc and copper, is derived from the heat in the metals. On the other hand, the *chemical theory* attributes the P.D. to the fact that the metals are in contact with air, the oxygen of which behaves as an electrolyte.

QUESTIONS ON CHAPTER XXV

1. Two clean zinc wires dip into a solution of sulphate of zinc and are respectively connected to the poles of a Daniell cell. What occurs? The liquid is then replaced by water acidulated with a little sulphuric acid. What occurs? The wires are then replaced by platinum wires. What occurs?

2. A battery of two Daniell's cells is used to send an electric current through an electrolytic cell, containing water acidulated with sulphuric acid, by means of two platinum plates A and B, the current entering by the former and leaving by the latter. After a time the battery is removed; and on connecting the electrolytic cell with a galvanometer it is found that a current is given out by the cell. Describe and explain what has happened in the cell and in which direction this current flows.

3. When currents are led by means of a pair of electrodes through an electrolyte, there frequently occurs a phenomenon called polarisation of electrodes. In what way does such polarisation depend on the size of the electrodes, on the strength of the current, and on the time that the current has been flowing?

4. A small accumulator has a capacity of 14 ampere-hours. What is theoretically the least weight of PbO_2 on its positive plates; given that the PbO_2 is reduced to PbO and that (*a*) the electro-chemical equivalent of hydrogen is 0.00001038 grams per coulomb; (*b*) the atomic weight of lead is 207, and that of oxygen 16?

Calculate how much heat is evolved during the whole discharge of this cell through a total resistance of 10 ohms, the average electromotive force being taken as 2 volts.

5. Give a general account of the chemical changes which occur in a storage cell during charge and discharge.

6. How do you explain the circumstance that in the electrolysis of a solution of sulphate of copper there is hardly any polarisation if a copper anode is used? How is it ascertained that the transfer of copper ions in one direction and of the corresponding SO_4 ions in the other are not accomplished at equal rates? Why would you expect some slight polarisation even with a copper anode?

7. How do you account for the fact that an E.M.F. of about 1½ volts is needed to electrolyse water at an appreciable rate? An accumulator, 2 volts E.M.F., maintains a current in a circuit of total resistance 2 ohms. An electrolytic cell with back E.M.F. 1.5 volts is then inserted, the resistance being adjusted again to 2 ohms. Compare the currents and the rate of working in the two cases.

8. Calculate the minimum E.M.F. necessary in order to decompose water, given the electro-chemical equivalent of hydrogen $= .0000105$ gm./ampere-second, the heat yield of 1 gm. of hydrogen in combining to form water $= 34500$ calories, $J = 42 \times 10^6$ ergs, 1 watt $= 10^7$ ergs/second.

9. Explain how it is possible to calculate approximately the E.M.F. of a cell if we know the chemical actions occurring in it, and their heat equivalents.

The electro-chemical equivalent of zinc is 0.00034 gram/ampere-second. Find the cost of the zinc used in a primary battery for each horse-power per hour, if zinc is p pence per kilogramme, and if the cell in which the zinc is used gives V volts [746 watts $= 1$ H.P.].

10. Describe generally the action which takes place in a lead storage cell during charge and discharge.

11. What do you mean by ionic velocities in electrolysis?

Explain how the ratio of the velocities of the ions of an electrolyte may be measured.

12. Show that the electromotive force of a voltaic cell may under certain conditions be calculated from thermo-chemical data.

CHAPTER XXVI

THE DISCHARGE OF ELECTRICITY THROUGH GASES

Phenomena of Discharge through a Rarefied Gas.—The discharge of electricity through a rarefied gas is observed by connecting metal electrodes, fixed inside the ends of a long glass tube connected to an air pump, to the terminals of an induction coil, the spark-gap of which is 3 or 4 inches long and forms an alternative path to the discharge. At first the whole of the discharge passes across the spark-gap ; but, if the air is rarefied partially, luminous streamers appear inside the tube, and the discharge across the spark-gap ceases. As the rarefaction increases the following phenomena are observed :

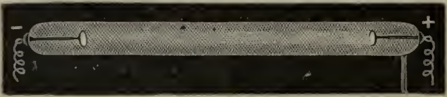

FIG. 333.

(i) The tube is filled with a crimson luminous column, which may extend along the entire length between the electrodes, or perhaps only from the anode nearly to the kathode—it is called the *positive column* (Fig. 333) ; also the spark-gap may be reduced to a fraction of an inch without any spark passing, which shows that the rarefied gas is now a good conductor.

(ii) The crimson colour disappears, and the column breaks up transversely into luminous discs or *striae* ; also a dark space, called *Faraday's dark space*, appears near to the kathode, which is itself covered with a luminous glow.

(iii) The striae thicken, the Faraday dark space becomes less distinct, and the luminous glow separates from the kathode, leaving a second dark space, known as *Crookes's dark space*, round the kathode. This condition of the tube is shown in Fig. 334, in which F and C are the Faraday and Crookes dark-spaces, and P is the positive column. The conductivity is now less than before, and the spark-gap must be lengthened.

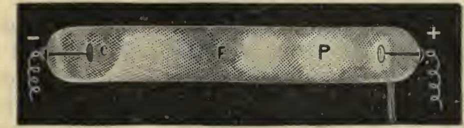

FIG. 334.

(iv) The glow disappears, the tube is filled with the Crookes dark space, and a phosphorescent light is distributed over the surface of the glass. The colour of the phosphorescence depends upon the composition of the glass; thus, it is bright green with soda glass and blue with lead glass. The most favourable pressure for this effect is one millionth part of one atmosphere.

(v) The resistance becomes extremely high, and the discharge can be made to traverse the tube only with difficulty.

It is interesting to note the effect of a transverse magnetic field on the luminous discharge through a comparatively low vacuum similar to the condition described under the heading (i) of a previous paragraph. The effect on the path of the discharge resembles that which would be observed if the electrodes were joined by a *flexible conductor* traversed by a current from the positive to the negative electrode. The direction of the deflection can be pre-determined by applying Fleming's *left-hand rule* (p. 262). It will be observed that the discharge is deflected only where it is traversing a strong magnetic field, and that it resumes its undeflected path on either side of the field. Fig. 335 represents the effect due to a horse-shoe electro-

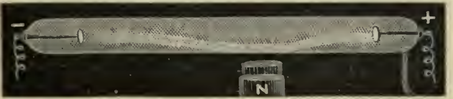

FIG. 335.

magnet placed immediately below the tube. In Fig. 336 the polarity of the magnet has been reversed. This phenomenon should be compared with the deflection of *kathode rays* (p. 494) in a magnetic field, in which case the deflection is permanent and can be observed by means of a comparatively weak field.

Kathode Rays.—When the degree of exhaustion within a vacuum tube is such that the glass walls are phosphorescent, the conditions are those which have been investigated fully by Crookes, Lenard, and others. Although the dark space is invisible, yet the effects obtained by introducing other bodies within the space proves that it has properties which are expressed by saying that it is a region of *kathode rays.* The phosphorescent light on the glass surface was observed first by Plücker, in 1859. Goldstein, in 1876, observed that a shadow of a solid object placed within the tube is thrown upon the glass surface; for this reason he regarded the discharge as *transverse ether waves* inside the tube.

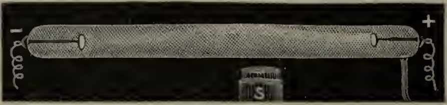

FIG. 336.

In 1879 Sir W. Crookes carried out a remarkable series of experiments on the electric discharge through high vacua, and revived the theory that "in these phenomena the particles constituting the kathode stream are not solid, liquid, or gaseous, but that they consist of *ultra-atomic corpuscles*, much smaller than the atom, which appear to be the foundation stones of which atoms are composed." In fact, Crookes concluded that this *radiant form of matter* was really a *fourth* state of matter, and quite distinct from the solid, liquid, and gaseous states.[1] Most of the properties of the kathode rays described in subsequent paragraphs were first announced by Crookes.

Shadows cast by Kathode Rays.—A mica cross fixed inside the tube transversely to the rays throws a shadow on the extreme end of the tube (Fig. 337); and even if the cross is fixed near to a kathode of considerable area the shadow is still well defined, which proves that *the rays are emitted along lines normal to the surface of the kathode*, and not in all directions, as is the case with the radiations of ordinary light. A further verification that radiant matter travels in straight lines is obtained by means of a **V**-shaped tube (Fig. 338); if the electrode at the apex of the **V** is made the kathode, then whichever of the remaining electrodes is used as an anode, the phosphorescent light is apparent only in the right-hand side of the tube. In fact, the discharge of

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[1] A full description of the experiments by which Crookes was led to this conclusion is given in the *Journal Inst. Elect. Engin.*, vol. xx., 1891, and in the *Electrician*, vol. xxvi., 1891.

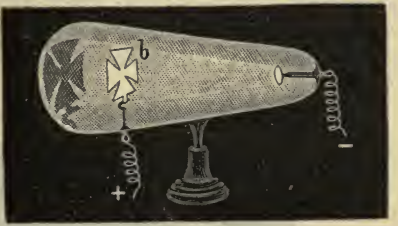

FIG. 337.

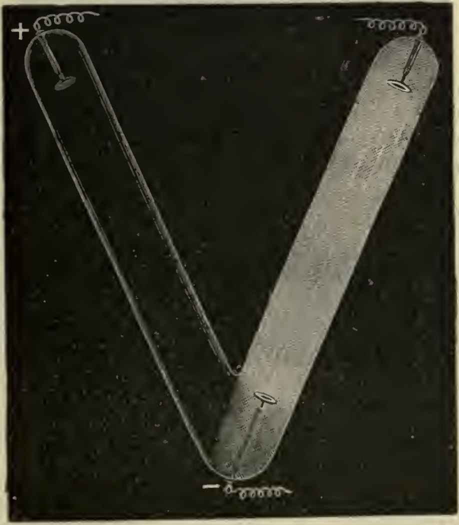

FIG. 338.

radiant matter from the kathode is analogous to a stream of bullets discharged from a large number of parallel guns, and phosphorescence is evident wherever the discharge strikes—on the glass if the vacuum is high enough, or on the residual air, and causing the luminous boundary to the dark space, if the vacuum is lower.

Heat Effects of Kathode Rays.—*The kathode rays heat bodies upon which they fall.* If the kathode is spherical, the rays may be concentrated so that a piece of platinum placed at the centre of curvature is heated to redness.

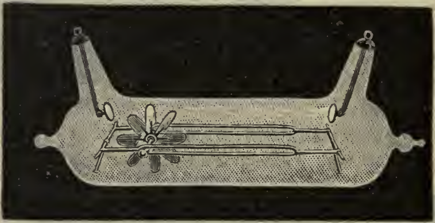

FIG. 339.

Mechanical Effects.—If kathode rays are allowed to impinge upon the mica vanes of a rotating wheel, the wheel is set in rotation if only one half of the wheel is situated within the path of the rays. In the apparatus represented in Fig. 339 the wheel will travel in one direction or the other according to which of the electrodes is used as a kathode.

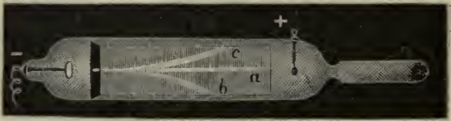

FIG. 340.

Action of a Magnetic Field on Kathode Rays.—Fig. 340 represents the form of tube with which Crookes proved that *the kathode rays are deflected by a magnetic field.* A mica screen with transverse slit is fixed near to the kathode, and the path of the rays is traced by the phosphorescence they excite on a longitudinal mica screen coated with chalk or other phosphorescent material. If a horse-shoe electro-magnet is placed with its poles below the tube so that the lines of force are at right angles to the path of the rays, the direction in which the rays are deflected agrees with the view that they mark the path of *negatively charged particles.* The particles would be acted upon by a force at right angles both to their direction of motion and to the direction of the magnetic field (p. 263); also the force would be proportional to the strength of the field and to the velocity of the particles. The discharge traverses the path *a* (Fig. 340) in the absence of a magnetic field; it is deflected along the path *b* by a transverse magnetic field directed from front to back, and along the path *c* when the field is reversed.

In the apparatus (Fig. 341) adopted by Sir J. J. Thomson to investigate this action of a magnetic field, a side tube is fixed to a glass bell-jar supported on a glass plate. The anode A consists of a solid well-fitting brass cylinder in which a horizontal slit is cut. C and A are joined to the terminals of an induction coil, and A is also earth-connected. The kathode rays pass through the slit and in front of a vertical glass plate, the surface of which is ruled into squares. The bell-jar is placed between two coils of wire fixed at a distance apart equal to the radius of either coil; by this means a uniform magnetic field is obtained (p. 276). A photograph of the phosphorescence in the residual gas can be obtained by prolonged exposure. With no magnetic field, a small phosphorescent patch is produced on the opposite side of the jar; but in the presence of a magnetic field the pencil of rays spreads out into a fan-shaped system which produces phosphorescence over two or three inches of the glass. It would seem therefore that the rays consist of a mixture of different rays which are not all deflected to the same extent. It was found, if the experiment is

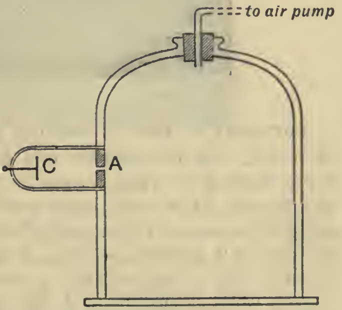

FIG. 341.

repeated with other gases, that the amount of deflection is the same as with air, *if the pressure is adjusted so that the P.D. between the electrodes is constant.*

The path of the rays is approximately circular. But if the rarefaction is diminished the curvature increases as the rays proceed further from the kathode. This would be expected, since a lower vacuum would increase the number of molecular collisions, and consequently would diminish the velocity of the particles. This effect is observed readily by means of a tube to which a potash-bulb is attached (Fig. 342): on heating the bulb the vacuum is diminished and the deflection of the discharge is increased, as indicated by the discharges *a* and *b*.

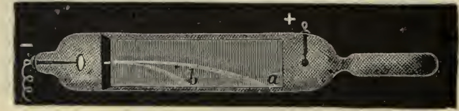

Fig. 342.

Curvature of Path of Kathode Rays due to a Magnetic Field.—Assuming that the kathode rays consist of electrically charged particles, then, if the charge on each particle is e electro-magnetic units, and if v is its velocity, each particle is equivalent to a current of strength ve units. If the path of the ray is at right angles to the lines of force of a uniform magnetic field of intensity H, a constant force equal to $ve\mathrm{H}$ (p. 263) will act on each particle, and the direction of the force is perpendicular both to that of the field and to that of the ray. Each particle therefore will trace out a circular path, and the radius of curvature of the path is given by the equation $f=mv^2/r$ (p. 24), where f is equal to $ve\mathrm{H}$. Hence,

$$r=\frac{mv}{e\mathrm{H}}.$$

Subsequent paragraphs (p. 499) will explain how the ratio m/e, which may be termed the *electro-chemical equivalent* of the charged particle, may be determined by measuring r, H, and v.

Kathode Rays impart Conductivity to Air.—The passage of kathode rays through a gas modifies it in such a manner that it becomes a conductor of electricity. This was proved by Hertz in an experiment which may be made with the apparatus of Fig. 341 with the addition of two parallel metal plates fixed to the vertical glass plate so that the rays pass between them after leaving the anode. If the metal plates are connected to the terminals of a battery, a transverse electric field is set up between the plates, and it would be anticipated that the negatively charged particles would be deflected upwards or downwards—but no deflection is observed, and this indicates that the space traversed by the rays is a conductor of electricity, which would have the effect of screening the particles from the action of the electric field. It was also observed that the P.D. which can be maintained between the plates is far less when the rays are passing than when the rays are absent.

Apparent Mutual Repulsion of Kathode Rays.—In the tube shown in Fig. 343, *a* and *b* are both kathodes, and *c* is the anode. The mica screen is provided with a horizontal slit opposite to each kathode. If the discharge is passed between *c* and *a* only, the path of the discharge is *df*; if between *c* and *b* only, the path is *ef*. If *a* and *b* are joined together, the two paths of discharge are transferred to *dg* and *eh* respectively. Crookes attributed this to the mutual repulsion of the charged particles in the rays; but it has been observed since that *dg* is still the path of the ray through *d* even when the slit at *e* was closed by a shutter so that no particles travelled along *eh*, and it is evident that the repulsion of the ray is due to the charge on the electrode *b*. Moreover, since the gas inside the tube is in a conducting state, an electric charge can have no electrostatic action on the ray itself, which is evidence in favour of the view that the gas *near to the kathode* is in an insulating condition and therefore different from the condition of the gas at a greater distance.

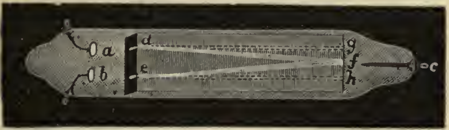

FIG. 343.

Negative Charge Conveyed by the Kathode Rays.—Fig. 344 represents Thomson's modification of an experiment, originally made by Perrin, which proves that a *negative* charge is conveyed by the kathode rays. The rays from the kathode C pass through a slit in the solid, brass, earth-connected anode A. B and D are two short, concentric cylinders insulated from each other, and each cylinder is perforated with a narrow transverse slit: B is earth-connected, and D is joined to an electrometer. Scarcely any charge is acquired by D when the rays are undeflected, but if the rays are deflected downwards by means of a magnetic field to such an extent that the rays strike upon the slit in B, the cylinder D acquires a considerable negative charge. If the intensity of the field is increased so that the rays are deflected below the slit in B, then D no longer acquires a charge. Also, the charge imparted to D soon acquires a maximum value: this would be expected, since the rays render the gas conducting, and the condition is reached soon when any increase in the charge is balanced by the loss due to the conductivity of the surrounding gas.

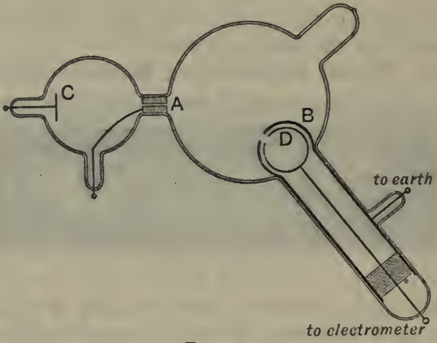

FIG. 344.

Lenard Rays.—Lenard has proved that, under suitable conditions, the kathode rays are capable of transmission through the walls of the tube to the outside; thus, if the end of the tube against which the rays strike consists of a strong metal cap perforated with a small hole covered with thin aluminium foil, the rays can be detected for a short distance outside the tube. Such rays, observed outside the tube, are termed **Lenard rays**; but the same rays, *within the tube*, are kathode rays.

If the negatively charged particles are of molecular dimensions, it is difficult to realise how they can penetrate through metal and also through a considerable layer of air without losing nearly all their momentum by reason of the frequent collisions which particles of such size would experience. On this point strong evidence is available in support of the view that the carriers are small compared with ordinary atoms or molecules.

Electrons.—More recent experiments have supported the assumption that elementary atoms are aggregations of more or less small particles, all similar to each other, into which the gaseous atoms within a vacuum tube are split up at the kathode, and that the kathode rays consist of such particles, negatively charged and moving with high velocity. These small particles are now identified with **electrons**, which are "atoms of electricity" and independent of gross matter: also, they are identical with J. J. Thomson's "corpuscles" and Sir O. Lodge's "disembodied ionic charges."

Dr. Johnstone Stoney in 1871 introduced the term electron to denote the most natural unit of electricity, viz.: the charge which is associated always with any monad ion. According to Sir Oliver Lodge's *Electronic Theory of Matter*, the monad ion is charged negatively or positively according as it is associated with one too many or one too few electrons, and a dyad ion will contain two too many or too few electrons.

Determination of the Electro-Chemical Equivalent and Speed of Kathode Rays.—It has previously been shown how the product $(m/e)v$ can be determined by measuring the radius of curvature of the path of the rays and the intensity of the deflecting field (p. 496). Sir J. J. Thomson, in 1897, carried out an experiment to determine the separate values of m/e and of v: he arranged that a narrow pencil of kathode rays, deflected by a magnetic field, should pass through a slit into a metal cylinder connected to a quadrant electrometer of known capacity, so as to measure the total charge on the particles collected in a given time. Also, a copper-iron thermo-junction, of known heat capacity, was fixed behind the slit, and could be joined by thin wires to a galvanometer; in this manner the total energy could be measured.

If N particles enter the cylinder in unit time, and if W is the mechanical equivalent of the heat given up, and Q is the charge imparted to the cylinder in unit time, then

$$\frac{1}{2}Nmv^2=W, \quad\text{..............................(1)}$$

$$Ne=Q. \quad\text{..............................(2)}$$

Also (p. 496) $$\left(\frac{m}{e}\right)v=Hr. \quad\text{..............................(3)}$$

Dividing equation (1) by equation (2), we obtain

$$\frac{1}{2}\cdot\frac{m}{e}\cdot v^2=\frac{W}{Q}.$$

Hence, $$\frac{m}{e}=\frac{1}{2}\frac{QH^2r^2}{W},$$

and $$v=\frac{2W}{QHr}.$$

The method adopted in this brilliant experiment is not so accurate as other more recent methods, since some of the charge imparted to the cylinder is lost owing to the conductivity acquired by the gas.

The value obtained for the ratio m/e was of the order 10^{-7} C.G.S. units. In ordinary electrolysis the value of the ratio m/e, *i.e.* the electro-chemical equivalent for hydrogen is about 10^{-4}. Hence, it was concluded that either the mass of each particle must be much smaller than that of the hydrogen atom, or the charge carried by each particle must be much greater than that carried by the hydrogen ion. In the same experiment the value of v was found to be 1.5×10^9 cms. per sec., or approximately one-tenth that of light.

Moreover, it was found that the results of the experiment were independent of the kind of residual gas and of the metal of which the electrodes consisted. The velocity of the particles depends upon the P.D. between the electrodes, but is independent of their distance apart: this would be expected if we consider that the velocity is due to the potential gradient between the electrodes, and that the actual velocity acquired is proportional to the product of the potential gradient and the distance between the electrodes.

Verification of the Value of m/e and of v.—In a simpler method, subsequently adopted by Sir J. J. Thomson, a narrow pencil of kathode rays was made to pass between two horizontal metal plates (P_1, P_2, Fig. 345) supported within the tube and maintained at a constant P.D. by connecting them to the terminals of a small battery, thus creating a vertical electrostatic field. At the same time a transverse magnetic field was maintained by means of external coils of wire conveying a current. The direction and magnitude of these fields can be adjusted so that the tendency of one to deflect the rays downwards is neutralised by the tendency of the other to deflect the rays upwards. The electrostatic force on each particle is Fe, where F is the intensity of the field; also, the magnetic force on each particle is Hev, where H is the intensity of the magnetic field. Hence, when the fields are adjusted so as to neutralise each other

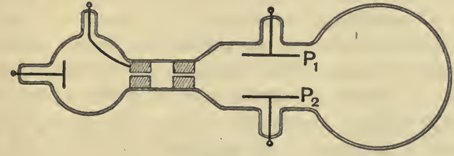

FIG. 345.

$$Fe = Hev$$

or

$$v = F/H.$$

The value of $(m/e)v$ is determined with the same tube by observing the deflection due to the magnetic field only. Hence the separate values of m/e and of v are found. The mean value found for m/e, when either air, hydrogen or carbon dioxide was used, was 1.3×10^{-7} (in E.M. units); and the value of v varied from 2×10^9 to 3×10^9 cms. per second.

More recent determinations of m/e, obtained by several available methods, give a mean value of 0.564×10^{-7} (in E.M. units). Also, the value of v has been found to vary up to a maximum of 6×10^9 cms. per second.

The Mass of an Electron.—The separate determination of m and e constitutes one of the most brilliant pieces of research work of modern times; it was conducted by Sir J. J. Thomson by a method based upon Mr. C. T. R. Wilson's observation that charged gaseous ions, produced when air is exposed to X-rays (p. 509), which are identical with the charged particles of kathode rays, may act as nuclei for the condensation of aqueous vapour: in this sense the ions serve the same purpose as that fulfilled by ordinary dust in the formation of clouds.

The upper part of a spherical glass vessel containing some water is closed by an aluminium plate, and the air within the vessel is ionised by exposing the aluminium plate to X-rays. By allowing the air to expand suddenly to a known extent the air is cooled, and water vapour is condensed into a mist; the quantity of vapour so condensed can be calculated from the extent to which the air is cooled. The radius of each drop of water constituting the mist is calculated by observing the rate at which the mist falls downwards.[1] Knowing the size of each drop and the total quantity of condensed moisture, the *number* of the nuclei can be calculated. The value of e was then determined by measuring the value of the product ne.

The principle of the method of measuring ne is as follows: Suppose that there are within the gas n particles charged positively and n charged negatively; the quantity of electricity of each sign is ne. If the aluminium plate is connected to an electrometer of known capacity, and if the water surface is insulated and then charged suddenly to a negative potential, all the negative particles will be driven against the metal plate, to which they will impart their charge; this charge is equal to ne, and is measured by means of the electrometer. The value of e is calculated by combining this result with the previous measurement of n.

It was found that $e = 3.4 \times 10^{-10}$ E.S. units $= 1.13 \times 10^{-20}$ E.M. units. More recent determinations give $e = 4.7 \times 10^{-10}$ E.S. units $= 1.57 \times 10^{-20}$ E.M. units. Hence, if $m/e = 0.564 \times 10^{-7}$,

$$m = (0.564 \times 10^{-7}) \times (1.57 \times 10^{-20}) = 0.886 \times 10^{-27} \text{ gram.}$$

Assuming the number of atoms in 1 gram of hydrogen to be 6.16×10^{23}, the mass of one atom is 0.162×10^{-23} gm. Hence, the ratio of the mass of an electron to that of the hydrogen atom is

$$(0.886 \times 10^{-27})/(0.162 \times 10^{-23}) = 1/1830 \text{ approximately.}$$

The value of the charge e can be compared with that on the hydrogen ion in electrolysis in the following manner: The passage of one electromagnetic unit of electricity through acidulated water liberates 1.23 c.c. hydrogen (at 76 cms. and 15° C.). If there are N molecules in each c.c. the total number of atoms in 1.23 c.c. is $2.46 \times \mathrm{N}$. Also, if E is the ionic charge,

$$2.46 \times \mathrm{NE} = 3 \times 10^{10} \text{ electrostatic units,}$$

$$\text{or } \mathrm{E} = (3 \times 10^{10})/(2.46 \times \mathrm{N}).$$

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[1] Sir G. Stokes has shown that the velocity v with which a drop of water falls is given by the formula,

$$v = \frac{2}{9} \times \frac{g \times (\text{radius})^2}{\text{coefficient of viscosity}}.$$

In order that E may be equal to e, then

$$(4.7 \times 10^{-10}) = (3 \times 10^{10})/(2.46 \times \text{N}),$$
$$\text{or N} = 2.6 \times 10^{19}.$$

The close agreement of this value of N with that deduced from the kinetic theory of gases is strong evidence that the charge of an electron and the ionic charge of the hydrogen atom are the same.

The Value of *m/e* for Positive Ions.—If a perforated kathode is used, luminous rays can be observed at the back of the kathode and proceeding from the perforations. Wien has proved that these rays consist of *positively* charged particles, and has found (by the method described on p. 501) that the value of m/e is never less than 10^{-4}, and that v is equal approximately to 3.6×10^7 cms. per second. This value of m/e is of the same order as that found in the electrolysis of liquids: it would seem, therefore, that the carriers of the positive charge are of the same magnitude as atoms of ordinary matter.

Motion of a charged sphere. It has been stated previously that a charged sphere in motion has similar properties to those of an electric current, and it can be proved that if a unit tube of electrostatic force moves with a velocity v at a given point (Fig. 346), a magnetic force equal to $4\pi v \sin\theta$ is set up at the point, where θ is the inclination of the tube to the direction of motion. The direction of the magnetic force is perpendicular both to the tube of force and to the direction of motion. The density of the tubes of force at a point distant r from the centre of a sphere charged with e electromagnetic units is $e/4\pi r^2$; hence the intensity H of the magnetic field at that point due to this charged sphere moving with velocity v will be

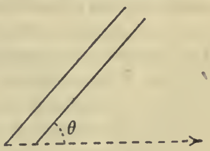

FIG. 346.

$$\frac{4\pi v \sin\theta \times e}{4\pi r^2} = \frac{ev \sin\theta}{r^2}.$$

But the energy per unit volume in a magnetic field of intensity H is $\text{H}^2/8\pi$ (p. 144); hence, in the above case, the energy per unit volume is $e^2v^2\sin^2\theta/8\pi r^4$. From this it can be proved that the total energy in the field outside the sphere and due to the charge is $e^2v^2/3a$, where a is the radius of the sphere. In addition to this, the kinetic energy of the sphere itself is $\frac{1}{2}mv^2$, where m is the mass of the sphere. Hence the total energy is

$$\frac{v^2}{2}\left(m + \frac{2e^2}{3a}\right);$$

or, the total energy is the same as if the mass had been increased by an amount $2e^2/3a$. The field has therefore a property equivalent to inertia.

If we assume that an electron is simply an "atom of electricity," and that its energy is entirely due to its charge, the size of an electron may be deduced from the fact that its "mass" is represented by the formula $2e^2/3a$. Hence,

$$a = \frac{2}{3} \cdot \frac{e}{m} \cdot e.$$

But $e/m = 10^7$ approximately, and $e = 10^{-20}$. Hence,

$$a = 10^{-13} \text{ cm.}$$

The volume of an electron would therefore be 10^{-39} c.c. approximately, and if there are 10^3 electrons in one hydrogen atom, the total volume occupied by these electrons would be 10^{-36} c.c. But, since the volume of the atom is about 10^{-23} c.c., the empty space within the atom is 10^{13} times as great as the filled space. Sir O. Lodge has suggested the analogy that "if the earth represented an electron, an atom would occupy a sphere with the sun as centre and five times the earth's distance as radius."

The distribution of charge on a sphere at rest is uniform; but Heaviside has shown that when the sphere is moving with great velocity the tubes of force become more or less concentrated in the plane through the centre of the sphere and at right angles to the direction of motion. In fact, if we attribute a real physical existence to the tubes of force it is easy to imagine that, when moving with high velocity, each tube will behave like a metal cylinder moving in a liquid; thus, a cylinder would not move end-on but would tend to set itself broadside-on, and thus carry far more of the liquid with it than if moving end-on. In the words of Sir J. J. Thomson, "the case is similar to that of falling leaves, which do not fall edge first, but flutter down with their planes more or less horizontal."

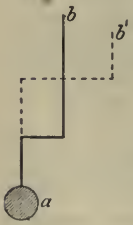

FIG. 347.

If a charged sphere, moving with high velocity, is brought suddenly to rest, then, since the electrostatic field has inertia, the tubes of force will tend to travel onwards. The outline traced by a given tube of force immediately after the sphere has been stopped will resemble *ab* (Fig. 347), and at a later instant its form will have changed to *ab'*. Thus the stoppage of the sphere gives rise to considerable modifications in the electric and magnetic forces in the field. The horizontal part of the tube of force travelling outwards will create in the space near to it a magnetic force in the opposite direction to that already existing; and it can be proved that this new force is greater than the previous force. These new forces constitute a pulse of energy travelling along the tubes with a velocity equal to that of light, and the amount of energy in the pulse depends upon the suddenness with which the sphere is brought to rest.

Applications of the Electron Theory.—Modern views are consonant with the opinion that all the phenomena of an electric current are due to electrons in motion.. Sir O. Lodge[1] describes the different modes of conduction in the following terms: "The possible modes of conduction are three, which may be called the bird-seed method, the bullet method, and the fire-bucket method respectively. The bird-seed method is adopted in liquids: it is exemplified in electrolysis; the bird carries the seed with it, and only drops it when it reaches an electrode. The bullet method is the method in rarefied gases: the space from kathode to anode represents the length between the breech and the muzzle of the gun, and the rest of the path is analogous to the trajectory of a bullet. . . . The fire-bucket method must be the method of conduction in solids, where the atoms are not susceptible of locomotion and can only pass electrons on from hand to hand, oscillating a little in one direction to receive them, and in another direction to deliver them up, and so getting thrown gradually into the state of vibration which we call heat."

The atom of matter may be regarded as a system of charged electrons; the negative charge of each electron is associated with an equal positive charge, and the equivalent charges are connected by lines of force. Although the lines of force near an electron are extremely crowded, owing to its small size, there is no reason why the positive charge may not be spread over a much wider area. If at rest, the lines of force would draw the opposite charges together; but there is reason for believing that the electrons within the atom are in a state of rapid movement—either vibrating to-and-fro or revolving in a minute orbit like a satellite. In either case the movement of the electron is undergoing acceleration continually (p. 24) and therefore giving rise to radiation (p. 504). Thus, if an electron describes an orbit 5×10^{14} times in one second, it will radiate yellow light corresponding to the D line of the spectrum. Since such moving electrons constitute an electric current, an external magnetic field of which the lines of force thread the path of the electron would cause an increase or diminution in the velocity of the electron, and consequently would raise or lower the refrangibility of the radiation. This effect was observed by Zeeman (of Amsterdam) in 1897, who observed a broadening of the characteristic D lines when a pure yellow flame is placed between the poles of a strong electro-magnet and examined spectroscopically: this is known as the **Zeeman effect.** More recently it has been found that some lines are separated into three distinct lines, others into four or even six lines. By measuring the changes in the lines it is possible to calculate the change in period of the electrons and therefore the amount of matter associated with the charge; or, in other words, to determine the ratio m/e. Zeeman found that the value derived from his observations agreed closely with that obtained from the phenomena of kathode rays.

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[1] *Jour. Inst. Elect. Engin.*, vol. xxxii. No. 159.

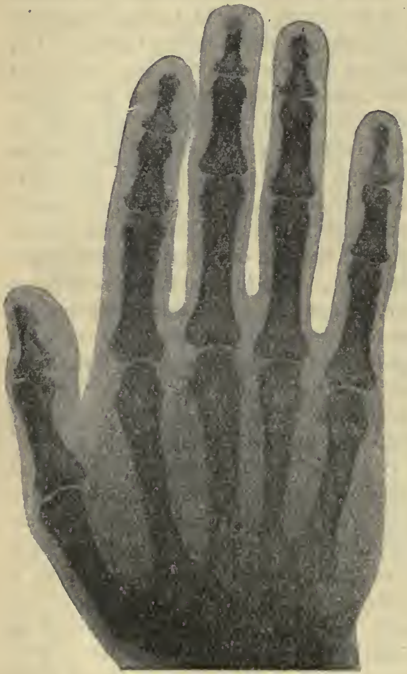

FIG. 348.

Röntgen Rays.—Prof. Röntgen, when investigating the phenomena of kathode rays in 1895, observed that a covered photographic plate, which was by chance lying near the apparatus, was affected just as though exposed to ordinary light.

He concluded that the effect must be due to some unknown form of radiation (since Lenard rays only penetrate a small thickness of air), and the uncertainty of their character led him to apply the provisional term "X-rays." The terms X-rays and Röntgen-rays are synonymous.

The rays differ from kathode rays in that they pass through many solid substances with comparatively little absorption, and are not deflected by a magnet; also, unlike light rays, they are not refracted when passing from one medium to another. The transparency of metals to X-rays depends upon the atomic weight of the metal: thus, aluminium is comparatively transparent and a thin sheet of lead is practically opaque. Also compounds of heavy metals are opaque: thus lead-glass is opaque, but soda-glass is fairly transparent. Röntgen also observed that the flesh of the hand is transparent, and more so than the bones. Consequently, if the hand is laid on a photographic plate protected by opaque paper and held in the path of the rays a "negative" or "shadow-graph" is obtained which clearly shows the details of the bones (Fig. 348).

X-rays excite powerful fluorescence in many substances, of which barium platino-cyanide is a striking example. If a paper screen, coated with such a compound, is held in the path of the rays and the hand is interposed, the fluorescence is less marked where the rays have been absorbed by the bones of the hand, and a shadow of the bones is visible on the screen.

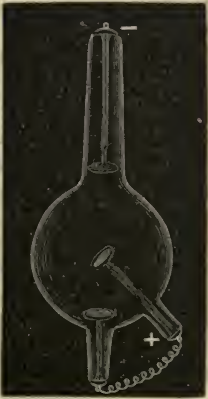

FIG. 349.

Fig. 349 represents the type of tube, known as a *focus tube*, originally devised by Mr. H. Jackson (of King's College, London), and now used for producing X-rays. The kathode is a concave disc of aluminium, and the anode is a round piece of platinum foil placed at the centre of curvature of the kathode and inclined at 45° to its axis. The focus-tube is exhausted through a side tube which afterwards is sealed. On passing a discharge through the tube, X-rays originate from the portion of the surface of the anode bombarded by the kathode rays.

It is accepted generally that X-rays are ether waves set up when the flying electrons are stopped suddenly by the anode (p. 504); they are, in fact, *ether pulses.* Each electron, when it strikes upon the anode, sends out a single pulse or ripple, the energy of the pulse depending upon the suddenness with which the electron is stopped. The higher the vacuum the more suddenly is the electron stopped and the more penetrating is the ray; this is the condition found in a *hard* tube, which is used in preference to one containing a lower vacuum when much penetration is required. Fig. 350 suggests the distinction between waves of ordinary light and X-ray pulses—which are single pulses of extremely short wave length.

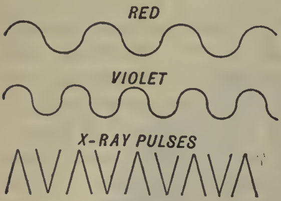

FIG. 350.

Becquerel Rays.—M. Becquerel observed, in 1896, when a compound of uranium is placed on a photographic plate protected by opaque paper, and the plate is exposed to sunlight, that the plate is affected as though exposed directly to light. Assuming that the effect might be due to phosphorescence of the compound caused by the exposure to light, he repeated the experiment in a darkened room and used a specimen of the compound freshly prepared by recrystallisation from an aqueous solution in darkness, and he found that the plate was still affected. This proved that the effect was characteristic of the compound, and was not due to exposure to light. He also found that a gas became a conductor when exposed to radiation from a uranium compound: this property can be shown by holding a fragment of the compound near to the disc of a charged electroscope.

Prof. Rutherford has found that the radiation is a mixture of three types:

(i) The α-radiation, which is absorbed readily by a thin layer of air, and having considerable ionising power p. (509).

(ii) The β-radiation, which has far more penetrating power but less ionising power; and

(iii) The γ-radiation.

The β-rays are deflected by a magnetic field in the same direction as kathode rays, and are therefore negatively charged electrons ; also their velocity is equal to two-thirds that of light. The α-rays are deflected by a magnetic field in the opposite direction and therefore carry a positive charge. The γ-rays are not deflected by a magnetic field.

This power of emitting radiation is termed **radio-activity.**

Radium.—In searching for radio-active substances, M. and Mme. Curie found that certain specimens of pitch-blende—a mineral which contains compounds of uranium—were more radio-active than the same bulk of pure uranium. By a laborious chemical process of elimination they succeeded in separating a compound of *radium*, which proved to be an extremely radio-active element. The radium obtained is associated with barium since the two elements are very similar in chemical properties, but they can be separated by a process which depends upon the fact that the chloride of radium is less soluble in water than the chloride of barium. The radiations from radium are, like those from uranium, of three distinct types, and denoted by the symbols α, β, and γ. Fig. 351 suggests the difference in the effect of a magnetic field on these radiations : the β-rays, resembling kathode rays, are deflected in the opposite direction to α-rays, while the γ-rays are unaffected.

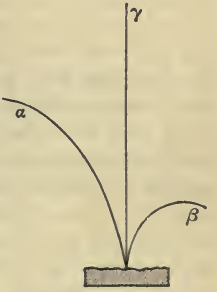

FIG. 351.

Ionisation of Gases.—A charged electroscope loses its charge more or less rapidly when held near to a flame or to a focus-tube in which X-rays are being generated : this effect is due to the fact that, in each case, the surrounding air thereby acquires a considerable degree of electrical conductivity.

The several methods by which any gas acquires conductivity may be enumerated thus :

(i) Kathode, Lenard, or X-rays.

(ii) Radiations from uranium, radium, and other radio-active elements.

(iii) Flames and hot metals.

(iv) An electric discharge.

The conductivity imparted to air by exposure to X-rays may be shown by means of the apparatus represented in Fig. 352.

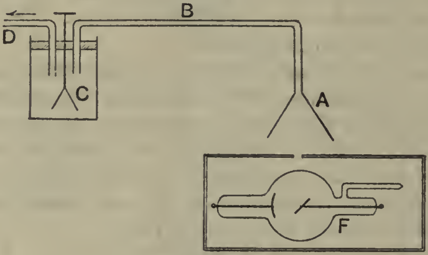

Fig. 352.

A focus-tube F is enclosed in a lead box fitted with a small window immediately under the glass funnel A, connected by the glass tube B to the inside of the vessel C in which a pair of insulated gold leaves are suspended; and the tube D is connected to a filter pump. The leaves are discharged rapidly when X-rays are generated and a slow stream of air is drawn through the apparatus. The conductivity, thus indicated, remains even when the tube B is replaced by a metal tube along the axis of which an insulated wire is fixed, if the tube and wire are at the same potential; but the conductivity is removed if an electrostatic field is set up by maintaining the tube and wire at different potentials. This proves that the conductivity is due to electrically charged particles, which may be termed ions. Also, since the ionised gas itself has no charge, there must be equal numbers of oppositely charged ions. Ionisation by means of a spark discharge is indicated by the greater readiness with which a discharge passes between two oppositely charged spheres after the first spark has passed. Also, if two vertical and slightly diverging wires (Fig. 353) are attached to the discharging knobs of an induction coil, the first spark passes between the lowest points

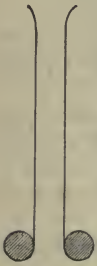

Fig. 353.

of the wires (where they are nearest together); but, since the hot ionised air tends to rise, the line of discharge slowly rises towards the top of the wires, where the discharge is broken by the increased length of the spark-gap, and the discharge re-starts from the base of the wires.

Conclusion—The electro-chemical equivalent of the negatively charged ions, obtained in any type of discharge through a gas, has been found to be 1.37×10^{-7}, a value which is independent of the nature of the gas and is approximately 10^3 times less than that for the hydrogen atom in ordinary electrolysis; and there is strong evidence that the negative carriers are detached portions of an atom and carrying the same charge as a hydrogen atom, but having scarcely more than one-thousandth part of its mass. It may be that the negative carriers have a material nucleus, though it is quite possible that they are purely electric charges free from matter, and that their inertia when in motion is due entirely to their charge. If this possibility is accepted these carriers may be *atoms of electricity*—a term which is synonymous with the recognised meaning of the term *electron.* With regard to the positive ions in gases, there is evidence that they consist of the natural unit charge (or electron) of positive electricity associated with an atom of matter.[1]

SUMMARY.

Discharge through a Rarefied Gas.—The following sequence of phenomena is observed when an electric discharge traverses a vacuum tube in which the pressure is being reduced gradually:

(i) A crimson column, known as the *positive column*, extends from the anode to, or nearly to, the kathode.

(ii) The positive column breaks up into striae, the *Faraday dark space* appears near to the kathode, and the kathode is covered with a luminous glow.

(iii) The Faraday dark space becomes less distinct, the striae thicken, and the luminous glow separates from the kathode, leaving a second dark space (known as *Crookes's dark space*).

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[1] The student is referred, for further information on the discharge of electricity through gases, to the following works:—Professor J. J. Thomson's *Conduction of Electricity through Gases* (Camb. Univ. Press); *Discharge of Electricity through Gases* (Constable); *Electricity and Matter* (Constable); and to Sir Oliver Lodge, *Journ. Inst. Elect. Engin.*, vol. xxxii., No 159.

(iv) The second dark space fills the entire tube, and a phosphorescent light is distributed over the surface of the glass.

The positive column, observed in (i), is deflected by a transverse magnetic field in the same direction as a flexible conductor would be if traversed by a current from the anode to the kathode.

Kathode Rays.—When the vacuum is so high that the second dark space completely fills the tube, the discharge consists of a stream of *negatively charged particles* which traverse paths normal to the surface of the kathode. The term *kathode rays* is applied to this stream of charged particles. The rays cast shadows of solid objects placed in their path; they also exhibit heating and mechanical effects. A transverse magnetic field deflects the rays in a direction which is in agreement with the conclusion that they consist of negatively charged particles proceeding from the kathode. The path of the deflected rays is approximately circular, and the radius of curvature depends upon the degree of vacuum and upon the intensity of the magnetic field.

Kathode rays impart conductivity to the gas through which they pass.

Lenard observed that kathode rays can pass through a thin sheet of aluminium forming part of the wall of a vacuum tube. The rays which can then be observed outside the tube are termed **Lenard rays.** This suggests that the charged particles are small compared with ordinary atoms or molecules, and recent investigation has identified these particles as **electrons**, or atoms of electricity which are independent of gross matter.

The Electro-chemical Equivalent, and the Speed of Kathode Rays.—The electro-chemical equivalent has been proved to be 10^{-7} C.G.S. units. Hence, since the E.C.E. of hydrogen is about 10^{-4}, either the mass of each particle must be much less than that of the hydrogen atom, or its charge must be much greater than that carried by the hydrogen ion. A recent experiment has shown that the mass of each particle constituting a kathode ray is 10^{-27} gram, or about $\frac{1}{1800}$th part of that of the hydrogen atom. The velocity of the charged particles is about one-tenth that of light.

There is strong evidence in favour of the view that the charge conveyed by a kathode ray particle is the same as the ionic charge of the hydrogen atom.

The Motion of a Charged Sphere.—The motion of the tubes of force originating from a charged sphere, which is itself in motion, creates a magnetic field in the surrounding space. The total energy in the system is due partly to the kinetic energy of the sphere and partly to the energy of the magnetic field. It can be proved that the total energy is the same as if the mass had been increased by an amount $2e^2/3a$, where e is the charge on the sphere, and a is its radius. If an electron is an atom of electricity free from ordinary matter, then its energy when in motion is due entirely to its charge ; and its mass—or, better, its *inertia*—is represented by the above expression. But the magnitudes of e/m and of e are known, hence that of a can be calculated. In this manner the volume of an electron has been calculated to be 10^{-39} c.c.

Röntgen Rays differ from kathode rays in that they pass through many solid substances with comparatively little absorption, and are not deflected by a magnet. Professor Röntgen termed them X-rays. The opacity of metals to these rays is approximately proportional to their atomic weights. The rays excite fluorescence in many substances, *e.g.* barium platino-cyanide. It is now accepted that the rays consist of *ether pulses* set up at the surface of an anode exposed to the bombardment of kathode rays.

Becquerel rays, observed to originate from compounds of uranium, have been proved to consist of a mixture of three types of radiation : (i) α-rays, which are readily absorbed by a thin layer of air, and carry a positive charge ; (ii) β-rays, which have a far greater penetrating power and carry a negative charge ; (iii) γ-rays, which are undeflected by a magnetic field.

The same types of radiation are observed with compounds of *radium*.

Ionisation of Gases.—Conductivity may be imparted to a gas by the following methods : (i) exposure to kathode, Lenard, or X-rays ; (ii) exposure to radiations from uranium or radium ; (iii) contact with flames and hot metals ; (iv) by the passage of an electric discharge. The conductivity is due to the generation of equal numbers of oppositely charged ions.

QUESTION ON CHAPTER XXVI

1. Give some account of the radio-activity of radium.

CHAPTER XXVII

UNITS, AND DIMENSIONS OF UNITS

Units.—Any physical quantity is expressed numerically in terms of some convenient magnitude of the same concrete kind, and the expression for the quantity involves two components, viz. : (i) a *numeric* or numerical factor, which is the ratio of the quantity to its unit, and (ii) the *unit* with which the quantity is compared. Thus, the statement that a length is '3' is incomplete since it does not include the unit with which the length is compared; but the statement that a length is '3 yards' is complete, and it implies that the length has been compared with a yard, which is here the unit, and the ratio has been found to be 3.

Any quantity may, of course, be re-expressed in terms of another unit, providing that the ratio between the units is known. Thus, if n_1 is the numeric of a given quantity in terms of a unit u_1, and if n_2 is the numeric of the same quantity in terms of another unit u_2, then

$$n_1 \times u_1 = n_2 \times u_2,$$

or,

$$n_2 = n_1 \times \frac{u_1}{u_2}.$$

The expression u_1/u_2 is termed the **change ratio** for the conversion from the first unit to the second, and is that factor which, when multiplied by the original numeric, gives the new numeric required. For example, if the quantity '3 yards' is to be expressed in terms of the *foot* as unit, then

$$n_2 = 3 \times \frac{1\ yard}{1\ foot} = 9.$$

Units may be chosen arbitrarily, each independent of any fact or circumstance, or each may be based upon some physical property of the universe which is least likely to change with time or place. Any complete system in which certain fundamental units are used, and in which all other units are *derived* from the fundamental units, is termed an **absolute system** of units. The term *absolute* was first introduced by Gauss, in 1832, in connection with his measurements of the intensity of the earth's magnetic field, and the same system was extended to electrostatic and electromagnetic quantities by Weber in 1851.

Fundamental Units.—Since all physical quantities can be expressed in terms of a *length*, a *mass*, and a period of *time*, the units of these three quantities are termed **fundamental units.** The units of other physical quantities are based upon these fundamental units, and are termed **derived units.** The system of fundamental units now universally adopted is that based upon the **centimetre,** the **gramme,** and the **mean solar second** as units of length, mass, and time; it is known as the C.G.S. (centimetre-gramme-second) system.

The **C.G.S. unit of length** is the $\frac{1}{100}$th part of the *metre*, which is defined as the distance between the extremities of a certain platinum bar at 0°C. The standard metre was constructed by decree of the French Republic in 1795, by whom it was enacted that the metre should be one ten-millionth part of the distance between the equator and the north pole. The arc of a meridian between Dunkirk and Barcelona was measured by Delambre, and the platinum standard was made by Borda. The following are the recognised multiples and sub-multiples of the metre:

The *decimetre*, or $\frac{1}{10}$ of the metre. The *dekametre*, or 10 metres.
,, *centimetre*, or $\frac{1}{100}$,, ,, ,, *hektometre*, or 100 ,,
,, *millimetre*, or $\frac{1}{1000}$,, ,, ,, *kilometre*, or 1000 ,,
,, *micron* (μ), or 10^{-3} millimetre.
,, *millimicron* ($\mu\mu$), or 10^{-6} millimetre.

The **C.G.S. unit of mass** is the *gramme*, and is defined as the mass of 1 cubic centimetre of water at 4°C. The gramme is really the one-thousandth part of the French standard of mass, which is a piece of platinum, called the "Kilogramme des Archives," made by Borda by decree of the Republic named above.

The **C.G.S. unit of time** is the mean solar second, and is defined as the $\frac{1}{86400}$ part of the mean solar day, which is the average time interval between successive transits of the sun across the meridian at any place. This unit is probably subject to a slow progressive lengthening due to a retardation caused by frictional resistance of the surrounding medium and to tidal effects.

The **British fundamental units** are the *foot*, *grain*, and *second*. These were used in British magnetic observations until recently, but they have now been superseded almost entirely by the C.G.S. system.

Dimensional Equations.—The equation for a velocity is usually written $v = s/t$, where s and t represent a length and a period of time respectively. Strictly speaking the symbols in this equation are merely *numerics*, and the equation would be written more accurately thus

$$v[\mathrm{V}] = \frac{s[\mathbf{L}]}{t[\mathbf{T}]},$$

where $[\mathrm{V}]$, $[\mathbf{L}]$, and $[\mathbf{T}]$ represent the units of velocity, length, and time respectively. If v, s, and t are each equal to unity, then

$$[\mathrm{V}] = [\mathbf{L}/\mathbf{T}].$$

This equation refers to the units only, and is known as a **dimensional equation.** It may be expressed in words thus: *a velocity is of one dimension in length and of minus one dimension in time.*

The dimensions of various derived units—dynamical, electrostatic, and electromagnetic—will be stated in the following paragraphs. Dimensional equations serve (i) to check the correctness of an equation and (ii) to aid the conversion of a quantity from one system of units to another.

DERIVED DYNAMICAL UNITS

Velocity.—*A body has unit velocity when it traverses unit length in unit time.* The C.G.S. unit is equal to *one centimetre per second.* The general equation is $v = s/t$, and the dimensional equation is

$$[\mathrm{V}] = [\mathbf{L}\mathbf{T}^{-1}].$$

Acceleration (a).—*A body has unit acceleration when its velocity changes by unity in unit time.* The C.G.S. unit is *one centimetre per second per second.* The general equation is

$$a = (v_2 - v_1)/t,$$

hence,

$$[a] = [\mathrm{V}]/[\mathbf{T}] = [\mathbf{L}\mathbf{T}^{-2}].$$

Force (f).—*Unit force creates unit acceleration in unit mass.* The C.G.S. unit of force is the *dyne*. The general equation is $f=ma$, hence $[f]=[\mathbf{MLT}^{-2}]$.
The dimensions of a **couple** are $[\mathbf{ML}^2\mathbf{T}^{-2}]$.

Work (w).—*Unit work is done when unit force acts through unit distance.* The C.G.S. unit of work is the *erg*, and is equal to the work done when the point of application of a force of one dyne moves in the direction of the force through a distance of one centimetre. The *Joule* is equal to 10^7 ergs. The general equation is $w=fs$, hence

$$[w]=[\mathbf{ML}^2\mathbf{T}^{-2}].$$

Energy is measured in the same units as *work*, and its dimensional formula is therefore the same.

Power (p).—*Unit power is capable of doing unit work in unit time.* The C.G.S. unit of power is capable of doing one erg of work per second. The *watt* is equal to 10^7 ergs per second, and the *horse-power* is equal to 550 foot-pounds per second or to 746 watts. The general equation is $p=w/t$, hence

$$[p]=[\mathbf{ML}^2\mathbf{T}^{-3}].$$

DERIVED ELECTROSTATIC UNITS

Quantity (q).—*The unit of quantity is that which, when concentrated at a point at unit distance from an equal and similar quantity, and surrounded with air as dielectric, is repelled with unit force.* The general equation is $f=q^2/\mathrm{K}d^2$, when K is the specific inductive capacity of the dielectric. The dimensional formula of an electrostatic quantity is therefore equal to the square root of the product of the dimension of a force, of inductive capacity, and of the square of a distance; hence

$$[q]=[\mathbf{MLT}^{-2}\times \mathrm{K}\mathbf{L}^2]^{\frac{1}{2}}=[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{3}{2}}\mathbf{T}^{-1}\mathrm{K}^{\frac{1}{2}}].*$$

Surface Density (σ).—*The surface density of an electrostatic charge is measured by the quantity on unit area.* Hence $\sigma=q/\mathrm{A}$, and

$$[\sigma]=[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{3}{2}}\mathbf{T}^{-1}\mathrm{K}^{\frac{1}{2}}]/[\mathbf{L}^2]=[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{-\frac{1}{2}}\mathbf{T}^{-1}\mathrm{K}^{\frac{1}{2}}].$$

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* Since the physical nature of specific inductive capacity is unknown, we are unable to express its dimensions in terms of the fundamental units; the symbol K is therefore allowed to remain in the dimensional equations, where it represents the unknown dimensions of this property of the medium.

Intensity of Field (F).—*The intensity at any point of a field is measured by the force with which unit quantity is acted upon if placed at the point.* Hence $f = \mathrm{F} \times q$, and

$$[\mathrm{F}] = [\mathrm{MLT}^{-2}]/[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}\mathrm{T}^{-1}\mathrm{K}^{\frac{1}{2}}] = \mathrm{M}^{\frac{1}{2}}\mathrm{L}^{-\frac{1}{2}}\mathrm{T}^{-1}\mathrm{K}^{-\frac{1}{2}}].$$

Potential Difference (v).—*Potential difference is measured by the work required in order to convey one unit of positive electricity from the point of lower to that of higher potential.* Hence $w = q \times v$, and

$$[v] = [\mathrm{ML}^{2}\mathrm{T}^{-2}]/[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}\mathrm{T}^{-1}\mathrm{K}^{\frac{1}{2}}] = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}\mathrm{T}^{-1}\mathrm{K}^{-\frac{1}{2}}].$$

Capacity (c).—*The capacity of a conductor is the quantity of electricity required to raise its potential by unity.* Hence $q = cv$, and

$$[c] = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}\mathrm{T}^{-1}\mathrm{K}^{\frac{1}{2}}]/[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{1}{2}}\mathrm{T}^{-1}\mathrm{K}^{-\frac{1}{2}}] = [\mathrm{LK}].$$

Current (C).—*The strength of a current is equal to the quantity of electricity which passes across a given cross-section in unit time.* Hence $C = q/t$, and

$$[C] = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}\mathrm{T}^{-2}\mathrm{K}^{\frac{1}{2}}].$$

Resistance (R).—*The resistance of a conductor is defined as the ratio of the potential difference between its ends to the resultant current.* Hence $R = v/C$, and

$$[\mathrm{R}] = [\mathrm{L}^{-1}\mathrm{TK}^{-1}].$$

DERIVED MAGNETIC UNITS

Unit Magnet Pole (m).—The unit pole is defined in the same manner as the unit quantity of electricity, except that the specific inductive capacity (K) of the medium is replaced by permeability (μ). Hence

$$[m] = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}\mathrm{T}^{-1}\mu^{\frac{1}{2}}].$$

Intensity of Field (H).—*The intensity at any point is measured by the force with which unit pole is acted upon if placed at the point.* Hence $f = \mathrm{H} \times m$, and

$$[\mathrm{H}] = [\mathrm{MLT}^{-2}]/[\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{3}{2}}\mathrm{T}^{-1}\mu^{\frac{1}{2}}] = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{-\frac{1}{2}}\mathrm{T}^{-1}\mu^{-\frac{1}{2}}].$$

Magnetic Moment (M).—*The moment of a magnet is the product of the pole-strength and the distance between the poles.* Hence $\mathrm{M} = m \times l$, and

$$[\mathrm{M}] = [\mathrm{M}^{\frac{1}{2}}\mathrm{L}^{\frac{5}{2}}\mathrm{T}^{-1}\mu^{\frac{1}{2}}].$$

Intensity of Magnetisation (I).—*The intensity of magnetisation is the magnetic moment per unit volume.* Hence $I = M/V$, and

$$[I] = [M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}\mu^{\frac{1}{2}}/L^3] = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}].$$

It is evident that the *surface density* of magnetism has the same dimensions.

Susceptibility (k).—*The susceptibility is defined as the ratio of the intensity of magnetisation to the field intensity*, or $k = I/H$. Hence

$$[k] = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}]/[M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}] = [\mu].$$

DERIVED ELECTROMAGNETIC UNITS

Current (C).—By definition (p. 255) *a current has unit strength when 1 cm. length of the circuit bent into the form of an arc, of 1 cm. radius, exerts a force of one dyne on a unit magnet pole placed at the centre of the arc* (or, in other words, creates at the centre a magnetic field of unit intensity). The intensity of the field at the centre of a complete circular circuit of radius a, and traversed by a current C, is (p. 255) equal to $2\pi C/a$. Hence, since 2π is a numerical constant, the dimensional formula of a current is equal to the product of the dimensions of a field-intensity and of a length, or

$$[C] = [M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}].$$

Quantity (Q).—*Unit quantity of electricity is conveyed by unit current in unit time.* Hence $Q = Ct$, and

$$[Q] = [C][T] = [M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}].$$

Potential Difference (E).—*Unit potential difference exists between two points when unit work is required in order to convey unit quantity of positive electricity from one point to the other against the electric force.* Hence $E = W/Q$, and

$$[E] = [ML^2T^{-2}]/[M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}] = [M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}\mu^{\frac{1}{2}}].$$

A similar formula may be derived from the following definition: *Unit potential difference is set up between the ends of a conductor which moves with such velocity that it cuts one magnetic line of force per second.* Thus, if a conductor of length l moves with a velocity v in a direction perpendicular to its length and to a magnetic field of intensity H and permeability $\mu=1$, then $e=lv\mathrm{H}$, and

$$[\mathrm{E}]=[\mathbf{L}^2\mathbf{T}^{-1}\times\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{-\frac{1}{2}}\mathbf{T}^{-1}]=[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{3}{2}}\mathbf{T}^{-2}].$$

Resistance (R).—Since $\mathrm{R}=\mathrm{E}/\mathrm{C}$,

$$[\mathrm{R}]=[\mathrm{E}]/[\mathrm{C}]=[\mathbf{L}\mathbf{T}^{-1}\mu].$$

Capacity (c).—Since $c=\mathrm{Q}/\mathrm{E}$,

$$[c]=[\mathrm{Q}]/[\mathrm{E}]=[\mathbf{L}^{-1}\mathbf{T}^2\mu^{-1}].$$

Inductance (L).—*A circuit has unit inductance when unit electromotive force is generated by the variation of the current at the rate of one unit of current per second.* Hence $\mathrm{L}=e\Big/\frac{d\mathrm{C}}{dt}$, and

$$[\mathrm{L}]=[\mathrm{E}]/[\mathrm{C}\mathbf{T}^{-1}]=[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{3}{2}}\mathbf{T}^{-2}\mu^{\frac{1}{2}}]/[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{1}{2}}\mathbf{T}^{-2}\mu^{-\frac{1}{2}}]=[\mu\mathbf{L}].$$

Ratio of the Electrostatic to the Electromagnetic Units.—For convenience of comparison the dimensions of the principal units in the two systems are given in the following table:

Unit	Electrostatic	Electromagnetic	$\frac{\text{Electrostatic}}{\text{Electromagnetic}}$
Quantity - -	$[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{3}{2}}\mathbf{T}^{-1}\mathrm{K}^{\frac{1}{2}}]$	$[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{1}{2}}\mu^{-\frac{1}{2}}]$	$[\mathbf{L}\mathbf{T}^{-1}\mathrm{K}^{\frac{1}{2}}\mu^{\frac{1}{2}}]$
Current - -	$[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{3}{2}}\mathbf{T}^{-2}\mathrm{K}^{\frac{1}{2}}]$	$[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{1}{2}}\mathbf{T}^{-1}\mu^{-\frac{1}{2}}]$	$[\mathbf{L}\mathbf{T}^{-1}\mathrm{K}^{\frac{1}{2}}\mu^{\frac{1}{2}}]$
Potential difference -	$[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{1}{2}}\mathbf{T}^{-1}\mathrm{K}^{-\frac{1}{2}}]$	$[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{3}{2}}\mathbf{T}^{-2}\mu^{\frac{1}{2}}]$	$[\mathbf{L}^{-1}\mathbf{T}\mathrm{K}^{-\frac{1}{2}}\mu^{-\frac{1}{2}}]$
Capacity - -	$[\mathbf{L}\mathrm{K}]$	$[\mathbf{L}^{-1}\mathbf{T}^2\mu^{-1}]$	$[\mathbf{L}^2\mathbf{T}^{-2}\mathrm{K}\mu]$

It will be seen that the dimensions assigned to any physical quantity in the two systems are not the same. But it must be borne in mind that the dimensions of K and of μ are not known, and that they may be such that the dimensions of a given unit are the same in both systems. Thus, if the dimensions for the unit of quantity are equated,

$$[\mathbf{L}\mathbf{T}^{-1}\mathrm{K}^{\frac{1}{2}}\mu^{\frac{1}{2}}]=1,$$

or

$$[\mathbf{L}\mathbf{T}^{-1}]=\left[\frac{1}{\sqrt{\mathrm{K}\mu}}\right].$$

Hence, if $\mathrm{K}^{-\frac{1}{2}}\mu^{-\frac{1}{2}}$ has the same dimensions as a velocity, the dimensions of each unit in the two systems are the same.

Until recently K and μ have been regarded as purely numerical quantities and having no dimensions in terms of **M**, **L**, or **T**. But it is now agreed that the dimensions of K and μ cannot be disregarded, and that their product must be capable of expression in terms of a velocity which is definite and characteristic of the medium. This velocity is usually denoted by the symbol 'v.'

Determination of 'v.'—Suppose that a definite quantity of electricity is measured by an electrostatic method and is found to be n_s E.S. units, and also by an electromagnetic method and found to be n_m E.M. units ; then

$$n_s[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{3}{2}}\mathbf{T}^{-1}\mathrm{K}^{\frac{1}{2}}]=n_m[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{1}{2}}\mu^{-\frac{1}{2}}],$$

or

$$\frac{n_s}{n_m}[\mathbf{LT}^{-1}]=\mathrm{K}^{-\frac{1}{2}}\mu^{-\frac{1}{2}}=v.$$

Here n_s and n_m are the numerics, or number of units, which are required in each case to make up the same total quantity of electricity ; and the greater the numeric the less must be the magnitude of the unit. Hence the ratio of the units must be equal to the inverse ratio of the numerics. If these units are Q_s and Q_m, then

$$\mathrm{Q}_m/\mathrm{Q}_s=v,$$

or 1 E.M. unit of quantity $=v$ E.S. units.

Also, by reference to p. 520,

1 E.M. unit of current $=v$ E.S. units,

1 „ „ capacity $=v^2$ E.S. units, and

v E.M. units of potential difference $=1$ E.S. unit.

Experimental Determination of 'v.'—The following methods have been applied to the experimental determination of 'v':

(i) **By Measurement of a Quantity of Electricity.**—Weber and Kohlrausch (1856) measured in E.S. units the charge in a condenser by transferring a known fraction of it to a Coulomb torsion balance. The charge was also measured in E.M. units by discharging the condenser through a ballistic galvanometer. The value of 'v' was found to be 3.107×10^{10} cms. per second.

(ii) **By Measurement of a Potential Difference.**—Lord Kelvin determined the potential difference in E.M. units between the ends of a wire by measuring the resistance of the wire and the current traversing it. The potential difference in E.S. units was measured by means of an absolute electrometer. The value of 'v,' by this method, was 3.004×10^{10} cms. per second.

(iii) **By Measurement of a Capacity.**—Ayrton and Perry (1879) calculated the capacity in E.S. units of a standard air condenser by measurement of its dimensions; and the capacity in E.M. units of the same condenser was measured by discharging it through a ballistic galvanometer. The ratio of the two measurements is equal to v^2, and v was found to be 2.980×10^{10} cms. per second.

The mean value of 'v' is approximately 3×10^{10} cms. per second—a value which agrees closely with the velocity of light in vacuo.

It may be added that Maxwell predicted from theory that an electrostatic charge of q E.S. units moving with a velocity u, is equivalent to a current of strength qu E.S. units or qu/v E.M. units; also, that if two similarly charged bodies are moving rapidly along two parallel paths, then, at a certain velocity, the electromagnetic attraction is balanced by the electrostatic repulsion between the charges, and that there is no force between them. Prof. Rowland, in 1875, verified the magnetic effect of a rapidly moving electrostatic charge by suspending an astatic needle near to a charged gilt ebonite disc which revolved at a high speed. Two fixed glass discs, gilt on the faces next to the ebonite disc, served to protect the needle from electrostatic effects and air currents. An appreciable deflection of the needle was observed, and the direction of the deflection was reversed when the sign of the disc's charge was reversed. This subject is also referred to on pp. 503 and 529.

Practical Units.—Several of the absolute units in the C.G.S. system would be inconveniently small, others would be inconveniently large, for practical purposes. The magnitudes of the *practical units* were agreed upon by the Paris Congress of 1881: those of resistance, electromotive force, and of current have been referred to already (p. 293), but they are given here for purposes of reference.

Resistance.—The *ohm* is equal to 10^9 C.G.S. electromagnetic units. It is represented theoretically by a velocity of 10^9 cms. per second (p. 431). For practical purposes it may be defined as the resistance at 0° C of a column of mercury, 1 sq. millimetre cross section and 106.3 cms. long (p. 430).

Electromotive Force.—The *volt* is equal to 10^8 C.G.S. electromagnetic units. As a practical standard the E.M.F. of a Clark cell is taken as 1.433 volt at 15° C.

Current.—The *ampere* is equal to the current which an E.M.F. of one volt will maintain in a circuit of one ohm resistance; hence, 1 ampere $= 10^8/10^9 = 10^{-1}$ C.G.S. unit. In practice, the ampere is defined sufficiently as the current which will deposit 0.001118 gram of silver per second.

Quantity.—The *coulomb* is the quantity conveyed in one second past any transverse section of a circuit traversed by a current of one ampere. It is therefore equal to 10^{-1} C.G.S. electromagnetic unit.

Capacity.—The *farad* is the capacity of a condenser which has a charge of one coulomb when the potential difference between its plates is one volt. Hence, 1 farad $= 10^{-1}/10^8 = 10^{-9}$ C.G.S. electromagnetic unit. The *micro-farad*, or millionth part of one farad, is equal to 10^{-15} C.G.S. unit.

Inductance.—The *henry* is the self-inductance of a circuit in which an opposing E.M.F. of one volt is set up when the current varies at the rate of one ampere per second. Hence, since $L = -e\Big/\frac{dC}{dt}$ (p. 418), 1 henry $= 10^8/10^{-1} = 10^9$ C.G.S. electromagnetic units.

Work.—The *joule* is equal to the electric energy expended when one coulomb passes between two points which differ in potential by one volt. Hence, 1 joule $= 10^{-1} \times 10^8 = 10^7$ C.G.S. units of work (*ergs*).

Activity.—The *watt* is the rate of work when one ampere flows between two points which differ in potential by one volt. Hence, 1 watt $= 10^{-1} \times 10^8 = 10^7$ C.G.S. units of power (ergs per second).

The prefixes *mega* and *micro* signify one million and one millionth part respectively; thus, 1 megohm $= 10^6$ ohms, and 1 microhm $= 10^{-6}$ ohm. The prefixes *kilo* and *milli* signify one thousand and one thousandth part respectively.

Extremely large or small numbers are written most conveniently in the system known as the *index notation*, in which the number is expressed as the product of two factors, one of which is a power of the number 10. Thus, 24600 may be written 24.6×10^3 or 2.46×10^4; also, 0.00246 may be written 24.6×10^{-4} or 2.46×10^{-3}. It is found convenient to place the decimal so that the exponent of the power of 10 is the *characteristic* of the logarithm of the number; thus, 64 millions would be written 6.4×10^7, and not 64×10^6.

Examples in the use of Dimensional Equations.—1. Express the acceleration due to gravity in terms of the mile and the hour as units of time, when its value is 981 in C.G.S. units.

If n is the numeric of an acceleration when L and T are the units of length and of time, and if n_1 is the numeric of the same acceleration when L_1 and T_1 are the units of length and time, then

$$n_1 \times \frac{L_1}{T_1^2} = n \times \frac{L}{T^2},$$

or
$$n_1 = n \times \frac{L}{L_1} \times \frac{T_1^2}{T^2}.$$

But $\quad n = 981$, and $L = T = 1$.

Also $\quad L_1 = (1760 \times 3 \times 30.5) = 1.6104 \times 10^5$ cms.,

$T_1 = 3600$ seconds;

$$\therefore \; n_1 = 981 \times \frac{(3600)^2}{1.6104 \times 10^5} = 78950 \text{ miles per hour per hour.}$$

2. If the horizontal intensity of the Earth's magnetic field is 0.18 C.G.S. units, what is its value in British (or foot-grain-second) units?

The dimensions of magnetic field intensity are $[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{-\frac{1}{2}}\mathbf{T}^{-1}\mu^{-\frac{1}{2}}]$. The quantities **T** and μ have the same value in the two systems; hence

$$n_1 = n \times \left(\frac{\mathbf{M}}{\mathbf{M}_1}\right)^{\frac{1}{2}} \left(\frac{\mathbf{L}_1}{\mathbf{L}}\right)^{\frac{1}{2}}.$$

But $\quad n = 0.18,$

and
$$\frac{\mathbf{M}}{\mathbf{M}_1} = \frac{1 \text{ gram}}{1 \text{ grain}} = 15.43,$$

and
$$\frac{\mathbf{L}_1}{\mathbf{L}} = \frac{1 \text{ foot}}{1 \text{ cm.}} = 30.5;$$

$$\therefore \; n_1 = 0.18 \times (15.43)^{\frac{1}{2}} \times (30.5)^{\frac{1}{2}} = 3.903 \text{ ft.-grain-sec. units.}$$

3. The time of swing of a magnet suspended so as to swing freely in a horizontal plane is equal to $2\pi\sqrt{I/MH}$, where I and M are the moment of inertia and the magnetic moment of the magnet. Verify that this formula has the dimension of a period of time.

Since $[\mathrm{I}]=[\mathbf{ML^2}]$,

and $[\mathrm{M}]=[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{\frac{5}{2}}\mathbf{T}^{-1}\mu^{\frac{1}{2}}]$,

and $[\mathrm{H}]=[\mathbf{M}^{\frac{1}{2}}\mathbf{L}^{-\frac{1}{2}}\mathbf{T}^{-1}\mu^{-\frac{1}{2}}]$.

Then $[\sqrt{\mathrm{I/MH}}]=[\sqrt{\mathbf{T}^2}]=[\mathbf{T}]$.

4. The time of swing of a simple pendulum depends only on its length and on the value of g. How does the time of swing depend upon the length?

Let $t \propto l^x g^y$, where x and y are unknown.

Then
$$[\mathbf{T}]=[\mathbf{L}]^x[\mathbf{LT}^{-2}]^y$$
$$=[\mathbf{L}]^{x+y}[\mathbf{T}]^{-2y}.$$

Hence $x+y=0$, and $-2y=1$;

or $y=-\frac{1}{2}$, and $x=\frac{1}{2}$.

Therefore $t \propto l^{\frac{1}{2}}g^{-\frac{1}{2}}$; or, *the time of swing is proportional to the square root of the length.*

SUMMARY

Units.—The expression for any physical quantity involves two components: (i) a *numeric*, and (ii) the *unit* in terms of which the quantity is expressed.

In order to express any quantity in terms of another unit, the first numeric must be multiplied by the ratio of the first unit to the second unit; this ratio is termed the *change ratio.*

The units of *length*, *mass*, and *time* are termed *fundamental units*; and any system of units derived from fundamental units is termed an *absolute system.* In the C.G.S. system, the fundamental units are the *centimetre*, the *gramme*, and the *second.*

Dimensional Equations.—Quantities which are of the same kind are said to be of the same *dimensions.* Thus, a straight line is said to be of *one dimension in length*, and a volume is of *three dimensions in length.* A velocity is of one dimension in length and of minus one dimension in time; and this may be written

$$[\mathrm{V}]=[\mathrm{LT}^{-1}].$$

The dimensions of all electromagnetic and electrostatic units may be expressed in terms of the fundamental units and the permeability μ in the former case, and in terms of the fundamental units and the specific inductive capacity K in the latter case.

The dimensions of K and of μ are unknown, but their product has the dimensions of a velocity, or

$$\left[\frac{1}{\sqrt{K\mu}}\right]=\left[LT^{-1}\right].$$

Determination of 'v'.—The expression $1/\sqrt{K\mu}$ is usually denoted by the symbol v. It is evident, from the dimensional formulae, that

1 *electromagnetic* unit of quantity $= v$ *electrostatic* units.
1 ,, ,, ,, current $= v$,, ,,
1 ,, ,, ,, capacity $= v^2$,, ,,
v ,, units of P.D. $= 1$,, unit.

The numerical value of v can be determined by either of the following methods:

(i) By measurement of a *quantity* of electricity in both electrostatic and electromagnetic units.

(ii) By measurement of a *potential difference* in both systems.

(iii) By measurement of a *capacity* in both systems.

Practical Units

Resistance:	The *ohm*	$= 10^9$	C.G.S. electromagnetic units.
E.M.F.:	The *volt*	$= 10^8$	,, ,, ,,
Current:	The *ampere*	$= 10^{-1}$	,, ,, ,,
Quantity:	The *coulomb*	$= 10^{-1}$	,, ,, ,,
Capacity:	The *farad*	$= 10^{-9}$	,, ,, ,,
	also, the *microfarad*	$= 10^{-15}$	,, ,, ,,
Inductance:	The *henry*	$= 10^9$	,, ,, ,,
Work:	The *joule*	$= 10^7$	C.G.S. units (or *ergs*).
Activity:	The *watt*	$= 10^7$	,, ,, (or *ergs per second*).

QUESTIONS ON CHAPTER XXVII

1. If the dimensions of a force are $\mathbf{MLT^{-2}}$, find the number of dynes in a poundal.

2. Calculate the factor for the conversion of kilowatts into horse-power units. (The dimensions of *power* are $[\mathbf{ML^2T^{-3}}]$; also 1 lb. = 453.6 grams, and 1 ft. = 30.48 cms.)

3. The intensity of the magnetic field at a point distant d cms. from the centre of a magnet, of moment M, and situated on its axis produced is equal to $2M/d^3$ approximately. Verify that this formula has the dimensions of a magnetic field intensity. What alteration must be made in the formula if the permeability of the medium is taken into consideration?

4. The E.M.F. set up in an earth-inductor of area A which is rotated with angular velocity ω round an axis perpendicular to the lines of force of a field of intensity F is equal to ωAF, the medium having a permeability equal to unity. If the medium has a permeability μ, verify that the formula for the E.M.F. generated has the same dimensions as a potential difference in the electromagnetic system.

5. Prove that the left and right-hand members of the equation, $Q = \frac{HT}{G\pi} \cdot \sin\frac{\theta}{2}$, for a ballistic galvanometer have the same dimensions.

6. What would be the values of the *practical units* of current, E.M.F., and resistance in C.G.S. units if the fundamental units were $T = 1$ sec., $L = 10^9$ cms., and $M = 10^{-11}$ gm.?

7. Describe the electrostatic and electromagnetic systems of units. Find on each of these systems the dimensions of (1) potential difference; (2) capacity of a condenser; (3) specific resistance.

8. If the magnetic moment of a bar magnet is 1.6×10^5 in the millimetre-milligram-second system, find its moment in (i) the C.G.S. system and (ii) in the foot-grain-second system.

9. If the centimetre and the gram are the units of length and mass respectively, find the magnitude of the unit of time so that the weight of one gram is equal to the absolute unit of force.

10. What are the practical units of quantity, current, resistance, E.M.F., capacity, energy and power? Give a table showing the connections between these units, and their value in C.G.S. absolute units.

CHAPTER XXVIII

ELECTRO-MAGNETIC THEORY

Transfer of Electric Energy through a Dielectric.—Recent advances in our knowledge of electric phenomena point to the conclusion that the transference of energy by means of an electric current is located as much in the surrounding medium as in the conductor itself, and that the function of the latter is largely to serve as a guide to the path which the flow of energy follows, and to serve as a place in which the energy may be dissipated. The passage of a current in a simple voltaic circuit involves the presence of a magnetic field in the surrounding dielectric, and we may imagine this field to be sub-divided into cells or compartments by the intersection of the magnetic equipotential surfaces and the magnetic tubes of force, the energy at any point due to this field being equal to $\mu H^2/8\pi$, where H is the magnetic intensity at the point and μ is the permeability of the medium (p. 384). At the same time, since the passage of a current involves a potential difference between different parts of the circuit, there must be,. of necessity, an electrostatic field of force which we may imagine to be subdivided into cells by the intersection of electrostatic equipotential surfaces and electrostatic tubes of force ; also, the energy at any point due to this field is equal to $KF^2/8\pi$, where F is the electrostatic intensity at the point and K is the specific inductive capacity of the medium (p. 181). In any simple voltaic circuit, the energy in the field is entirely electrostatic before the circuit is completed ; but, as soon as the circuit is closed, the energy in the dielectric becomes partly electromagnetic and partly electrostatic; it is then imparted gradually to the conductor, and there becomes dissipated as heat. Fig. 354 represents the electrostatic field in the medium enclosed by a voltaic circuit in which a uniform conducting wire, which connects the poles of a voltaic cell, is bent twice at right angles. If the point A is earth-connected, corresponding points on opposite sides of the circuit will have equal and opposite potentials, and the equipotential surfaces are indicated by the dotted lines. The electrostatic tubes of force are everywhere normal to the equipotential surfaces (p. 148), and may be represented by the thin continuous lines. At points near to the battery the P.D. between corresponding points on the flow and return circuit will be greater than the P.D. between points at a greater distance from the battery: hence, as the distance from the battery increases, the number of equipotential surfaces between the flow and return wire will become smaller, as shown in the diagram; also, for the same reason, the electrostatic tubes of force will be less crowded between the corresponding points more distant from the battery. Since the electrostatic tubes of force are always perpendicular to equipotential surfaces, they cannot meet the surface of the conductor normally; hence they are not in equilibrium, and their ends travel along the surface of the conductor from left to right (Fig. 354). This process sets up a magnetic field, of which the tubes of force are perpendicular to the plane of the diagram and are directed, within the boundary of the circuit, through the paper and away from the observer; also, the magnetic equipotential surfaces are planes passing through the axis of the conductor. The relative direction of the magnetic field may be remembered by applying a rule which resembles Fleming's *right-hand* rule (p. 413):

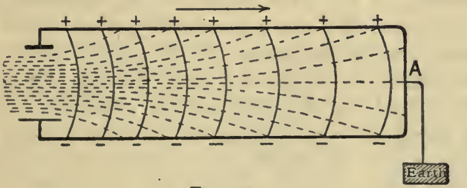

FIG. 354.

if the extended thumb of the right hand represents the direction of motion of the electrostatic tubes of force, and if the first finger represents the direction of the electrostatic force, the second finger (bent at right angles to the palm) then represents the direction of the magnetic field. Prof. Poynting has shown that **the paths along which the energy flows into the circuit are perpendicular to the plane containing the lines of electric force and the lines of magnetic force**—or, in other words, the paths coincide with the intersections of the magnetic and electrostatic equipotential surfaces.

The energy thus transferred from the medium to any given length of the circuit is frittered away as heat within that portion of the circuit; and we may regard the whole process as involving the gradual transference of energy from the voltaic cell to the dielectric, and the simultaneous transference, along the *energy paths*, of energy from the dielectric to the circuit.

The current in a simple voltaic circuit assumes its maximum value only after a short, but appreciable, interval of time. During this interval, the energy of the current represents only a part of the energy withdrawn from the cell, the remainder being transferred to the surrounding medium. When the circuit is broken, the energy in the medium returns into the circuit, and gives rise to the *extra current* (p. 417)—at least this happens if the change in the current is comparatively slow; but, if the change is sufficiently rapid, much of this energy is thrown off from the circuit in the form of electromagnetic waves.

Displacement Currents.—A conductor which connects the poles of a battery is traversed by a steady current, and the total flow of electricity across any cross-section is the same at all points of the circuit. If a simple air-condenser is included in the circuit (Fig. 355) then, during a short interval after closing the circuit, each wire is traversed by a current which rapidly diminishes to zero. The space between the plates is now traversed by Faraday tubes, and is in a condition of electrostatic strain. Maxwell supposed that the electrostatic force between the plates causes an **electric dis-**

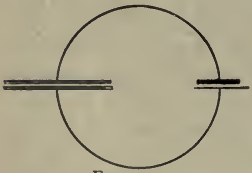
FIG. 355.

placement which is proportional to the force, positive electricity being displaced in the direction of the force, and that the quantity of electricity displaced across any cross-section of the dielectric between the plates is equal to the quantity of electricity transmitted across any cross-section of the circuit. The electric displacement is equivalent to a *strain*, which opposes the *stress*, and the displacement ceases as soon as the strain is sufficient to balance the stress which gives rise to it.

Maxwell also supposed that this electric displacement is equivalent to an electric current, which is measured by the rate of change of the displacement, and creates a magnetic field in the surrounding dielectric. A magnetic field, created in this manner, would disappear as soon as the displacement current ceases; but if the charged plates are discharged, the magnetic field momentarily reappears in a reverse direction: in fact, a magnetic field is only set up when the electrostatic field is undergoing change.

Dr. H. A. Wilson[1] has recently proved that an E.M.F. is induced in a *dielectric* when it is rotated in a magnetic field in such a manner as to cut the lines of force, and that the direction of the electric displacement is the same as that which is created in a *conductor* moving under similar conditions. The method employed was to rotate a cylinder of ebonite in a magnetic field of which the lines of force coincided in direction with the axis of the cylinder. The inner and outer surfaces of the cylinder were each covered with a metal plate, and sliding brushes making contact with the plates were connected to one pair of quadrants of an electrometer and to earth respectively. The electrometer was calibrated previously by means of a guard-ring condenser. The E.M.F. set up between the cylinder's surfaces and calculated from the observed deflections was in close agreement with the value calculated from theory, viz.:

$$E = n\pi H(r_2^2 - r_1^2)\left(\frac{K-1}{K}\right),$$

where r_2 and r_1 are the external and internal radii of the cylinder, n the number of revolutions per second, K the specific inductive capacity of the dielectric, and H the intensity of the field.

Magnetic Force caused by the Motion of Electrostatic Tubes of Force.—A steady current traversing a wire is equivalent to a series of insulated conductors charged electrostatically and moving with considerable velocity along a path traced out by the wire. In the surrounding dielectric there would be (i) *Faraday tubes*, radiating outwards from the charged conductors, and (ii) *magnetic lines of force* at right angles to the Faraday tubes and to the direction of motion of the conductors. Faraday tubes at rest do not create a magnetic field, but Rowland's experiment (p. 522) shows that a magnetic field is created by tubes in motion, *even when the intensity of the electrostatic field remains constant.* Hence, a magnetic field may be due either to displacement currents or to Faraday tubes in motion. In Rowland's experiment, if the disc is charged positively and rotated in an anti-clockwise direction, it is evident, by applying the rule suggested on p. 530, that the direction of the magnetic field above the disc is from the centre to the circumference.

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[1] *Proc. Roy. Soc.*, vol. 73, 1904.

Electric Elasticity.—The coefficient of elasticity of any elastic material is defined as the ratio of the applied mechanical stress to the strain produced by the stress. The electric stress to which a non-conducting medium may be subjected by an electric field of force, and the resulting electric strain, are analogous to this; and we may speak of the *electric elasticity* of the medium. Thus, consider the case of a charged insulated sphere and the electric stress and strain produced in a thin concentric spherical shell of the dielectric surrounding the sphere. If a charge $+Q$ is imparted to the sphere, charges equal to $-Q$ and $+Q$ will be induced on the inner and outer surfaces of the shell. If the radius of the shell is r cms., then

$$\textbf{Stress} = \textit{electric force per unit area} = Q/Kr^2, \text{ and}$$

$$\textbf{Strain} = \textit{quantity displaced per unit area} = Q/4\pi r^2\ ;$$

hence,

$$\textbf{Electric elasticity} = \frac{\textbf{Stress}}{\textbf{Strain}} = \frac{4\pi}{K}.$$

Maxwell's Electro-magnetic Theory.—The question which naturally arises is whether the lines of force in the neighbourhood of a conductor conveying a current appear instantaneously, or whether the final condition follows a sequence of changes propagated through the medium. Thus, supposing the current to acquire its maximum value instantaneously, we can imagine magnetic lines of force spreading outwards from the conductor with a definite velocity—some expanding to a more distant position than others ; also, an electric displacement is set up in the medium, the direction of the displacement being perpendicular to the magnetic lines and parallel to the conductor. If the current now increases gradually, we can imagine how each small increment of current is accompanied by the generation of fresh lines of force, spreading outwards and increasing the concentration of lines at all points of the field, and how each such increment creates a commensurate displacement. If the current is rapidly periodic, varying from zero to a maximum and back to zero, the medium will be traversed by *shells* of magnetic force and of electric displacement travelling outwards from the conductor—like waves of light from a candle flame. The velocity with which an electric disturbance travels through a medium may be deduced from the parallel case of the propagation of a transverse vibration through an elastic medium. In the latter case, the velocity of propagation is given by the formula

$$v=\sqrt{\frac{e}{\mathrm{D}}},$$

where e is the elasticity of the medium, and D is its density. If we substitute for e the *electric elasticity* $\frac{4\pi}{\mathrm{K}}$ (p. 532), and for D the electric density $4\pi\mu$ (p. 434), we obtain for the velocity of propagation of an electric disturbance

$$v=\sqrt{\frac{4\pi}{\mathrm{K}}\Big/4\pi\mu}=\frac{1}{\sqrt{\mathrm{K}\mu}}.$$

It is impossible to determine the separate values of K and of μ for the ether which is concerned in the propagation of an electric disturbance—the values usually given to these quantities are *ratios* and not absolute values. But the value of their product can be determined by the methods described on p. 521.

By forming equations which express the changes in the field due to an assumed propagation of electromagnetic effects, Maxwell (in 1867) deduced an equation for the velocity of propagation which is analogous to that for the velocity of a disturbance in an elastic solid and is in exact agreement with that deduced above. Maxwell's elaborate treatment cannot be reproduced here, but the process of reasoning may be understood from the following paragraphs.

Let LR (Fig. 356) represent a small rectangular prism of the dielectric, and let the dimensions of the prism be LM = 1 cm: LO = δx, and LP = δy (where δx and δy are extremely small lengths); also suppose the prism to be so placed that the edge LM is parallel to the axis of a straight conductor AB. In order to investigate the electromagnetic effects set up in LR by the establishment of a current in AB we may consider either that LR is fixed and that the effects are due to the propagation through it of lines of force due to a current which is *suddenly* established, or we may regard the field as already established and that the effects are produced by a movement of LR towards AB and along a path perpendicular to LM, the velocity being equal to that with which the electromagnetic effects are propagated through the dielectric. In the reasoning which follows, we will adopt the latter assumption.

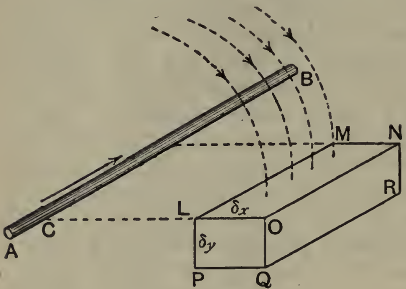

FIG. 356.

The movement of the prism of dielectric will create in it an electromotive intensity in just the same manner as though the prism were of conducting material. If **B** is the flux-density, or magnetic induction through unit area, in the region occupied by LM, the electromotive intensity set up along LM is equal to $\mathbf{B}v$, where v is the velocity with which LM moves towards AB, or $E = \mathbf{B}v$.

Differentiating this with respect to x, we have

$$\frac{dE}{dx} = v\frac{d\mathbf{B}}{dx}.$$

But, since $v = dx/dt$, $dx = v\,.\,dt$,

Therefore $$\frac{dE}{dt} = v^2\frac{d\mathbf{B}}{dx},$$

or $$v^2 = \frac{dE}{dt}\Big/\frac{d\mathbf{B}}{dx}. \qquad \text{..............................(1)}$$

Both dE/dt and $d\mathbf{B}/dx$ may be expressed in terms of the electric and magnetic constants, K and μ, of the dielectric. Thus, it has been shown (p. 532) that $E/D=4\pi/K$, where D is the *electric displacement.* Therefore

$$\frac{dE}{dt}=\frac{4\pi}{K}\cdot\frac{dD}{dt}. \quad \text{.............................}(2)$$

Similarly $d\mathbf{B}/dx$ may be resolved into terms involving the displacement current and a constant, by applying the principle that *the line integral of the magnetic force round a given path of a current is equal to* 4π *times the surface integral of the total current through the area bounded by that path.*[1] The magnetic induction along LP is equal to **B**, and directed downwards; also the induction along OQ is equal to

$$\left(\mathbf{B}+\frac{d\mathbf{B}}{dx}\cdot\delta x\right)$$

and directed downwards. Since $B=\mu H$, the magnetic forces along LP and OQ are $\frac{\mathbf{B}}{\mu}$ and $\frac{1}{\mu}\left(\mathbf{B}+\frac{d\mathbf{B}}{dx}\cdot\delta x\right)$ respectively; and, since the induction along the sides LO and PQ is zero, the line integral of the magnetic force round the path LPQO is

$$\frac{1}{\mu}\cdot\frac{d\mathbf{B}}{dx}\cdot\delta x\cdot\delta y.$$

The surface integral of the current is equal to the product of the intensity of the displacement current and the area of the surface LPQO; hence, since the displacement *current* is equal to the time rate of change of the displacement, *i.e.* to dD/dt,

$$\frac{1}{\mu}\cdot\frac{d\mathbf{B}}{dx}\cdot\delta x\cdot\delta y=4\pi\times\frac{dD}{dt}\cdot\delta x\cdot\delta y$$

or

$$\frac{d\mathbf{B}}{dx}=4\pi\mu\cdot\frac{dD}{dt}. \quad \text{.............................}(3)$$

The displacement current therefore can be expressed, as in equations (2) and (3), either in terms of the electromotive intensity set up in the prism or in terms of the magnetic induction in a plane perpendicular to the displacement. Whichever expression is adopted, the displacement current is the same; and therefore

$$\frac{dD}{dt}=\frac{K}{4\pi}\cdot\frac{dE}{dt}=\frac{1}{4\pi\mu}\cdot\frac{d\mathbf{B}}{dx},$$

or

$$\frac{dE}{dt}\bigg/\frac{d\mathbf{B}}{dx}=\frac{1}{\mu K}.$$

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[1] For example, the magnetic force at a point distant r cms. from an infinitely long straight wire conveying a current C C.G.S. units is equal to $2C/r$ (p. 257). The *line integral* of the magnetic force is therefore $2\pi r\times 2C/r=4\pi C$.

Hence, from equation (1),

$$v = \frac{1}{\sqrt{K\mu}}.$$

Thus an electromagnetic wave consists of an electric force and of a magnetic field set up in the dielectric in directions at right angles to each other and both at right angles to the direction of propagation. By comparison of the electromagnetic and electrostatic units of quantity, and by other methods (p. 521), it has been proved that $1/\sqrt{K\mu}$, when air is the dielectric, is equal to the velocity of light in the same medium. Maxwell concluded that the same medium is involved in the propagation of both optical and electromagnetic phenomena, and that both are identical in character. This conclusion was verified, at a much later date, by the experiments of Hertz (p. 553).

Relation between $\frac{1}{\sqrt{K\mu}}$ and the Refractive Index of a Dielectric.—Maxwell stated, as a further deduction from his electromagnetic theory, that *the specific inductive capacity of any dielectric should be equal to the square of its refractive index.* Thus, if K_1, μ_1, and K_2, μ_2, are the electrostatic and magnetic constants for two media, and if v_1 and v_2 are the velocities of propagation of an electromagnetic wave in the media, then

$$v_1 : v_2 :: \frac{1}{\sqrt{K_1\mu_1}} : \frac{1}{\sqrt{K_2\mu_2}},$$

or

$$v_2/v_1 = \sqrt{K_1\mu_1}/\sqrt{K_2\mu_2}.$$

If the media are non-magnetic, then $\mu_1 = \mu_2$; also, if v_1 is the velocity of propagation in air, then $K_1 = 1$. Hence

$$v_1/v_2 = \sqrt{K_2},$$

and if v_1 and v_2 also represent the velocity of *light* in air and in another medium, the ratio v_1/v_2 is termed the *refractive index* (n) of the latter medium.

The following table indicates that several substances give results which are in accord with this deduction; yet these are less numerous than the substances which, at first sight, appear to be exceptions to the deduction. It must be borne in mind that the value of K has usually been determined by a method involving the application of a *steady* electric force, whereas the refractive index is measured by means of visible radiations which, according to theory, consist of a rapidly alternating electric force of frequency varying from (4×10^{14}) to (10×10^{14}) per second. It may be assumed, therefore, that if K could be determined when the applied electric force is rapidly alternating, and if the refractive index could be measured for extremely long wave-lengths, the apparent exceptions to the principle would disappear. The refractive index n_∞ for waves of infinite length may be calculated approximately by means of Cauchy's formula from the value of n obtained with visible parts of the spectrum, but we have no evidence that the formula can be extended over such a wide range of wave-lengths. The values of K given in the table are obtained by means of an alternating electric force having a frequency of 120.

TABLE I

Substance.	K.	n^2_∞.
Paraffin,	2.29	2.022
Petroleum oil,	2.07	2.075
Turpentine,	2.23	2.13

In Table II the value of n is that observed for the sodium, or D, line of the spectrum.

TABLE II

	K.	n^2.
Carbon bisulphide,	2.67	2.68
Benzine,	2.38	2.26
Glass (dense),	10.1	2.92
Water,	80.0	1.78*
Quartz,	4.55	2.41
Ethyl alcohol,	25.80	1.83

Prof. Fleming has used a method of measuring the dielectric constant in which the condenser is charged and discharged many times per second. One prong of an electrically driven tuning fork closes and opens a battery circuit, which includes an electromagnet and vibrator. In one position of the vibrator the condenser is charged by means of a battery; in the other position the condenser is discharged through a high resistance galvanometer, which has been calibrated previously. If the rate of vibration of the tuning fork is known, the charge on the condenser can be calculated from the observed deflection of the galvanometer; and the capacity can be calculated from the charge and from the known E.M.F. of the charging battery.

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* Fleming and Dewar have found the refractive index of water for electric waves of considerable length to be 8.9. The square of this number closely approximates to 80.

With reference to the numerous cases in which Maxwell's law is not obeyed, it is important to bear in mind that the value of the dielectric constant is influenced largely by temperature as well as by the time of application of the electric force. Thus Fleming and Dewar[1] have found the value of K for liquid oxygen to be 1.491, and its magnetic permeability is 1.0041; and the product of these approximates to the value, 1.48, or the square of its refractive index for waves of infinite length. In these experiments the condenser consisted of two co-axial cones, one fitting within the other, and the capacity could be varied by raising or lowering the inner cone. The same observers have found that the value of K for ice at 0° is about 76, but that it falls to 2.4 at the temperature of liquid air. Similarly, the value for alcohol drops from 25 to 3.12.

Rotation of the Plane of Polarisation by a Magnetic Field.—Faraday appears to have had, in 1845, an instinctive idea that there was a close relationship between electricity and light. He allowed a beam of polarised light to pass through holes in the pole-pieces of a powerful electromagnet, and so that the direction of the beam was parallel to the lines of force; he placed a mass of dense lead glass between the poles, and adjusted an analyser so as to cut off all the light when the magnet was not excited, then, on exciting the magnet, the field appeared luminous, and a slight rotation of the analyser was necessary in order to restore the darkness of the field. The magnetic field, required for this observation, may also be obtained by surrounding the medium with a coil traversed by a strong current.

The amount of rotation of the plane of polarisation depends upon the material and upon the intensity of the magnetic field; and, since it depends upon the length of the material traversed, and since it is unaltered in direction when the direction of the field is reversed, the observed rotation is increased by successively reflecting the rays from the silvered ends of the transparent material. The rotation δ may be expressed by the formula

$$\delta = \gamma LH,$$

where γ, known as **Verdet's constant**, depends upon the material and upon the wave-length of the light employed, and where L is the length of path and **H** is the intensity of the field.

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[1] *Proceedings Roy. Soc.*, vol. 60, 1896, vol. 60, 1897, vol. 61, 1897.

In the case of all diamagnetic substances the value of γ is *positive*, *i.e.* in the same direction as the current which would produce the same magnetic field. Verdet's constant may be defined as *the rotation produced by unit length of the given medium when placed in a magnetic field of unit intensity.* The constant is expressed usually in minutes of arc.

VERDET'S CONSTANT FOR D-LIGHT

Glass (dense flint)	15° C.	0.065
Water	20° C.	0.0131
Carbon bisulphide	18° C.	0.0420

The value diminishes with increase of temperature; thus, with most substances except water, the general formula

$$\gamma_t = \gamma_0 (1 - 0.00104t - 0.0000014t^2)$$

may be used.

This phenomenon may be used in order to measure the strength of a current in absolute units; for, if a length l of a material, of which Verdet's constant is known, is placed with its length parallel to the axis of a coil consisting of n_1 turns per unit length and traversed by a current C, then

$$\delta = \gamma l \times 4\pi n_1 C.$$

Dr. Kerr, in 1877, observed that the plane of polarisation is rotated when polarised light is reflected from the polished pole of an electromagnet, and that the direction of rotation is reversed when the polarity is reversed.

SUMMARY

The Energy-paths in a Simple Circuit.—The energy in a dielectric surrounding a simple circuit, traversed by a current, is partly electrostatic and partly magnetic. So long as the current remains constant this energy is transferred at a constant rate to the conducting circuit, and the total energy in the dielectric is kept constant by a steady flow of energy from the source of current (*e.g.* a voltaic cell). The paths along which the energy flows are perpendicular to the plane containing the lines of electric force and the lines of magnetic force.

Displacement Currents.—The application of an electromotive force to a dielectric causes a momentary transfer of electricity in the direction of the force. This electric displacement is equivalent to an electric current, and it is termed a *displacement current*.

Electric Elasticity may be defined as the ratio of an electric stress to the electric strain produced. For any given medium, of which the *dielectric constant* is K, the elasticity is equal to $4\pi/K$.

Maxwell's Theory.—The velocity of propagation of a mechanical disturbance, through a medium of elasticity e and density D, is equal to $\sqrt{e/D}$. The *electric density* of a dielectric may be written $4\pi\mu$, where μ is the permeability; hence the velocity v of an electromagnetic disturbance through the dielectric may be written,

$$v=\sqrt{\frac{4\pi}{K}\Big/4\pi\mu}=\frac{1}{\sqrt{K\mu}}.$$

This velocity has been proved experimentally to be equal to that of light in the same medium. Maxwell concluded that the same medium is concerned in the propagation of both optical and electromagnetic phenomena, and that both are identical in character.

The fact that, with several dielectrics, *the specific inductive capacity is equal to the square of its refractive index* is evidence in support of the identity of optical and electromagnetic phenomena.

Verdet's Constant is defined as *the amount of rotation of the plane of polarisation, of a beam of light, produced by unit length of the given medium when placed in a magnetic field of unit intensity.*

CHAPTER XXIX

ELECTRIC OSCILLATIONS. WIRELESS TELEGRAPHY

Electric Oscillations.—The indefinite polarity acquired by a steel needle when magnetised by means of a Leyden jar discharge (p. 224), observed by Henry in 1842, was probably the first experimental evidence of the oscillatory nature of a spark discharge. Lord Kelvin, in 1853, predicted by means of mathematical analysis that a discharge might, under suitable conditions, be oscillatory; and, in 1865, Maxwell formulated his theory of light, in which the radiations known as *light* are attributed to electric oscillations in the ether.

The principle of an electric oscillation may be explained by Fig. 357, in which a conductor shaped like a dumb-bell has opposite charges imparted to opposite ends. If the connecting shaft is a good conductor the charges not only flow towards each other but they *overshoot the mark* and recharge the ends with charges of reversed sign. This surging to-and-fro may be repeated several times before the charges finally come to rest. On the other hand, if the shaft is of badly conducting material, the charges slowly neutralise each other, and no surging takes place. Fig. 358 represents the discharge of a charged Leyden jar when the tin-foil surfaces are connected by a bad conductor (*e.g.* wet string). But if the surfaces are connected by a short thick wire, the discharge, although it may

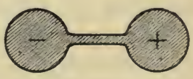

FIG. 357.

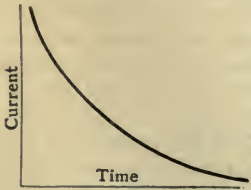

FIG. 358.

appear to be instantaneous, usually consists of a number of electric oscillations (Fig. 359). If the circuit consists of a good conductor and a short air gap, the first portion of the discharge sets up a conducting path for the surgings that follow, the amplitude rapidly becoming less. Immediately before the discharge takes place the energy is entirely electrostatic; but during the discharge it is partly electromagnetic and partly electrostatic, and is gradually frittered away in the form of heat (*except in certain cases, where a considerable portion of the energy may be thrown off as electric radiation*).

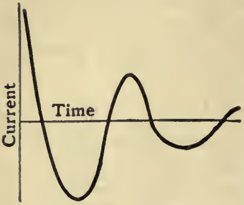

FIG. 359.

The Leyden jar discharge, and the conditions which determine its character, may be compared to the release of compressed air contained in one limb (B, Fig. 360) of a U-tube which is partly filled with mercury: if the tap is opened but slightly, the mercury column returns gradually to its position of rest; but if the tap is opened fully the mercury only comes to rest after a series of oscillations. In the same manner, a pendulum (or a flexible steel strip clamped at one end) in its deflected position has energy which is potential only and which becomes kinetic only on passing its position of rest; if swinging in air, inertia carries it beyond this position, the kinetic energy gradually becoming potential, and, during a sequence of oscillations, the energy is dissipated as heat in the frictional resistance of the air; but, if swinging in a viscous medium, the pendulum may return to its position of rest without describing any oscillations. In the case of the flexible spring the time of vibration depends upon two factors, (i) *the elasticity of the spring*, and (ii) *the inertia of the spring*. In the electric case, it can be proved that the *inductance* and the reciprocal of the *capacity* of the circuit are comparable to the inertia and the elasticity respectively of the mechanical analogue. If the ohmic resistance is negligible, the **period of oscillation** (p. 24) is given by the equation

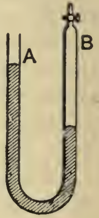

FIG. 360.

$$T = 2\pi\sqrt{LC},$$

where L and C are the inductance and capacity of the circuit; the period is therefore increased by making L and C as large as possible. The frequency (n) is given by the equation

$$n = \frac{1}{T} = \frac{1}{2\pi\sqrt{LC}}.$$

If L[*] is measured in electromagnetic (E.M.) units, and C in electrostatic (E.S.) units, then, since C E.S. units $= \frac{C}{9 \times 10^{20}}$ E.M. units,

$$n = \frac{3 \times 10^{10}}{2\pi\sqrt{LC}}.$$

On the other hand, if C is measured in microfarads, then, since 1 microfarad $= 10^{-15}$ E.M. unit,

$$n = \frac{1}{2\pi\sqrt{LC \times 10^{-15}}} = \frac{5.033 \times 10^{6}}{\sqrt{LC}} \quad \dots\dots\dots\dots\dots\dots (1)$$

where **L** is expressed in absolute units and C in microfarads. Fig. 361 (i) represents a Leyden jar and discharge circuit. The oscillations will be rendered more rapid if the capacity is reduced, as in Fig. 361 (ii), and still more so if altered to Fig. 361 (iii) which is practically of the same form as the *dumb-bell oscillator* (Fig. 357).

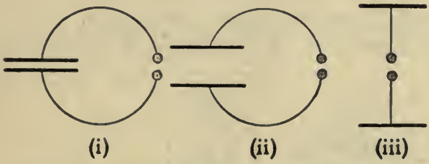
(i) (ii) (iii)

FIG. 361.—Discharging circuit of variable capacity.

We may regard either end of an oscillator as alternately the seat of static pressure and rarefaction; and if the length of the oscillator is adjusted so that the time required for a static impulse to traverse the length of the oscillator is equal to half the natural period of oscillation, there is a close resemblance

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[*] Neumann has given the following formula for the inductance of a straight wire of length l and radius r: $L = 2l\left(\log_e \frac{2l}{r} - 1\right) = 2l\left(2.3026 \log_{10} \frac{2l}{r} - 1\right)$. The same formula is approximately correct in the case of a wire bent into the form of a circle, providing that l is large compared with r; thus, suppose a wire, of radius 0.25 cm. and length 100 cms., bent into the form of a circle, then

$$L = 200(2.3026 \log_{10} 800 - 1) = 1136.8.$$

between the electric phenomena and the acoustic phenomena of a closed organ pipe which is emitting its fundamental note (Fig. 362). In the organ pipe, the closed end is a region of maximum variation of pressure and of minimum velocity of air particles, while at the open end these conditions are reversed. Similarly, the ends of the oscillator are regions of maximum variation of electric pressure and minimum variation of current; the perpendicular distance of the dotted line *aob* from the oscillator may represent the variation of pressure at individual points. The point *o*, corresponding to the open end of the organ pipe, is a region of maximum variation of current and of minimum variation of pressure.

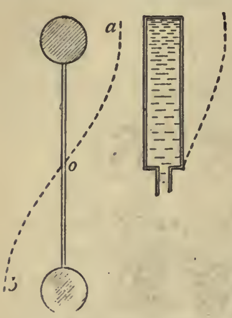

Fig. 362.—Analogy of an oscillator to a closed organ pipe.

The analogy may be carried still further: for, just as the length of the sound waves from the organ pipe is equal to four times the length of the pipe, so also are the electromagnetic waves generated (under certain conditions, p. 551) by the oscillator equal to twice the total length of the oscillator.

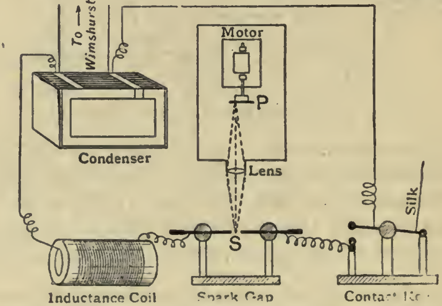

Fig. 363.—Method of photographing an oscillatory discharge.

Photography of an Oscillatory Discharge.—Fig. 363 represents an arrangement[1] suitable for obtaining direct evidence of the oscillatory character of a spark discharge. An air condenser is charged by means of a Wimshurst machine, and is discharged through an induction coil across a spark-gap S by means of a contact key; an image of the spark is thrown upon a photographic film which is fixed to the axle of an electric motor rotating at high speed. If the discharge were continuous an unbroken band of light would be obtained, but Fig. 364 proves that the spark consists of a rapid sequence of separate discharges.

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[1] For Figs 363 and 364 the author is indebted to Mr. C. J. Watson (Acocks Green, Birmingham).

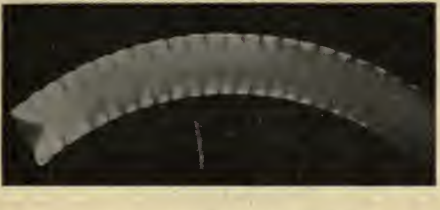

FIG. 364.—An Oscillatory Discharge.

The Period of Oscillation of a Leyden Jar Discharge.—If, at any instant, the charge in a condenser is q, and if the current in the discharge circuit is i, then

$$\textit{the electrostatic energy} = \frac{1}{2}\frac{q^2}{C}, \text{ and}$$

$$\textit{,, electromagnetic ,,} = \frac{1}{2}Li^2, \qquad \text{(see p. 433)}$$

where C is the capacity of the condenser, and L is the inductance of the circuit. Hence, *the total energy* E may be written

$$E = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2$$

$$= \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}L\left(\frac{dq}{dt}\right)^2.$$

The rate at which energy is being dissipated is

$$\frac{dE}{dt} = \frac{q}{C}\left(\frac{dq}{dt}\right) + L\left(\frac{dq}{dt}\right)\frac{d\left(\frac{dq}{dt}\right)}{dt}.$$

If *the resistance of the circuit is practically zero*, there is no loss of energy, and the rate of loss of energy is zero. In this case

$$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0,$$

$$\text{or } \frac{d^2q}{dt^2} = -\frac{1}{LC} \cdot q. \quad \text{..............................(2)}$$

The method of solving this equation, in order to find the manner in which q changes with time, is suggested by the fact that the second differential of q is equal to the original quantity multiplied by a constant. For, if $q = Q \sin \omega t$, where Q is the initial charge in the condenser, then

$$\frac{dq}{dt} = Q\omega \cos \omega t, \text{ and}$$

$$\frac{d^2q}{dt^2} = -Q\omega^2 \sin \omega t$$

$$= -q \cdot \omega^2. \quad \text{.......................................(3)}$$

Equations (2) and (3) are identical if $\omega = 1/\sqrt{LC}$. Since $q = Q \sin \omega t$, the value of q varies harmonically between the limits $+Q$ and $-Q$, and the period of one complete cycle of changes is $2\pi/\omega$. Hence the condenser discharges by a series of oscillations, and the periodic time T is given by the equation

$$T = 2\pi\sqrt{LC}.$$

But, *if the resistance of the circuit is not negligible*, we must take into account the energy dissipated as heat in the conductor: the rate of this dissipation is equal to Ri^2, where R is the resistance of the conductor. In this case, the rate of loss of energy in the electrostatic field must be equal to the sum of the rates at which electromagnetic energy is being taken up by the medium and at which energy is being dissipated as heat. Hence

$$-\frac{q}{C} \cdot \frac{dq}{dt} = L \cdot \frac{dq}{dt} \cdot \frac{d^2q}{dt^2} + R \cdot \left(\frac{dq}{dt}\right)^2, \text{ or}$$

$$\frac{d^2q}{dt^2} + \frac{R}{L} \cdot \frac{dq}{dt} + \frac{1}{LC} \cdot q = 0, \text{ which may be written}$$

$$\frac{d^2q}{dt^2} + A\frac{dq}{dt} + Bq = 0. \quad \text{.......................................(4)}$$

In order to solve this equation, we may assume that the relation between q and t is expressed by the equation $q = a \cdot e^{xt}$, where a and x are suitable constants; then $dq/dt = ax \cdot e^{xt}$, and $d^2q/dt^2 = ax^2 \cdot e^{xt}$. Hence, equation (4) becomes

$$ax^2 \cdot e^{xt} + A \cdot ax \cdot e^{xt} + B \cdot a \cdot e^{xt} = 0, \text{ or}$$

$$a \cdot e^{xt}(x^2 + Ax + B) = 0.$$

Hence $q=a \, . \, e^{xt}$ satisfies the equation when

$$x^2+Ax+B=0, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$\text{or when } x=-\frac{A}{2}\pm\sqrt{\frac{A^2}{4}-B}.$$

The roots of equation (5) are real when $A^2>4B$, *i.e.* when

$$R^2/L^2>4/LC, \text{ or when } R>\sqrt{4L/C}.$$

If these roots are denoted by x_1 and x_2, then

$$q=a \, . \, e^{x_1 t}+b \, . \, e^{x_2 t}.$$

This indicates that when $R>\sqrt{4L/C}$ **the charge disappears gradually and the discharge is unidirectional.**

If, on the other hand, $4B>A^2$, the roots of equation (4) are unreal, but they may be written

$$x=-\frac{A}{2}\pm\sqrt{B-\frac{A^2}{4}}\sqrt{-1}$$

$$\text{or } x=\alpha\pm\beta i$$

$$\text{where } \beta=\sqrt{B-\frac{A^2}{4}}=\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}, \text{ and } i=\sqrt{-1}.$$

Hence
$$q=a \, . \, e^{(\alpha+\beta i)t}+b \, . \, e^{(\alpha-\beta i)t}$$
$$=e^{\alpha t}(a \, . \, e^{\beta it}+b \, . \, e^{-\beta it}).$$

But, since the exponential values of $\sin\theta$ and $\cos\theta$ are $(e^{i\theta}-e^{-i\theta})/2i$ and $(e^{i\theta}-e^{-i\theta})/2$ respectively, the above equation may be written

$$q=e^{\alpha t}(X \, . \, \cos\beta t+Y\sin\beta t), \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

where X and Y are constants.

Equation (6) **indicates that** q **varies harmonically**, and

$$\text{the } \textit{periodic time } (T)=\frac{2\pi}{\beta}=2\pi\Big/\sqrt{\frac{1}{LC}-\frac{R^2}{4L}}, \text{ and}$$

$$\text{the } \textit{frequency } (n)=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L}}. \quad \ldots\ldots\ldots\ldots\ldots(7)$$

In nearly all practical cases R is so small that the ratio $R^2/4L$ may be neglected, and then $T=2\pi\sqrt{LC}$.

EXAMPLE. A Leyden jar, of diameter 10 cms. and in which the height of the tin-foil coatings is 12.5 cms. and the thickness of the glass (S.I.C. = 6) is 0.2 cm., is charged and then discharged through a copper wire of diameter 0.5 cm. and length 100 cms. bent into the form of a circle. Determine the frequency of the oscillatory discharge.

The area of the coated surface of the jar $=150\pi$ sq. cms.

The capacity of the jar $= \frac{150\pi \times 6}{4\pi \times 0.2} = \frac{4500}{4}$ electrostatic units,

$$\text{or } C = \frac{4500}{4 \times 9 \times 10^5} = \frac{1}{800} \text{ microfarad.}$$

The inductance (L) of the copper wire $= 1140$ cms. approximately. Hence, *if the resistance of the wire is disregarded*, then by equation 1 (p. 543), $n = 5.033 \times 10^6 \Big/ \sqrt{1140 \times \frac{1}{800}} = 4.21 \times 10^6$.

But, if the resistance is taken into consideration, then

$$R = 0.00083 \text{ ohm} = 8.3 \times 10^5 \text{ C.G.S. units, and}$$

by equation 7 (p. 547), $n = 4.17 \times 10^6$.

The Skin Effect.—We have assumed here that the resistance of the wire is the same for a rapidly alternating current as for a steady current; but it can be proved that the resistance to an alternating current is greater than that to a steady current, and that it varies as the square root of the frequency of alternation. This is due to the fact that a rapidly alternating current does not penetrate far into the interior of the conductor, but is restricted to a thin layer on the outside. In the case of a copper wire traversed by oscillatory currents of frequency approaching 10^6, this so-called *skin effect* only extends to a depth of about 0.01 mm., and the *effective* resistance of the conductor is equal to that of the thin layer which is concerned in the transfer of electricity. Nor, in the case of oscillatory discharges, is the inductance constant: Lord Rayleigh has shown that it decreases as the frequency of the discharge increases.

Impulsive Discharges.—If a P.D. is set up slowly between the ends of a conductor, the strength of the resultant current is determined chiefly by the magnitudes of the P.D. and of the conductor's resistance; but if the P.D. *is set up with great suddenness*, the inductance is more important than the resistance in determining the current strength. The inductance introduces a quality which may be termed an inertia effect, and a current cannot be set up instantaneously in a circuit possessing inductance—in just the same way that a velocity cannot be imparted instantaneously to a mass.

The subject of impulsive discharges has been fully investigated by Sir Oliver Lodge, and the following experiments are due to him. The inner coatings of two Leyden jars (Fig. 365) are connected to the terminals of a Wimshurst machine; the jars stand on a badly conducting material (*e.g.* wood), and their outer coatings are connected to discharging knobs B, which are short-circuited by a wire C. On turning the machine, the inner coatings acquire opposite charges, and the outer coatings also acquire induced charges of opposite sign, but the potentials of the outer coatings remain practically zero. When a spark passes at A, the charges on the outer coatings are immediately liberated, and, instead of the resultant flow taking place along C, nearly the whole of it passes as a spark at B: this is due to the fact that self-induction makes C practically a non-conductor when the imposed E.M.F. is so sudden. This is termed the experiment of *the alternative path.* It is interesting to note that the A-spark has strong physiological action, while the B-spark is scarcely felt. If the outer coatings are connected only by a spiral of thick wire, the momentary current through the spiral when the A-spark passes is great: the total quantity of electricity is small, but the period may be as short as 10^{-6} sec., and the current may amount therefore to several thousand amperes. The P.D. between two different points of the spiral may be seen by touching the points with the ends of a short bent wire.

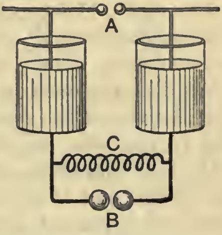

FIG. 365.—Experiment of the *Alternative Path.*

If a long copper wire is attached as an extension to either the A or B circuit (as in Fig. 366), then the passage of a spark at A sets up electric *pulses* in the extension; and if the wire is bent round so as to form a nearly closed loop, the pulses may give rise to sparks at the gap C (or C′). In explanation of this we may assume

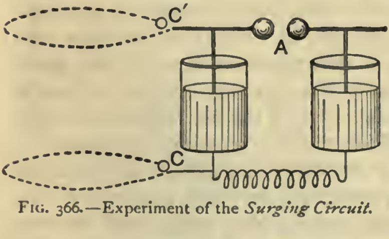

FIG. 366.—Experiment of the *Surging Circuit.*

that the passage of the spark involves an *impulsive rush* of electricity; as soon as the spark ceases, the electricity rebounds, followed by increased pressure at the free end of the extension. This experiment is described as the *surging circuit.* If the extension is straightened out to a long wire (or, for convenience, it may be coiled), and if its length is adjusted so that it corresponds to a quarter- (or half-) wave length for the oscillations of the discharge circuit, violent oscillations are set up in the wire, on the principle of sympathetic resonance.[1] The fixed end of the wire becomes a seat of maximum changes of current and of minimum changes of potential, while the free end is a region of minimum current and maximum changes of potential; and, when the A-spark occurs, the electric pulses are so violent that a brush discharge is formed at the free end. If the extension is made considerably longer, a glow discharge is visible at definite parts only, and these represent the antinodes of a stationary electric oscillation (p. 555). This is known as the phenomenon of the *recoil kick.*

Electric Waves generated by an Oscillatory Circuit.—*When the electric oscillations in a conducting circuit are extremely rapid, the phenomena are not restricted to the circuit, but the surrounding medium is traversed by a system of electromagnetic waves which travel outwards with the velocity of light, and the frequency of the waves is equal to that of the oscillations within the circuit.*[2] The conditions which determine the generation of these waves are analogous to those obtained in the vibration of a steel strip clamped at one end: if immersed in a viscous medium, its oscillations are slow, and the energy of the spring is dissipated as heat in the space traversed by the spring—the movement only creates vortices in the liquid near to the spring; but, if the oscillations are sufficiently rapid, the inertia of the medium modifies its behaviour, and the movement gives rise to waves of alternate compression and rarefaction, which are propagated outwards as sound waves.

We may regard the changes taking place in the medium near to an oscillating circuit in the following manner: suppose that the oscillator (Fig. 367) is just beginning to discharge, *and that the rate of discharge is not very sudden*; the electric tubes of force contract and, owing to their motion (p. 529), give rise to circular lines *m* of magnetic force. Since the discharge is oscillatory, the tubes of electric force will re-appear with their direction reversed; and the direction of the magnetic field will remain the same until the electric field has acquired its maximum development in this reversed direction. The reverse field will now collapse and give rise to a magnetic field similar to that shown in Fig. 367, but with its direction reversed. Moreover, the magnetic field will have maximum intensity when the electric field has zero intensity, and *vice versâ*. In this case the energy of the discharge is dissipated entirely within the circuit and none is radiated to a distance.

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[1] This is analogous to the air vibrations set up in a closed organ pipe when the period of vibration of a reed at the open end is equal to the period required for an air-wave to traverse the pipe four times.

[2] Prof. G. F. Fitzgerald first suggested, in 1883, that the discharge of a Leyden jar should give rise to electromagnetic waves in the surrounding medium.

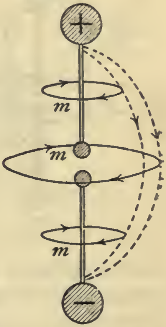
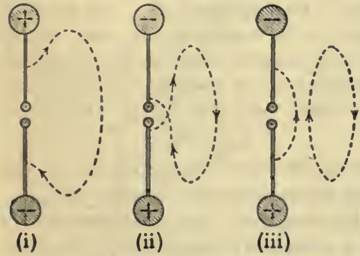


Fig. 367.

Electric tubes of force are comparable to vortex rings, which are known to possess properties of inertia and of elasticity: tubes of force, in themselves, represent potential energy, and they may also possess, in virtue of velocity, more or less kinetic energy. Hence, if the oscillations are sufficiently rapid, so as to bring into effect the inertia quality of the tubes of force, the simple phenomenon described in the previous paragraph may be modified. The tubes may not contract inwards as rapidly as the opposite charges on the oscillator approach, and the tubes will assume a pear shape; at a later instant the ends of the tubes have overlapped, and the main length of the tube is separated and thrown off as an independent loop of electric strain, resembling a vortex ring in character (Fig. 368).



(i) (ii) (iii)

Fig. 368.—Generation of Loops of Electric Strain.

Thus a series of such loops will be distributed uniformly round the oscillator: the next alternation will give rise to a similar set of independent loops, which push outwards the loops previously developed, and the direction of the electric strain in successive loops is in opposite directions. These loops travel outwards with the velocity of light and give rise to magnetic forces which are in opposite directions through consecutive loops. Fig. 369 represents the distribution of electric and magnetic force when two independent loops have been generated. The two sets of strain, electric and magnetic, constitute an electromagnetic wave; the energy of the wave is partly electric and partly magnetic, and is transmitted in a direction perpendicular to the plane containing the directions of the electric and magnetic forces.

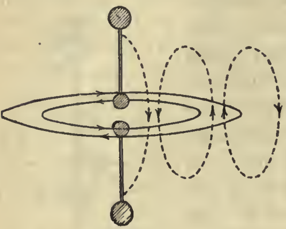


FIG. 369.

Thus, if the oscillator is fixed at the centre of the earth with its axis in the polar axis of the earth, we may imagine that the meridians of longitude represent the direction of the electric force, and that the parallels of latitude represent that of the magnetic force. The direction of propagation would be normal to the earth's surface; and, at any point on the surface, the electric and magnetic effects would be reversed twice during each complete oscillation. It may be added that the radiated energy would be a maximum round the equator and zero at the poles.

The application of the principle of *resonance* is one of the simplest methods of detecting electromagnetic waves.

Electric Resonance.—In acoustics, the phenomenon of resonance is observed when a stretched string is set in vibration by air-waves proceeding from another source of sound, providing that the period of vibration of the string is the same as that of the air-waves striking it. Similarly, an electric conductor may be adjusted, or *tuned*, to respond to vibrations set up by the discharge of a neighbouring Leyden jar. Sir O. Lodge demonstrated the phenomenon by means of two Leyden jars (A and B, Fig. 370); A is provided with a spark-gap C, and is connected to a Wimshurst machine, and the two coats of the jar B are connected by a wire circuit the self-induction of which can be adjusted by the sliding-contact *w*. A strip of tinfoil *t* is connected to the inner coating of B and terminates within a millimetre of the outer coating. When a spark passes at C, the waves originating from the circuit of A will set up oscillations in the circuit of B, providing that the latter is adjusted so as to be *in tune* with A, and the oscillations will cause a spark to pass between *t* and the outer coating of B.

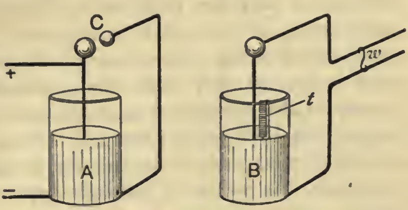


Fig. 370.—Demonstration of Electric Resonance.

Hertz's Experiments.—The investigations of Hertz, in 1888, constitute the most remarkable experimental confirmation of Maxwell's theory: he proved that electromagnetic waves travel with the same speed, and exhibit the same properties of reflection, refraction, and polarisation as those of light.

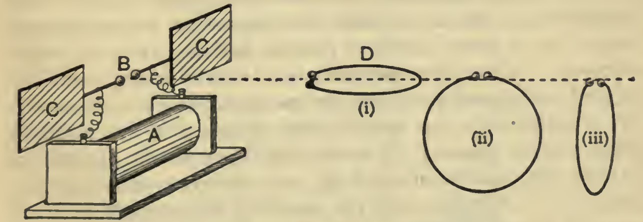


Fig. 371.—Hertz's Oscillator and Resonator.

Fig. 371 represents the method adopted by Hertz to obtain and to detect the electromagnetic waves: the secondary terminals of an induction coil A are connected to the radiator (or *oscillator*), which consists of a pair of metal rods terminating in highly polished brass knobs B and zinc plates C (40 cms. square). The zinc plates constitute the two conducting surfaces of a condenser of small capacity, and the capacity can be adjusted by varying the distance apart of the plates. The waves were detected by means of a *resonator* D consisting of a wire bent into the form of a circle (radius, 35 cms.): and terminating in two small knobs, the distance apart of which could be adjusted. The capacity of the oscillator was modified until it was *in tune* with the resonator. The length and intensity of the resonator spark depend upon its position: thus, in position (i) (Fig. 371) the axis of the spark gap, *i.e.* the line joining the centres of the spheres, is parallel to the axis of the oscillator, the waves of electric displacement set up an alternating potential-difference between the terminals of the resonator, and if this is sufficiently great a spark discharge occurs. In position (ii) no spark is obtained, since the axis of the spark gap is not in a direction to be affected by the waves. In position (iii) sparks are obtained if the gap is either at the top or at the bottom of the resonator.

The crucial experiment, to determine whether the effects are transmitted with finite velocity, was based upon the well-known principle in acoustics that if waves of sound are reflected from a plane surface, the reflected waves interfere with the direct waves and give rise to stationary waves, in which the vibrating anti-nodes are separated by equidistant fixed nodes. Non-conducting material is quite transparent to electromagnetic waves, but a metal sheet is opaque and serves as a perfect reflector. In Hertz's experiment, one wall of a long room was covered with sheet zinc, and the oscillator was placed near to the opposite wall; no sparks were observed in the resonator (supported in position iii) when held near to the conducting surface; but maximum sparking was observed when held at a distance of 1.8 metre, and the sparking disappeared at a distance of 4 metres: in this manner a series of nodes and anti-nodes was observed. Assuming the velocity of propagation to be the same as that of light, the wave-length of 9.3 metres (observed in one experiment) showed that the period of oscillation was 1.55×10^{-8} sec., which was in close agreement with the period (1.4×10^{-8} sec.) calculated from the dimensions of the oscillator.

In order to prove that electromagnetic waves obey the ordinary laws of reflection, Hertz constructed a mirror from a large sheet of zinc, bent so that its cross-section was parabolic. The oscillator was mounted with the axis of its spark gap coinciding with the focal line of this mirror. The resonator was fixed in the focal line of a similar mirror. The mirrors were placed side by side and so that their axes converged at a distance of 3 metres: sparks were only observed in the resonator when a sheet of zinc was held at the converging point and with its normal bisecting the angle between the axes of the mirrors. The phenomenon of refraction was observed by passing the waves through a large prism of pitch.

An important difference between Hertzian waves and light waves is obtained in the fact that the former are always plane polarised, the vibrations being parallel to the spark gap of the oscillator, whereas in ordinary light the vibrations are not restricted to one such plane. The effect of tourmaline, which absorbs those vibrations of ordinary light which take place in a certain plane, may be imitated in the case of Hertzian waves by means of a grid consisting of a number of thin wires stretched close together and parallel to each other; if the mirrors are placed face to face and the grid held so that its wires are parallel to the focal lines of the mirrors, the waves are reflected by the grid and no sparks are observed in the resonator; but sparks are observed when the grid is rotated in its own plane through 90°.

The wave-length of Hertzian waves varies from 1.3 cms. to 1300 cms., while the wave-length of the visible spectrum varies from 3×10^{-5} cms. to 7.5×10^{-5} cms.

Stationary Vibrations in Wires.—Lecher's modification of Hertz's method of setting up vibrations in wires is represented in Fig. 372. Two metal plates, A and B, are fixed opposite and parallel to the plates of the oscillator, and long wires attached to A and B are stretched outwards in parallel directions. The rapid alternations of potential in the plates of the oscillator set up equally rapid alternations of potential in the plates A and B, and this gives rise to electric oscillations

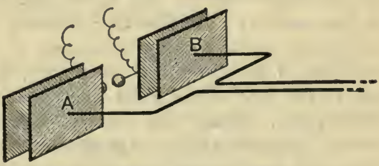


FIG. 372.

in the wires. The reflection of the oscillations from the ends of the wires gives rise to stationary oscillations—the nodes and anti-nodes of potential coinciding respectively with the anti-nodes and nodes of current. If the distant ends of the wires are not joined together, the ends must be anti-nodes of potential. Hertz determined the distribution of the potential oscillations by means of his resonator, held with its spark gap parallel to the wires, and with its plane perpendicular to the axis of the oscillator spark gap. Lecher bridged the space between the wires by means of an electrodeless vacuum tube, which can be slid along the wires: the potential anti-nodes are detected by the luminosity set up in the tube. It has been proved that the wave-length of the oscillations in the wires is the same as that observed in air when the wires are absent; hence the velocity of propagation along the wires is the same as that through air.

Blondlot's Experiment.—The following method has been devised by M. Blondlot to determine the velocity of propagation of an electromagnetic wave along a wire: the inner coatings of two small Leyden jars, A and B (Fig. 373) are connected to the secondary terminals of a Ruhmkorff coil. The outer coating of each jar consists of two separate rings; the upper rings, a_1 and b_1, are joined to a short spark gap s; the lower rings, a_2 and b_2, are connected by means of wires w_1 and w_2, over 1 kilometre long, to opposite sides of the spark gap; and the upper rings, also the lower rings, are joined by wet string (indicated by dotted lines). When the jars are charged the outer coatings will acquire opposite charges, the redistribution taking place along the strings. But when the jars discharge across the gap S, the induced charges on the rings are set free *suddenly*; the discharge between the rings does not traverse the strings, owing to the suddenness of the applied potential difference, but, in the case of the upper rings, it proceeds across the gap s, and, in the case of the lower rings, proceeds along the wires w_1, w_2 to the gap s. Hence two distinct discharges, separated by the time interval required for the charge on the lower rings to traverse the wires w_1, w_2 will cross the gap s, and this time interval can be determined by means of a rotating mirror. In this manner Blondlot proved that the velocity is 2.98×10^{10} cms. per second approximately.

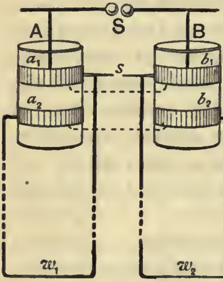


FIG. 373.

Wireless Telegraphy.—The first successful attempts to detect Hertzian waves at a distance from this source were made by Sir Oliver Lodge[1] in 1894; and Sir E. Rutherford, at Cambridge in 1896, detected the waves at a distance of half a mile. The remarkable developments in the efficiency of wireless telegraphy during recent years has been due largely to Signor Marconi, a pupil of Prof. Righi, of Bologna. It is possible here to give only a brief description of the technical details of a modern equipment.

The source of radiation consists of a long vertical conductor, termed the **aerial**, which is connected through an adjustable coil to the earth; in principle it resembles the *upper half* of the double oscillator shown in Fig. 369 (p. 552). The radiation consists therefore of half loops of electrostatic strain and of circular magnetic lines of force; these are indicated by the broken lines and the continuous lines respectively in Fig. 374.

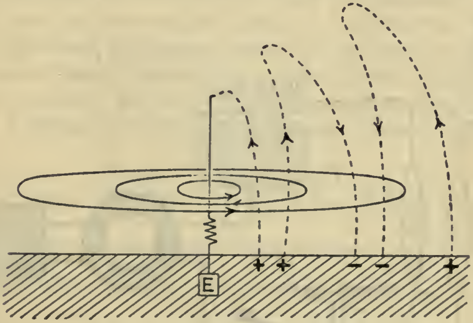


FIG. 374.—Radiation from an Aerial.

The electrostatic loops are vertical, and if a single aerial is used they are uniformly distributed in all directions round the aerial as a centre. The *direction* of the electric force round consecutive loops is evidently opposite. The free ends of the loops terminate on the earth's surface, and they travel outwards with a velocity of 3×10^{10} cms. per sec. The higher conductivity of water, as compared with that of dry land, explains why long distance wireless telegraphy is more successful when the stations are separated by the sea only. Each loop of electric force has associated with it a system of magnetic lines of force ; and in consecutive loops the direction of the magnetic force is opposite. When these magnetic lines of force cut a vertical wire at any distant point, an oscillatory potential difference is set up corresponding in time-period to that which has previously occurred in the transmitting aerial.

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[1] Brit. Assoc. Meeting, Oxford, 1894.

In its simplest form, the aerial consists of a single straight wire, of copper or aluminium, at least 150 ft. long, supported from the top of a mast by means of an insulator. As the aerial has a definite electrostatic capacity and also self-induction, it will have a natural *time-period of oscillation*, which is expressed by the equation $T = 2\pi\sqrt{LC}$ (see p. 546) ; similarly, the *frequency* (n) is expressed by the equation $n = 1/2\pi\sqrt{LC}$. In these equations L is expressed in *henrys*, and C in *farads*. When L and C are expressed in microhenrys and microfarads, the equation for the frequency becomes

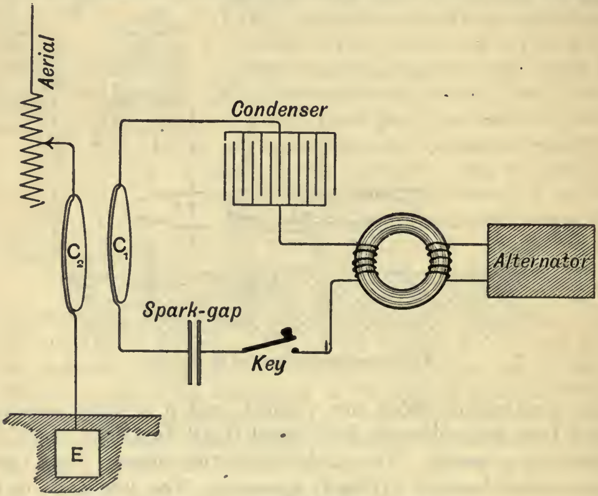

FIG. 375.—Wireless Telegraphy Transmitter.

$$n = 10^6/2\pi\sqrt{LC}.$$

In short-distance telegraphy the apparatus is designed so that the wave length shall not exceed (6×10^4) cms.; then, since $v = 3 \times 10^{10}$, the frequency n must be $(3 \times 10^{10})/(6 \times 10^4) = 500{,}000$ per sec. For long-distance work the wave length adopted is about ten times as great; and the frequency therefore is about 50,000 per sec.

Fig. 375 represents, in rough detail, the method of setting up an oscillatory condition in the transmitting aerial. An alternator supplies current to a transformer, which raises the potential difference supplied by the alternator. The secondary circuit of the transformer is connected in series with an air-condenser, a coil of wire C_1, a spark gap (which in its simplest form may consist of two parallel metal plates separated by a small air space) and a make-and-break key. The coil C_1 is fixed near, and parallel to, a corresponding coil C_2 in series with the aerial. Above C_2 is an adjustable coil, by which the period of oscillation can be adjusted as desired. Similarly the condenser circuit is adjusted so as to have the same period of oscillation. When the key is depressed, a stream of sparks passes across the spark gap, and each spark consists of an oscillatory discharge of definite time-period: this may be represented in diagram by Fig. 376. A message is sent by breaking up the train of waves into long and short periods, according to the Morse Code of telegraphy.

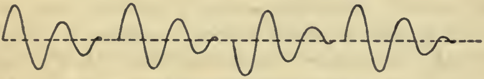

FIG. 376.—A Stream of Oscillatory Sparks.

When a train of electric waves falls upon a similar aerial at a distant point, the inductance and capacity being adjusted so that it has a natural time-period of oscillation corresponding to that of the transmitting aerial, electric oscillations are set up in it; and these are made evident by means of one of the several available detectors.

Detectors of Radiation.—The **coherer** was the earliest form of detector. Sir O. Lodge, in 1889, observed that two brass knobs which are so lightly in contact that an electric current does just *not* pass will become conducting when an oscillatory spark occurs in the neighbourhood. Branly, in 1890, observed the same effect with copper filings packed loosely in a glass tube; and he observed that a slight tap sufficed to make the filings *de-cohere*. An oscillatory discharge *through* the filings has the same effect as a discharge near to them. The modern coherer (Fig. 377) due to Marconi, consists of a mixture of nickel and silver filings between two silver plugs (S) contained in an exhausted glass tube. The filings are made to decohere by connecting the coherer through an auxiliary circuit including a voltaic cell and a 'tapper' worked by an electromagnet: this latter might be attached to a Morse inker, thus obtaining a permanent record of the oscillatory discharges passing through the coherer. The practical difficulties are considerable, and the rate of transmission slow; for these reasons it is now more usual to modify the arrangement, so that the signals may be read by means of a telephone.

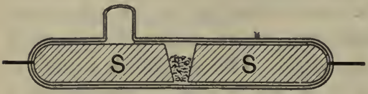

FIG. 377.—Marconi's Coherer.

As a telephone is not sensitive to very rapid oscillatory currents, it cannot be used if connected directly to the condenser circuit of a receiving station; but when the circuit includes any appliance which allows current to pass *in one direction only* a telephone is then sensitive. Such an appliance may be termed a *valve* or *rectifying contact*. A typical example consists of two crystals of copper pyrites and of zincite in contact. This contact possesses the peculiar property of allowing current to pass in one direction only. Fig. 378 represents the main details of a receiving station fitted with a 'valve' and telephone (T).

In order to reduce the induced E.M.F. and to increase the current, through the telephone, the valve is connected in series with the secondary terminals of a Rhumkorff coil, and the telephone is connected to the primary terminals.

Another, and very efficient, valve is the *Fleming oscillation valve*, which depends upon the principle that negative electrons are freely discharged from the incandescent filament of a tungsten-filament lamp: this transfer of electrons is equivalent to a current in one direction only, and it does not facilitate a current in the reverse direction. The filament is partly surrounded by a metal cylinder which is supported from a platinum wire fused through the glass bulb; this wire is connected through the secondary wire of an induction coil to the negative terminal of the lamp. The primary terminals of the induction coil are connected to the telephone receiver. Fig. 379 represents the method of connecting the lamp and telephone to the condenser circuit of the receiving station, the points A and B corresponding to the same points in Fig. 378.

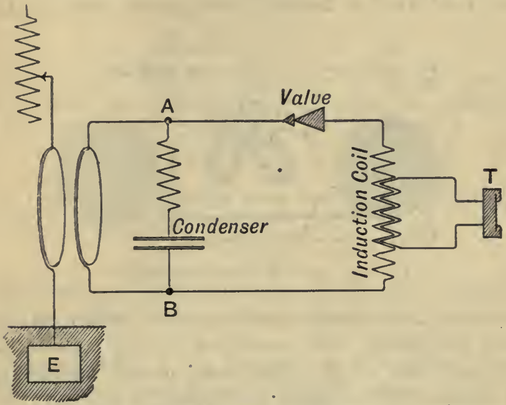

FIG. 378.—A Receiving Station, with Valve and Telephone.

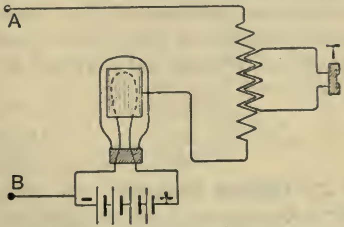

FIG. 379.—Fleming's Oscillation Valve.

The Marconi **magnetic detector** (Fig. 380) is now frequently used. In principle it depends upon the fact that an oscillatory discharge in a spiral surrounding a piece of soft iron in a magnetic field tends to destroy the *hysteresis* effect in the iron.

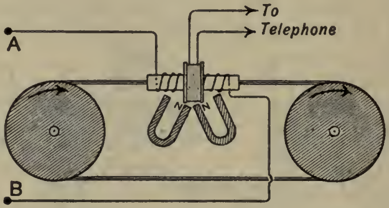

FIG. 380.—Marconi's Magnetic Detector.

It consists of two ebonite grooved discs, driven at constant speed by clockwork, and connected by an endless band of stranded soft iron wires. This band passes through a short narrow glass tube on which are wound a few turns of wire connected in series with the condenser circuit of Fig. 378. An outer coil of wire, surrounding the first, is connected to a telephone. Two permanent horse-shoe magnets are fixed near to the glass tube, and the soft iron wire is magnetised as it passes. With the poles placed as shown, the magnetisation of the iron does not respond at once to the change in direction of the magnetic field, owing to hysteresis. But when an oscillatory discharge passes through the inner coil, the removal of hysteresis causes magnetic disturbance in the soft iron, and the induced currents thus set up in the telephone circuit give rise to audible sound.

SUMMARY.

The Discharge of a Charged Condenser.—If a charged condenser is discharged through a *high* resistance, the discharging current is in one direction only, and gradually diminishes in magnitude to zero. But if the discharge takes place through a *low* resistance, the discharge is oscillatory, the amplitudes of the oscillations rapidly diminishing to zero.

If the resistance R of the discharging circuit is negligibly small as compared with the inductance L, the periodic time T of an oscillation is given by the equation

$$T = 2\pi\sqrt{LC},$$

$$\text{or,}\quad n = \frac{1}{2\pi\sqrt{LC}}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where C is the capacity of the condenser and n is the frequency of oscillation. But, *if the resistance is not negligible*, the periodic time is given by the equation

$$T = 2\pi\Big/\sqrt{\frac{1}{LC} - \frac{R^2}{4L}}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

If L is measured in absolute units and C in microfarads, then equation (1) becomes

$$n = \frac{5 \cdot 033 \times 10^6}{\sqrt{LC}}.$$

The Skin Effect.—A rapidly alternating current does not penetrate far into the interior of the conductor, but is restricted to a thin layer on the outside.

Impulsive Discharges.—The phenomenon of impulsive discharges may be classed under three headings: (i) the *alternative path* discharge, (ii) the *surging circuit*, and (iii) the *recoil kick.*

Electric Waves.—When the oscillations in a conducting circuit are extremely rapid, the surrounding medium is traversed by a system of electromagnetic waves which travel outwards with the velocity of light, and the frequency of the waves is equal to that of the oscillations within the circuit.

Hertz's Experiments.—Hertz, in 1888, proved that electromagnetic waves are propagated with finite velocity by detecting the presence of stationary waves formed by the interference between the incident waves and the waves reflected from a plane conducting surface. By measuring the wave-length, Hertz proved that the velocity of propagation is the same as that of light. He also proved that electromagnetic waves obey the ordinary laws of reflection and refraction.

Wireless Telegraphy is the technical application of the propagation and detection of electromagnetic waves.

Questions on Chapter XXIX

1. Give some description of the production and detection of electromagnetic waves.

2. Discuss the production of electric oscillations in a circuit containing capacity and self-induction, and find the effect of damping on the frequency.

APPENDIX

Solid Angles.—Imagine each point of the edge of a disc AB (Fig. 381) to be joined by straight lines to the point O. These lines will constitute a solid cone of which O is the apex. We say that the surface AB subtends a **solid angle** at O. In order to obtain a numerical measure of this solid angle, a sphere of 1 cm. radius is drawn round O as centre, and the **area** on the surface of this sphere which is enclosed within the cone is a numerical measure of the solid angle.

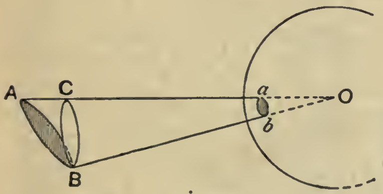

Fig. 381.

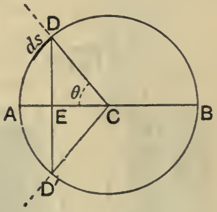

Fig. 382.

Thus, the solid angle subtended at O (Fig. 381) by the surface AB is measured by the area *ab* which is intersected on the sphere of unit radius. If a sphere is described round O as centre and with radius equal to OB, the surface BC will be enclosed within the cone, and the surfaces AB and BC have the same solid angle. Since the area of the surfaces BC and *ab* are proportional to the square of the radii of their generating spheres, then, if ω is the solid angle subtended at O by the surface AB,

$$\text{Area BC}/(OB)^2 = \text{Area } ab = \omega.$$

When the areas are small we may regard BC as the orthogonal projection of AB; and if θ is the angle between the normals to the two surfaces,

$$\text{Area BC} = \text{Area AB} \times \cos\theta.$$

Hence,

$$\omega = \text{Area AB} \times \cos\theta/(\mathrm{OB})^2.$$

If the point O *is situated on the normal drawn from the centre of any circular area, the solid angle subtended by the area may be proved to be equal to* $2\pi(1-\cos\theta)$, where θ is the angle subtended at O by the radius of the circular area. Thus, let ADB (Fig. 382) represent a sphere of unit radius, and DCD′ the solid angle subtended by a circular area to the left of C. If a small element ds of the surface at D revolves round E as centre and in a plane perpendicular to the paper, the area of the surface of revolution is equal to

$$2\pi\mathrm{DE}\,.\,ds = 2\pi\,.\,\frac{\mathrm{DE}}{\mathrm{DC}}\,.\,\frac{ds}{\mathrm{DC}} = 2\pi\sin\theta\,.\,d\theta.$$

Hence, the total area on the sphere's surface enclosed within the solid angle is

$$2\pi\int_0^\theta \sin\theta\,.\,d\theta = -2\pi\Big[\cos\theta\Big]_0^\theta = 2\pi(1-\cos\theta).$$

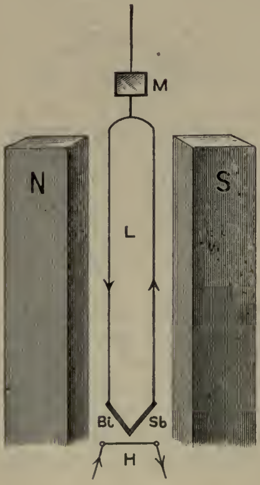

FIG. 383.—
Duddell's Thermo-galvanometer.

Duddell's Thermo-galvanometer.— In this instrument the heating of a wire, traversed by a current, is measured by an adaptation of the principle of Boys's radio-micrometer (p. 369). The lower ends of a loop L (Fig. 383) of fine silver wire terminate in a thermo-junction Bi – Sb. The loop is suspended vertically between the poles of a permanent magnet by means of a glass rod and quartz fibre. M is a mirror for observing the deflections. Immediately below the thermo-junction is stretched horizontally a very fine wire H which is traversed by the current to be measured, and which serves as a heater. Heat is transmitted by radiation and convection currents to the thermo-junction, thus giving rise to a current in the loop L. The deflection is found to be proportional to the square of the current; and, as the heating effect is independent of the direction of the current, both direct and alternating currents may be measured. According to the purpose for which the instrument is to be used, the heater may be of low resistance and consist of a thin metal wire; or, if a high resistance is required, a quartz fibre with platinized surface is substituted. The sensibility varies from 20–700 micro-amperes per 1 cm. deflection on a scale distant one metre from the instrument.

Fleming's Thermo-ammeter. — Prof. J. A. Fleming, by taking advantage of the position of bismuth and tellurium on the thermo-electric scale, has devised a thermal method of measuring an extremely small current, whether direct or alternating. The instrument (Fig. 384) resembles two glass test-tubes, the rims of which are fused together. Four wires pass through the bottom of the inner tube; the ends, A and B, of two of these are joined by a thin wire of constantan; the others are connected to a bismuth-tellurium thermocouple, the junction of which is soldered to the middle point C of the constantan wire. The air is removed from the space between the tubes. The deflection observed on a low-resistance galvanometer G, joined in series with the thermocouple, is proportional to the temperature of the point C, and therefore to the square of the current transmitted along AB. In the case of an alternating current the deflection is proportional to the square of the *virtual* current (see p. 444). Both this instrument and the Duddell thermo-galvanometer are appropriate for measuring the extremely small oscillatory currents associated with wireless telegraphy.

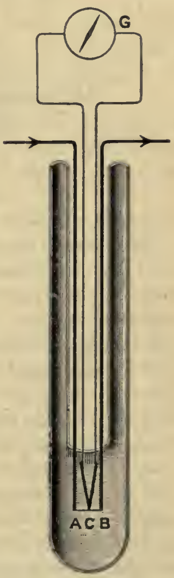

FIG. 384.—Fleming's Thermo-ammeter.

MISCELLANEOUS QUESTIONS FROM EXAMINATION PAPERS

Magnetism

1. Explain what is meant by the magnetic axis of a magnet.

How would you propose to determine the direction of the magnetic axis of a magnetised steel sphere?

2. Explain how to determine the intensity of a magnetic field by observing the time of oscillation of a needle. A magnetic needle under the influence of the earth alone makes 10 oscillations per minute; when placed at a point A it makes 25 oscillations per minute; find the intensity of the field at A in terms of the earth's field.

3. What is an isoclinal line? Give a general account of the arrangement of the isoclinal lines in a magnetic map of the earth's surface.

4. A thin uniform magnet, 1 metre long, is suspended from the north end so that it can turn freely about a horizontal axis which lies magnetic east and west. The magnet is found to be deflected from the perpendicular through an angle θ, such that $\sin\theta = .1$ and $\cos\theta = .995$. If the weight of the magnet is 10 grammes, the horizontal component of the earth's field $= .2$ C.G.S. units, and the vertical component .4 C.G.S. units, find the moment of the magnet.

5. Two exactly equal magnets are fastened together at their mid points, so that their axes are at right angles, and the combination pivoted so that the axes of the magnets are horizontal, and they can turn freely about a vertical axis. How will the system set itself under the influence of the horizontal component of the earth's field? If the moment of each magnet is M, and the moment of inertia of each about the axis round which it can turn is K, what will be the period of oscillation of the system?

6. A magnet of moment 1000 C.G.S. is balanced on a central knife edge. How must it be weighted if it is to remain horizontal in a place where the earth's horizontal field is .2 C.G.S. and the dip 45°?

7. Two magnets are placed in an aluminium frame with their axes horizontal and parallel, one being vertically over the other. The frame is suspended, and oscillates in the earth's horizontal field, making 20,

and 5, vibrations per minute, respectively, when the similar poles of the magnets are together or opposed. If the moment of the stronger magnet is 300, what is that of the other?

8. Define the magnetic elements at any place in the earth's surface. The magnetic elements at a given spot are: declination 30° W., dip 60°, horizontal force .2 C.G.S. Calculate the components of the earth's field in the directions true north, true west, and vertically downwards.

9. Two bar-magnets, respectively 10 and 12 centimetres long, are placed at right angles to the magnetic meridian; the first 10 centimetres north, and the other 12 centimetres south, of a compass needle. Compare the moments of the magnets if the needle is not deflected.

10. Give a short account of how it is possible to neutralise the disturbing action of the magnetised iron and steel of a ship on a compass needle.

11. Explain carefully how you would measure the effect of an increase of temperature on the magnetism of a bar-magnet.

Describe shortly how the moment of a magnet would change as it was gradually heated from 0° C. to about 400° C., and then cooled to 0° again.

12. Find approximately the force of attraction or repulsion between two short bar-magnets of moments 10 and 20 C.G.S. respectively, with their centres at a distance 20 centimetres apart, and their axes pointing in the same direction along the same straight line.

13. Two short magnets, with their axes horizontal and perpendicular to the magnetic meridian, are placed with their centres 30 cms. east and 40 cms. west respectively of a compass needle. Compare the moments of the magnets if the needle remains undeflected, and show how to derive the formula employed in the calculation.

14. Describe some method of comparing H and V, the horizontal and vertical components of the earth's magnetic field.

Show that the ratio H/V would be equal to $(\cot \theta)/2$, where θ is the latitude, if the magnetic field were due to a magnet at the centre of the earth with its axis pointing north and south.

Electrostatics

1. Two Leyden jars, each having a capacity of 1000 centimetres, are charged in series to a difference of potential of 10 electrostatic units. Calculate the energy of discharge, and state in what units it is expressed.

2. Discuss the changes of charge, potential, and energy that occur when a sheet of glass is inserted between the plates of a condenser, (1) when the plates of the condenser are connected to the poles of a

battery, and (2) when the condenser is charged and disconnected from the battery.

3. The capacity of a spherical condenser is .0033 microfarad, the diameters of the inner and outer surfaces of the dielectric are 20 and 20.5 cms. respectively. What is the specific inductive capacity of the dielectric?

4. An insulated sphere having a diameter of 20 cms. is charged. It is then connected to an electrometer by a fine wire, the deflection being 50 divisions. An insulated and uncharged sphere of 16 cms. diameter is then joined to the first by a long wire, and the deflection falls to 32. Calculate the capacity of the electrometer.

5. Two parallel conducting plates at a distance of two centimetres apart are connected with the terminals of a battery of 60 volts E.M.F. Express in electrostatic units the electric force in the air space between them, and find the mechanical force in dynes per square centimetre tending to draw the plates together. State how these forces are affected when the space between the plates is filled with a liquid of Specific Inductive Capacity = K.

6. Find an expression for the energy of an insulated sphere due to its charge. What is the energy of a sphere of 20 cms. radius charged with 2.6 units of positive electricity?

Voltaic Electricity

1. A tangent galvanometer having a coil of one turn of 34 cms. radius gives a deflection of 45° with a current of 10 amperes. Calculate the strength of the earth's magnetic field at the centre of the coil.

2. On sending a current through a tangent galvanometer, the needle of which is oscillating about a certain mean position, no deflection is produced, but the time of oscillation is reduced. What deductions can be made as to the situation of the galvanometer, and what additional facts would have to be supplied before the current could be calculated from the two times of oscillation?

3. What is the magnitude and direction of the force acting on a straight conductor, 10 cms. long, placed at right angles to a magnetic field of 50 lines per square centimetre, the current through the conductor being 5 amperes? In what units is your result expressed?

4. Two batteries of E.M.F. 10 volts and 8 volts and internal resistance 3.5 ohms and 2 ohms respectively are connected in parallel, and their poles are joined with a wire of 14 ohms resistance. Find the current passing through each battery.

5. Describe a method of measuring the resistance of a battery.

When two batteries, A and B, are joined in turn to a galvanometer, it is found that A gives the greater current, but when another galvanometer is used B gives the greater current. Explain how this may occur.

6. State Faraday's laws of electrolysis, explaining the meaning of the term *electro-chemical equivalent* of a substance.

Calculate the amount of copper deposited by the passage of 1 ampere for 1 second through a depositing tank, given the weight of hydrogen liberated in electrolysis is $\frac{1}{100000}$ gramme per coulomb. Atomic weight of copper 63, valency 2.

7. A U-shaped loop of fine wire hangs down between the poles of a horse-shoe magnet, and on sending a current through the loop its lower end is observed to move with respect to the upper. State more precisely the nature of the motion, and explain by what forces it is brought about.

8. A rectangular coil of length 20 cms. and breadth 10 cms. contains 100 turns, and is placed with its plane parallel to the lines of force of a magnetic field. The coil can turn about an axis passing through the middle of the shorter sides, and the couple acting on the coil is found to be 4000 dyne-cms. when the coil is traversed by a current of 10 amperes. What is the strength of the field?

9. Assuming Ohm's law, prove the relation used in determining a resistance by means of a Wheatstone's bridge. Compare the amounts of heat developed per sec. in the four arms of a balanced Wheatstone's bridge when the arms have resistances 100, 10, 300, and 30 ohms respectively.

10. Five cells, each having an electromotive force of 2 volts and an internal resistance of 0.04 ohm, are arranged in series, and drive a current between platinum electrodes through a vessel filled with dilute sulphuric acid. The resistance of the acid between the electrodes is 4 ohms, and the electromotive force of polarisation may be taken as 1.5 volts. Calculate the mass of water decomposed in one hour. The electro-chemical equivalent of hydrogen is about 10^{-5} grammes per ampere-second.

11. The current in a circuit containing a storage cell and a resistance is 5 amperes. The difference of potential at the terminals of the cell is 1.85 volts. When the circuit is broken the difference of potential rises to 1.95 volts. Find the resistance of the cell.

12. A copper disc having a diameter of 40 centimetres is rotated about a horizontal axis perpendicular to the disc and parallel to the magnetic meridian. Two brushes make contact with the disc, one at

the centre and the other at the edge. If the value of the horizontal component of the earth's field is .2 C.G.S. units, find the potential difference in volts between the two brushes when the disc makes 3000 revolutions per minute.

13. How would you connect two equal constant cells of internal resistance 5 ohms each, if you wished to deposit copper as rapidly as possible in a voltameter of 7 ohms resistance?

14. State Faraday's (or Neumann's) law of electro-magnetic induction. The North pole of a magnet is moved rapidly downwards towards a horizontal metal plate. In which direction will the induced currents flow?

15. You are provided with three resistance coils capable of carrying a large current, each having a resistance of 0.1 ohm, and with a voltmeter reading up to 5 volts. Explain how with these instruments you would measure a current of 120 amperes.

16. An electric current of 1 ampere flows round a circular metal ring, the radius of which is 10 centimetres. Determine the strength and direction of the magnetic field at a point on the line drawn through the centre of the ring perpendicular to its plane, and 10 centimetres distant from the plane of the ring.

17. Distinguish between the mean value and the root mean square value of an alternating current, and find the relation between them. Prove that the power absorbed by a coil traversed by an alternating current is EC cos ϕ, where E and C are the root mean square values of the E.M.F. and the current respectively, and ϕ is the difference in phase between these two quantities.

18. Describe experiments which show that cathode rays are small particles charged with negative electricity. How has the velocity of these rays been determined?

19. Give the leading features of the phenomenon of electrolysis, and give some account of methods which have been employed in determining the velocity of ionic migration.

20. Describe the nature of the magnetic forces in the neighbourhood of a long straight wire through which a current is passing.

Calculate the force of attraction between two parallel wires of a metre length and two centimetres apart when a current of one ampere is sent through each wire.

21. Describe the action of the moving coil galvanometer.

If the coil moves in a uniform radial field so that the deflection due to a steady current is proportional to the current, show that when the galvanometer is used ballistically the quantity of electricity Q which produces a swing of amplitude θ is given by the equation $Q = Ti/2\pi$, when T is the period of oscillation of the coil and i the continuous current which gives a steady deflection equal to the angle of swing θ.

22. Discuss critically Ampère's formula $F = cl \sin \theta / r^2$ for the magnetic force due to an element of length l of a conductor carrying a current c, when θ is the angle between the direction of the current and the joining line of length r. Calculate the magnetic force at a point on the axis of a circular coil of 10 turns and of 5 cms. mean radius. The point is 20 cms. from the plane of the coil, and the current through the coil is 0.5 ampere.

23. Prove as fully as possible that the magnetic force inside a solenoid is equal to $4\pi n C$, where n is the number of turns of wire per cm. and C the current.

24. Describe the moving coil galvanometer, and explain how it may be constructed so that the angular deflection is proportional to the current.

25. State the law of interaction of a magnetic pole and a current element, and deduce the magnitude of the force with which a circular coil of one turn is repelled from a magnetic pole placed on its axis.

26. What are the special characteristics of a ballistic galvanometer?

Show that the sine of half the angle of throw of the needle is equal to $qG\pi/HT$, where G is the 'galvanometer constant,' H the horizontal component of the earth's magnetic field, T the time of one complete oscillation of the needle, and q the total electric transfer through the instrument.

27. Two liquid resistances, A and B, of 5 and 10 ohms respectively, are connected in parallel, and a battery of electromotive force 8 volts and 2 ohms internal resistance is used to send a current through them. Find the currents in the two liquids, being given that the electromotive force of polarisation is 0.1 volt in A and 1.8 volts in B.

28. A circular coil of wire is mounted on a vertical axis and rotated with uniform speed in the earth's magnetic field. Trace the changes in current during one complete revolution.

If the radius of the coil be 5 cms., the number of turns of wire 30, the resistance of the circuit 2 ohms, the horizontal component of the earth's magnetic field 0.18, and the number of revolutions per second 4, calculate the average current in the coil irrespective of sign.

ANSWERS TO EXAMPLES

Chapter I. (p. 11.)

4. 1000 grams.

Chapter II. (p. 27.)

1. 83 min. 20 sec. **2.** 13,823. **3.** 49.05×10^7 ergs.
4. 2.5×10^8 ergs. **5.** 981×10^8 ergs.
6. 8×10^{10} ergs; 8×10^8 dynes. **7.** 4561; 7.456×10^9.
8. 392,500 foot-poundals; 20,070,000 foot-poundals.
9. 103×10^7 foot-pounds; 52 H.P.

Chapter III. (p. 37.)

3. 14.56 oscillations per minute. **5.** 2. **7.** 17. **8.** 13.8.
9. 1.712 C.G.S. units; 4.27 C.G.S. units.
11. 1584 H units; 2020 H units.

Chapter IV. (p. 72.)

3. 6.7 cms. from N. **4.** 7.5.

Chapter V. (p. 88.)

2. $\frac{2M^2(3r^2 - l^2)}{r^2(r^2 - l^2)^2}$. **4.** 1 : 1.225. **5.** 9; 18.
6. 160 ergs. **8.** 270°. **10.** 1 : 1.412.
14. 0.5 dyne; 0.57 dyne. **15.** 360°.
21. 304. **22.** 2.717. **23.** 1.331 : 1.
24. 14.42 cms. **26.** 0.01. **27.** 1296.
28. 0.134 MH ergs.

Chapter VI. (p. 116.)

7. 0.143 C.G.S. units. **11.** 1 : 1.95. **12.** 30°.
16. $m = \frac{2\mathrm{I}\tan\theta/2}{g}$ (where I = total intensity, and m = mass of weight attached).
22. 1 : 1.18. **23.** 1.27 : 1. **24.** 31.98. **27.** 0.158.

Chapter VII. (p. 132.)

2. 0.5 milligram. **3.** 121.3. **4.** 89.45.
5. $1 : \sqrt[3]{2}$, or (1 : 1.259).
7. $10\sqrt{2}$ cms. on opposite side ; $10\sqrt{2}$ cms. on same side.
9. 1 : 2 ; 4 : 7. **10.** 1 : 1.95.
12. 37 : 13, if the charges are similar ; 7 : 1, if the charges are opposite.

Chapter VIII. (p. 153.)

3. (a) 2.41 cms. to left of charge 1 ; (b) 0.33 cm. and 1 cm. to right and left of charge 1 respectively. **7.** + 15.
8. $\frac{1}{10}$. **11.** $2\sqrt{2}$; − 0.3 erg. **12.** 2.553.
15. 500. **21.** 100 ; 10. **22.** 4.905 E.S. units.

Chapter X. (p. 196.)

1. 20. **2.** 5. **3.** 1 : 5 ; 1 : 5. **5.** 0.01125 cm.
8. $\frac{3\mathrm{V}}{5}$. **9.** 26. **10.** From 1 to 2.
11. Inner coating, from V to $\mathrm{V}\left(\frac{\mathrm{B}-b}{\mathrm{B}}\right)$; outer coating, from $\frac{\mathrm{V}b}{\mathrm{B}}$ to *zero*.
12. Inner coating, from 9 to 3 ; outer coating, no change.
13. 3 + 7 = 10. **16.** 3. **17.** 121 : 400.
18. 2686 C.G.S. units. **19.** 1327 C.G.S. units. **21.** 84 : 325.
25. $\frac{3}{11}$. **26.** 2 : 3. **27.** 16.53 ; 19.83.
28. 4 : 9. **30.** $\frac{40}{3}$ ergs. **31.** Yes ; 0.07 erg.
32. 1 : 0.96. **33.** 5 : 4 ; equal. **34.** 2 : 1.
35. 4 : 1 : 1. **37.** 6.17 cms. **38.** 708.
40. 2.94 microfarads. **42.** (a) P.D. is reduced 25 % ; (b) 12.5 cms.

Chapter XI. (p. 211.)

1. 35. **2.** 5 electrostatic units.

Chapter XV. (p. 270.)

14. 0.6 C.G.S. unit.
15. $\frac{\sqrt{3}}{5}$ dynes towards a point mid-way between the other wires.
16. 0.0503 C.G.S. unit. **17.** 100.53 C.G.S. units. **18.** 5.03 C.G.S. units.

Chapter XVI. (p. 289.)

2. 0.034 ; 0.34. **4.** 2.38 amp. **5.** 30°.
6. 17 : 8. **8.** 248. **9.** 0.14 amp.
10. 0.0113 ; 0.174 C.G.S. unit.

Chapter XVII. (p. 316.)

2. 20 cells. **3.** 0.2 volt ; 5 ohms.
4. $\frac{1}{3}$ amp. **5.** 4.39 milliamps. **6.** 0.02 ohm.
7. 0.25 volt. **8.** 1 volt. **9.** 1.50 volt.
11. One cell is reversed. **12.** 0.33 amp. ; $\frac{4}{3}$ volt.
13. $+0.25$; -1.45 ; -0.95 ; -0.25. **14.** $2\sqrt{3} : 1$.
16. 4 to 3. **17.** $\frac{1}{7}$ ohm. **18.** 1 : 9.
19. 1.622×10^{-6} ohm. **20.** 94.07×10^{-6} ohm.
21. 227 cms. ; 4.551 ohms. **22.** 975.2 cms. ; 0.039 %.
23. $\frac{1}{3} \times 10^{3}$ microhms. **25.** 0.202 ohm ; 98 % ; 19.8 ohms.
26. 0.176 milliamps. **27.** 58. **28.** 14.054 ohms.
29. 6.802×10^{-8} amp. per scale division.
30. Three rows of six each.

Chapter XVIII. (p. 334.)

1. 30°. **3.** 3.9 amps. **10.** 6 ohms ; 22 volts.

Chapter XIX. (p. 355.)

1. 2 : 1. **2.** $l_1/l_2 = d_1^2/d_2^2$. **3.** 28°.68 C. **4.** 0.9165 amp.
5. (i) 1 : 0.1 : 1.1 ; 1 : 10^{-1}. (ii) 0.62 amp. and 0.31 amp. ; 2 : 1.

6. 2.03 calories. **10.** 1 : 0.1 : 0.180. **11.** 1.66 amp.; 2.66.
12. Four rows of three cells each. **13.** 1.66 amp.
14. 6 ohms. **15.** 114.3.
16. If $d\mathrm{C}$ is a portion of the current in one wire transferred to the other, the increase of heat generated is $\overline{d\mathrm{C}}|^{2}t(r_1+r_2)$.
17. 3.1. **18.** 1.05. **19.** 857×10^3.
20. 746. **21.** 1*s.* 3¾*d.* **22.** 252; 198.
23. 288 amps. **26.** 90.43%.
27. 1.6, 20%; 2.0, 50%; 1.84, 69.2%; 1.60, 80%; 1.20, 90%; 0.648, 97.2%.
28. 0.248 *d.*; 1.49 kilowatt. **29.** 1 : 3; 1 : 2. **30.** 45°.45 C.
33. Insert 120 ohms in series; 55 watts; 25 watts. **34.** 9°.52 C.

Chapter XX. (p. 381.)

8. 790 microvolt; 1260 microvolt. **10.** 204° C.

Chapter XXI. (p. 406.)

4. 240; 120. **5.** 5.4; 8.48. **6.** 10; 6333 C.G.S. units.
7. 4964 C.G.S. units. **8.** 14.32. **9.** 21.08.
10. 11,220. **11.** 129.7. **12.** 4.31 amps.
13. 0.002, 0.0027, 0.004; 324.5, 509, 871.
14. 5024; 0.0105; 480,000; 48000; 0.1103; 45600.

Chapter XXII. (p. 436.)

11. 1.28×10^{-3} volt. **19.** 6×10^{-5} coulomb.
20. 1.885×10^{-4} volt. **21.** 5.89×10^{-5} volt.
22. zero; 7.385×10^{-3} volt; 8.527×10^{-3} volt.
23. 1.414×10^{-5} volt. **24.** 80 ohms.
25. 0.00296 henry. **26.** 0.000252 henry.
27. 0.322; 0.000805. **28.** 0.5; 39.35; 91.79 : 100.
29. 6.08 henry; 0.0608 henry.
33. $l^2\theta\mathrm{H}/2r$, where l = length, and θ = angle of swing.

Chapter XXIII. (p. 453.)

1. 16.06 ohms; 51°28′. **2.** 16.6 amps.; 37°. **3.** 13.54 ohms.
4. 16.8 ohms; 44°.4. **5.** 100.6. **6.** 96.
7. 14.85 and 19.09 scale divisions.

Chapter XXIV. (p. 471.)

4. 0.001 gram per coulomb. **5.** 0.000828 gram per coulomb.
6. 0.3354 gram; 50 min. 40 secs. **7.** 122 min. 22 secs.
8. 0.8. **9.** 0.0095 mm. **10.** 3.88.
11. Increase in ratio $1 : \sqrt{2}$. **12.** 0.0104 gm.
13. 31.4 hours. **14.** 0.427 amp.
15. 0.3406 gram; 142.5 calories; 69.7 and 52.3 calories.
16. 0.505 amp. **17.** 0.1588 gram; 30964 calories. **18.** 1.53 ohms.

Chapter XXV. (p. 488.)

4. 62.52 grams; 24×10^3 calories. **7.** 4 : 1; 16 : 1.
8. 1.52 volt. **9.** $0.913p/V$.

Chapter XXVII. (p. 526.)

1. 1.383×10^4. **2.** 0.7463. **8.** 1.6×10^3; 387.5. **9.** 0.0319 sec.

Miscellaneous Questions. (p. 568.)

Magnetism.

2. 6.25 H. **4.** 2.025×10^5 C.G.S. units. **5.** $2\pi\sqrt{\frac{K\sqrt{2}}{MH}}$.
6. Moment of weight must be 200 dyne-cms.
7. 264.7 C.G.S. units. **8.** 0.1732, 0.1, 0.3464 C.G.S. units.
9. 1 : 1.73. **12.** 0.0075 dyne. **13.** 27 : 64.

Electrostatics.

1. 25000 ergs. **3.** 7.24. **4.** 4.2 E.S. units.
5. Electric force = 0.1 E.S. unit; 0.000398 dynes/cm^2.
6. 0.169 C.G.S. units.

Voltaic Electricity.

1. 0.185 C.G.S. units.
3. 250 dynes, at right angles to the wire and to the magnetic field.
4. $\frac{4}{7}$ ampere; 0. **6.** 0.000315 gm. **8.** 0.2 C.G.S. units.
9. 1 : 10 :: 3 : 30; or, 30 : 3 :: 10 : 1. **10.** 0.6545 gm.
11. 0.02 ohm. **12.** 1.26×10^{-4} volt. **13.** In series.
16. 0.0222 C.G.S. units. **20.** 1 dyne. **22.** 0.009 C.G.S. units.
27. 1.03 amp.; 0.345 amp. **28.** 3.39×10^{-5} amp.

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